

Mr. *Wingate's* Arithmetick :
CONTAINING
A PLAIN and FAMILIAR
M E T H O D
For Attaining the
Knowledge and Practice
O F
Common Arithmetick.

Composed by EDMUND WINGATE,
of Gray's-Inn, Esq;

And, upon his Request, Inlarged in his Life-Time ; also since his
Decease carefully Revis'd, and much Improv'd ; as will appear by
the *Preface* and *Table of Contents* : By JOHN KERSEY,
late Teacher of the *Mathematicks*.

With a NEW SUPPLEMENT, Of *Easy Contractions* in the
Necessary Parts of *Arithmetick* ; Useful *Tables of Interest*, and
Flemish Exchanges ; as also *Practical Mensuration*.

By GEORGE SHELLEY, late WRITING-MASTER
of CHRIST'S-HOSPITAL.

The SIXTEENTH EDITION :
Accurately Revised and Corrected.

L O N D O N :

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1608/5294



TO THE
RIGHT HONOURABLE
THOMAS,
Earl of *Arundel* and *Surrey*,
Earl Marshal of
ENGLAND, &c.

Right Honourable,



THE Good Affection You bear to all Kind of Learning, and in Particular to the Mathematicks, makes me adventure to Present Your Lordship with this Tractate of Arithmetick ; because that ART, compared with other Mathematical Sciences, is as the Primum Mobile, in Respect of the other inferiour Orbs : For as the Poets us'd in Times pass'd to say of Venus, Sine Cerere & Baccho, Friget Venus ; so may I also confidently averr of them, without Arithmetick, they are Poor, and without Motion. Presuming therefore that Your Lordship, loving the ART, cannot disaffect the Artist, nor his Intention to do Good in that Kind, I am bold to shelter this Treatise under Your Lordship's Protection, humbly entreating Your Gracious Acceptation, and earnestly desiring for ever to remain,

Your Honour's, in all Service,
Affectionately Devoted,
EDM. WINGATE.

T H E
P R E F A C E
O F
J O H N K E R S E Y.



ABOUT the Year 1629, our Learned Country Man, EDMUND WINGATE, Esquire, Publish'd a Treatise of *Arithmetick*, divided into Two *BOOKS*, the one Intituled *Natural Arithmetick*, the other *Artificial Arithmetick*; and in regard his principal Design in that *Treatise* was, to remove the Difficulties which ordinarily arise in the Practice of *Common Arithmetick*, by the Help of Artificial or

Borrowed Numbers, called *Logarithms*, (whose proper Work is to perform *Multiplication* by *Addition*; *Division* by *Subtraction*, &c.) he did then in his said First Book omit divers Pieces of *Common* or *Practical Arithmetick*, which for the perfect and Universal Understanding of it, were necessary to be inserted: But, after the first Impression of both those Books was spent, our Author being importuned to take Care of the *Second Edition*, he promised his Assistance therein: Yet his other necessary Employments not permitting him to pursue his said Purpose, he was pleased to impart his Thoughts concerning the same to me, together with his Request, That I would peruse the said First Book, and supply it with such Pieces of *Practical Arithmetick*, as, for the Reasons aforesaid, were wanting in the *First Edition*.

In Pursuance of this Request, I have contributed my Talent towards Perfecting the said *Treatise*, upon our Author's Foundation; partly in his Life-time, to his good Liking, and partly since his Decease, in several Editions committed to my Care to be prepared for the Press; wherein I have used my best Endeavours, as well to preserve this Book as a Monument of our said Author's Worth, as also to make it a Compleat Store House of *Common Arithmetick*; from whence the
Ingenious

The P R E F A C E.

Ingenious may be furnish'd with the Excellencies of that Art, in Reference as well to *Common Affairs* as to the *Practical Parts of the Mathematicks*. And in order to these Ends, I have maed these following Alterations and Additions ; namely,

First, For the Ease and Benefit of those Learners, that desire only so much Skill in *Arithmetick*, as is useful in Accompts, Trade, and such like ordinary Employments ; the Doctrine of whole Numbers, (which, in the *First Edition*, was intermingled with Definitions and Rules concerning Broken Numbers, commonly called Fractions,) is now entirely handled a-part. And to the End the full Knowledge of *Practical Arithmetick* in whole Numbers might more clearly appear, I have explained divers of the old Rules in the First Five Chapters, and framed a-new the Rules of *Division*, *Reduction*, and the *Golden-Rule*, in the Sixth, Seventh, Eighth, and Ninth Chapters : So that now *Arithmetick* in Whole Numbers is plainly and fully handled before any Ent'rance be made into the craggy Paths of Fractions, at the Sight of which some Learners are so discouraged, that they make a stand, and cry out, *Non plus ultra*, There's no Progress farther.

Secondly, To assist such young Students as would lay a good Foundation for the attaining of a general Knowledge in the *Mathematicks*, I have in a familiar Method delivered the entire Doctrine of Fractions, both *Vulgar* and *Decimal*, which was omitted in the *First Edition* ; and have also newly framed the Extraction of the Square and Cube-Roots, in a Method which by Experience is found to be much easier than that commonly used heretofore, and is exactly suitable to the Construction or Composition of Square and Cube Numbers.

Lastly, I have added an *APPENDIX*, furnished with Variety of choice and delightful Knowledge in Numbers, both *Practical* and *Theoretical*. In all which Performances, I have earnestly aimed at Truth, Perspicuity, and exact Correction, both of the Text and Numbers : So that I hope this Book is now supplied with all Things necessary to the full Knowledge and Practice of *Common Arithmetick*, the Usefulness whereof is so generally known, that there will be no need of Arguments to excite any one that is desirous of his own or the Publick Good, to be acquainted with so excellent an ART.

But if the more curious Artist, after he is well exercis'd in *Vulgar Arithmetick*, desires farther Inspection into the Mysteries of Numbers, his best Guide is the admirable Art call'd *Algebra* ; the *Elements* of which I have explained at large in a *Treatise* lately Publish'd.

JOHN KERSEY,



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Where those CHAPTERS of Mr. *Wingate's*, which have been altered and framed a-new by *John Kersey*, are distinguished by this Mark, : And those Chapters that have been entirely composed by the said *J. K.* may be discovered by this Afterisk *.

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A
TREATISE
 OF
Common Arithmetick.

The First Book.

C H A P. I.

Concerning Notation of Numbers.

I.  *R I T H M E T I C K* is the *A R T* of Accounting by Number. As Magnitude, or Greatness, is the Subject of *Geometry*, so Multitude, or Number, is that of *Arithmetick*.

II. Number is that by which every Thing is numbered ; or that which answers this Question, *Number.*
 How many? (unless it be answered by nothing :) So if it be asked, How many Days are in a Week? The Answer is Seven, which is called Number.

B

III, The

The Characters by which Number is expressed. **III.** The Notes or Characters, by which Number is ordinarily expressed, are these, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 nothing.

IV. These Notes or Characters are either significant Figures, or a Cypher.

V. The significant Figures are the first Nine; viz. 1, 2, 3, 4, 5, 6, 7, 8, 9. The first of these is more particularly called an Unit or Unity, and the rest are said to be composed of Units: So 2 is composed of two Units, 3 of three Unities, &c.

VI. The Cypher is the last, which tho' of it self it signifies nothing, yet, being annexed after any of the rest, it encreases their Value: As will appear in the following Rules.

VII. Arithmetick has two Parts, Notation and Numeration.

VIII. Notation teaches how to express, read, or declare the Signification or Value of any Number written; and also to write down any Number proposed, with proper Characters, in their due Places.

The Places or Degrees of any Numbers. **IX.** A Number is said to have so many Places, or Degrees, as there are Characters in the Number; viz. When divers Figures, whether they be intermix'd with a Cypher or Cyphers or not, are placed together, like Letters in a Word, without any Point, Comma, Line, or other Note of Distinction interposed; all those Characters make but one Number, which consists of so many Places as there are Characters so placed together; so this Number 205 consists of 3 Places, and this 30600 of 5 Places, &c.

X. Notation consists in the Knowledge of two things, viz. the Order of Places, and the Value of every Place in every Number.

The Order of Places in any Number. **XI.** The Order of the Places is from the Right-hand towards the Left: So in this Number 465, the Figure 5 stands in the first Place, 6 in the second, and 4 in the third; likewise in this Number 7560, a Cypher stands in the first Place, 6 in the second, 5 in the third, and 7 in the fourth.

The Value of Places in any Number. **XII.** The first Place of a Number, which, as before, is the outermost towards the Right-hand is called the Place of Units or Unities; in which Place any Figure signifies its own simple Value: So in this Number 465, the Figure 5 standing in the first Place signifies five Units, or five.

XIII. The



XIII. The second Place of a Number is called the Place of Tens, in which Place any Figure signifies so many Tens as the Figure contains Units: So in this Number 465, the Figure 5 in the first Place signifies simply 5, but the Figure 6 in the second Place denotes six Tens, or Sixty.

XIV. The third Place of a Number is called the Place of Hundreds; in which Place any Figure signifies so many Hundreds as there are Units contained in the Figure: So in this Number 465, the Figure 4 in the third Place expresses four Hundred: Wherefore if it be required to read or pronounce this Number 465, you are to begin on the Left hand; and according to the aforesaid Rules to pronounce it thus, Four hundred sixty five; likewise this Number 315 is to be pronounced thus, Three hundred and fifteen: And this Number 205, Two hundred and five; also this Number 500, Five hundred. Whence it is manifest, That although a Cypher of it self signifies nothing, yet being placed on the Right-hand of a Figure it encreases the Value of it, by advancing such Figure to an higher Place, than that wherein it would be seated, if the Cypher were absent.

The true reading or pronouncing the Value of any Number written, as also the writing down any Number proposed, depends principally upon a right Understanding of the three first Places before mentioned, and therefore I shall advise the Learner to be well exercised therein, before he proceeds to the following Rules.

XV. The fourth Place of a Number is called the Place of Thousands (that is, any Number of Thousands under Ten thousand;) the fifth Place Tens of thousands; the sixth Place Hundreds of thousands; the seventh Place Millions; (a Million being ten hundred thousand;) the eighth Place Tens of Millions; the ninth Place Hundreds of Millions; the tenth Place Thousands of Millions; the eleventh Place Tens of Thousands of Millions; the twelfth Place Hundreds of Thousands of Millions: And in that Order you may conceive Places to be continued infinitely from the Right-hand towards the Left, each following Place being ten times the Value of the next preceeding Place; but to give Names to them would be both a troublesome and unnecessary Task.

XVI. From the Rules aforesaid, an easie way A brief way may be collected to read or express the Value of a of Notation. Number propounded; viz. Let it be required to

read or pronounce this Number, 521426341. *First*, Distinguish by a Comma, or Point, every Three Places, beginning at the Right-hand, and proceeding towards the Left, so will the aforesaid Number be distinguish'd into

A Period. Parts, which may be called, *Periods*, and stands thus, 521, 426, 341. Where you may note the first *Period* towards the Right hand to consist of these Figures, 341, the second of these 426, and the third of these 521. *Secondly*, Read or pronounce the Figures in every *Period* as if they stood a-part from the rest; so will the first *Period* be pronounced Three hundred forty one, the second, Four hundred twenty six, and the third, Five hundred twenty one. *Thirdly*, to every *Period*, except the first towards the Right-hand, a peculiar *Denomination* or *Sirname* is to be apply'd, viz. The *Sirname* of the second *Period*, is *Thousands*; of the third, *Millions*; of the fourth, *Thousands of Millions*, &c. Therefore beginning to pronounce at the highest *Period*, which in this Example is the third, and giving every *Period* its due *Sirname*, the said Number will be pronounced thus, *Five hundred twenty one Millions, Four hundred twenty six Thousand, Three hundred and forty one.*

NOTE, When a Number is distinguished into *Periods*, as before, the highest *Period* will not always compleatly consist of three Places, but sometimes of one Place, and sometimes of two; nevertheless after such *Period* is pronounced as if it stood a-part, the due *Sirname* is to be annexed; so this Number 3204689, after it is divided into *Periods* will stand thus, 3, 204, 689. and is to be pronounced thus, *Three Millions, Two hundred and four Thousand, Six hundred eighty nine.*

XVII. The aforesaid Rules for the right pronouncing or reading of a Number which is written down, being well understood, will sufficiently inform the Reader how to write down any Number propounded to be written.

The TABLE of Notation.

The Order of Places.		The Value of Places.	
	℞c.		℞c.
Fourth Period	Twelfth Place	3	Hundreds of Thousands of Millions.
	Eleventh Place	2	Tens of Thousands of Millions.
	Tenth Place	1	Thousands of Millions.
Third Period	Ninth Place	9	Hundreds of Millions.
	Eighth Place	8	Tens of Millions.
	Seventh Place	7	Millions.
Second Period	Sixth Place	6	Hundreds of Thousands.
	Fifth Place	5	Tens of Thousands.
	Fourth Place	4	Thousands.
First Period	Third Place	3	Hundreds.
	Second Place	2	Tens.
	First Place	1	Units.

Notation

Notation of Numbers by Latin Letters.

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	IIII. or thus IV.	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIII. or thus LIX.
7	VII.	60	LX.
8	VIII. or thus IIX.	89	LXXXIX.
9	VIIII. or thus XIX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII. or thus IXX.	500	D. or thus IƆ.
19	XVIIII. or thus XIX.	600	DC. or thus IƆC.
20	XX.	700	DCC. or thus IƆCC.
		1000	CIƆ. or thus M.
		2000	CIƆ. CIƆ.
		3000	CIƆ. CIƆ. CIƆ.
		5000	IƆƆ.
		10,000	CCCIƆC.
		50,000	IƆƆƆ.
		100,000	CCCCIƆƆƆ. or thus CM.
		500,000	IƆƆƆƆ.
		1,000,000	CCCCCI, ƆƆƆƆ.
		1726.	{ CIƆ. IƆCC, XXVI. or, MDCCXXVI.

C H A P. II.

Concerning English Money, Weights, Measures, &c.

I. **T**HE Things expressed by Numbers are principally Money, Weight, Measure, Time, and Things accounted by the Dozen. Of the three first of these, there are infinite Kinds and Varieties according to the Diversity of the several *Common-wealths* wherein they are used, all which here to produce were both endless and needless: Wherefore we intend here to treat only of such Money, Weights, Measures, &c. as are used in this Nation, being indeed only necessary for our present Purpose.

II. The

II. The least Piece of Money used in England, is a Farthing, from whence this following Table is produced.

Of English Money.

1 Farthing	}	make	{	1 Farthing.
4 Farthings				1 Penny.
12 Pence				1 Shilling.
20 Shillings				1 Pound.

English (or Sterling) Money, is ordinarily written down with Figures after this manner :

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
34	13	05	2
09	05	10	1
69	00	06	3
00	12	11	0
00	00	07	2

The first Rank of the said Numbers signifies thirty four Pounds, thirteen Shillings, five Pence, two Farthings; the second Rank expresses Nine Pounds, five Shillings, ten Pence one Farthing; the third Rank, Sixty nine Pounds, no Shillings, six Pence, three Farthings, &c.

III. The smallest Weight used in England is a Grain, that is, the Weight of a Grain of Wheat, well dried, and gathered out of the Middle of the Ear, of which thirty two make another Weight, called a Penny-weight, and twenty Penny-weight make an Ounce Troy. *Vide Stat. de Compositione Ponderum, 51. Hen. III.*

Here observe, That by the *Grains* quoted in the Margins the Weight of two and thirty Grains of Wheat, make a Penny-weight, which Weight being once discovered by Thirty two such Grains, the said Penny-weight (being the Twentieth Part of an Ounce Troy) is usually subdivided into Twenty four Parts only, called also Grains, as appears by the ensuing Table.

A TABLE of Troy-Weights.

Troy Weight.

32 Grains of Wheat	}	make	{	24 Artificial Grains.
24 Grains				1 Penny-weight.
20 Penny-weights				1 Ounce.
12 Ounces				1 Pound Troy.

Troy

Troy-Weight is ordinarily set down with Figures after this manner :

lb.	oz.	p.w.	gr.
17	05	13	13
00	11	07	06
00	00	05	20

The first Rank of the said Numbers expresses Seventeen Pounds, five Ounces, thirteen Penny-weights, thirteen Grains of *Troy-weight*: The second Rank, no Pounds, eleven Ounces, seven Penny-weights, six Grains; and the third no Pounds, no Ounces, five Penny-weights, and twenty Grains.

Now this *Troy-weight* serves only to weigh Bread, Gold, Silver, and Eleſtuaries. And here obſerve alſo by the way, That *Troy-weight* regulates and preſcribes a Form how to keep the Money of England at a certain Standard: For about two Hundred Years before the Conqueſt, *Osbright* a Saxon, being then King of England, cauſ'd an Ounce *Troy* of Silver to be divided into 20 Pieces, at the ſame time called Pence; and ſo an Ounce of Silver at that Time was worth no more than Twenty Pence, or one Shilling eight Pence, which continu'd at the ſame Value until the time of *Henry* the Sixth, who (in regard of the enhancing of Money in Foreign Parts) valu'd the ſame at Thirty Pence; ſo that then there were accordingly Thirty Pieces made out of the Ounce, and the old Pieces went then for Three half-pence, till the time of *Edward* the Fourth, who valu'd the Ounce at Forty Pence, and then the old Pieces went at Two Pence a piece. After this *Henry* the Eighth valu'd the Ounce of Sterling-Silver at Forty five Pence, which Value continu'd till *Queen Elizabeth's* Time, who valu'd the ſame old Pence at Three Pence the Piece, ſo that all Three-pences coined by the ſame *Queen* weigh'd but a Penny-weight and every Six-pence two Penny-weights; and ſo in like manner the Shilling and other Pieces accordingly; which made the Ounce *Troy* of Silver, to be valued at Sixty pence, or Five Shillings, as it now remains at this Day without Alteration.

IV. The Weights uſed by Apothecaries are derived from a Pound *Troy*, which is ſubdivided, as in the following Table.

A Table of Apothecaries Weights.

℔	A Pound Troy	} is equal to {	12 Ounces.
℥	An Ounce		8 Drams.
ʒ	A Dram		3 Scruples.
ʒ	A Scruple		20 Grains.

So that if you were to express in Figures Twelve Pounds, ten Ounces, five Drams, two Scruples, and sixteen Grains; also three Pounds, five Ounces, seven Drams, one Scruple, and Two Grains, the ordinary way to write them down is briefly thus :

lb	3	3	2	gr.
12	10	5	2	16
3	05	7	1	02

V. Besides *Troy-weight* before-mentioned, there is another Kind of Weight used in *England*, called *Avoirdupois weight*, a Pound whereof is equal to 14 Ounces Twelve-penny weight *Troy*. This *Avoirdupois-weight* serves to weigh all Kind of Grocery-ware as also Butter, Cheese, Flesh, Tallow, Wax, and every other thing that bears the Name of *Garbel*, and from which issues a Refuse or Waste. *Malynes, ib. pag. 49.*

VI. *Avoirdupois-weight* is either Greater, or Less.

VII. The Greater is, when One hundred and twelve Pounds *Avoirdupois* are considered as one entire Weight, commonly called an *Hundred-weight*, and then such *Hundred-weight* is subdivided first into Four Quarters, and each Quarter into Twenty eight Pounds: Again each Pound into Four Quarters, or (if you will be more exact) into Sixteen Ounces, and if you please each Ounce into Four Quarters. But ordinarily a Pound is the least Quantity that is taken notice of in *Avoirdupois Gross Weights*. *Avoirdupois greater Weight.*

A Table of *Avoirdupois Greater Weight*.

28 Pounds	}	make	{	A Quarter of 112 lb.
4 Quarters				An Hundred-weight, or 112 lb.
20 Hundred weight				One Tun.

So that if you were to express by Figures Eight Hundred, three Quarters, and five Pounds; likewise, Seven Hundred, one Quarter, and seventeen Pounds; the ordinary way to set them down is briefly thus :

C.	q.	lb.
8	3	05
7	1	17

VIII. The lesser *Avoirdupois Weight* is, when a Pound is the highest Name or Integer, each Pound being subdivided into Sixteen Ounces, and each Ounce again into Sixteen Drams, and *Avoirdupois lesser Weight.*

if you please each Dram into Four Quarters, as by the subsequent Table is manifest.

A Table of Avoirdupois Lesser Weights.

4 Quarters of a Dram	} make {	1 Dram.
16 Drams		1 Ounce.
16 Ounces		1 Pound.

So that if you were to express by Figures Eighteen Pounds, twelve Ounces, five Drams, and three quarters of a Dram ; likewise Five Pounds, no Ounces, twelve Drams, and one quarter of a Dram ; the ordinary way to write them down is briefly thus :

℥	ʒ	ʒ	q.
18	—	12	—
5	—	00	—
		12	—
			1

IX. The Measures used in *England* are either of Capacity, or Length.

X. The Measures of Capacity are those which are produced from Weight, and they are either Liquid, or Dry.

Liquid Measures.

XI. The Liquid Measures are those, in which all Kind of Liquid Substances are measur'd, and they are expressed in the following Table.

A Table of Liquid Measures.

1 Pound of Wheat Troy Weight.	} make {	1 Pint.
2 Pints		1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
8 Gallons		1 Firkin of Ale, Soap, Herring.
9 Gallons		1 Firkin of Beer.
10 Gallons and an half		1 Firkin of Salmon or Eels.
2 Firkins		1 Kilderkin.
2 Kilderkins		1 Barrel.
42 Gallons		1 Tierce of Wine.
63 Gallons		1 Hogshead.
2 Hogsheads		1 Pipe or But.
2 Pipes or Buts		1 Tun of Wine.

XII. Dry Measures are those in which all kind of Dry Substances are meted, as Grain, Sea-coal, Salt, and the like ; their Table is this that follows :

A Table

A Table of Dry Measures.

1 Pint	}	make	1 Pint.
2 Pints			1 Quart.
2 Quarts			1 Pottle.
2 Pottles			1 Gallon.
2 Gallons			1 Peck.
4 Pecks			1 Bushel Land-measure.
5 Pecks			1 Bushel Water-measure.
8 Bushels			1 Quarter.
4 Quarters			1 Chaldron.
5 Chaldrons			1 Wey.
2 Weys			1 Last.

XIII. Long Measures are expressed in this Table following.

Long Measures.

3 Barley Corns in length	}	make	1 Inch.
12 Inches			1 Foot.
3 Foot or 16 Nails			1 Yard.
3 Foot 9 Inches			1 Ell.
6 Foot or two Yards			1 Fathom.
5 Yards and an half			1 Pole or Perch.
40 Poles or Perches			1 Furlong.
8 Furlongs			1 English Mile.

A L S O,

16	}	4	}	1	}	Yard.
20		5		1		Ell English.
12		3		1		Flemish Ell or Auln.

Note. That a Yard is usually subdivided into four Quarters, and each Quarter into Four Nails.

And each Ell into Four Quarters, but each Quarter of an Ell contains Five Nails.

XIV. Superficial, or Square Measures of Land, Land Measures are such as are express'd in the following Table:

40 Square Poles, or Perches	}	make	1 Rood or Quarter of an Acre.
4 Roods			1 Acre.
640 Acres			1 Mile.

So that if you would express by Figures these Quantities of Land, viz. Thirty six Acres, three Roods, twenty Perches; also Seven Acres, no Roods, thirty two Perches, the ordinary way to set them down is, thus:

$$\begin{array}{r}
 A. \quad R. \quad P. \\
 36 \text{ --- } 3 \text{ --- } 10 \\
 7 \text{ --- } 0 \text{ --- } 32
 \end{array}$$

Time. XV. A Table of Time is this that follows :

1 Minute	} make {	1 Minute.
60 Minutes		1 Hour.
24 Hours		1 Day natural.
7 Days		1 Week.
4 Weeks		1 Month of Twenty eight Days.
13 Months		1 Year very near.
1 Day, and		
6 Hours		

But in ordinary Computations of Time, the whole Year consisting of Three hundred sixty five Days, is divided either into Twelve equal Parts or Months ; every Month then containing Thirty Days and Ten Hours ; or else into twelve unequal *Kalendar-Months*, according to the ancient Verse :

*Thirty Days hath September, April, June, and November ;
February hath Twenty eight alone, and each of the rest,
Thirty One.*

Note, That every *Leap-Year* (which happens once in Four Years) contains Three hundred sixty six Days, and in such Year, *February*, contains Twenty nine Days.

XVI. Of Things accounted by the Dozen, a Gross is the Integer, consisting of Twelve Dozen, each Dozen containing again twelve Particular. So that if you would express in Figures, Seven Gross, four Dozen, and five Particulars ; also four Dozen, and eight Particulars, they may be briefly written thus :

$$\begin{array}{r}
 G. \quad D. \quad P. \\
 7 \text{ --- } 04 \text{ --- } 05 \\
 0 \text{ --- } 04 \text{ --- } 08
 \end{array}$$

C H A P. III.

Addition of Whole Numbers.

I **C**ONCERNING Notation of Numbers, and how thereby the Quantities of Things are usually expressed, a full Declaration has been made in the preceeding Chapters : Numeration

meration ensues, which comprehends all Manner of Operations by Numbers.

II. In Numeration, the Four primary or fundamental Operations (commonly called Species) are these, *Addition, Subtraction, Multiplication, and Division.*

III. Addition is that by which divers Numbers are added together, to the end that their Sum, Aggregate, or Total may be discovered.

IV. In Addition, place the Numbers given, one above another, in such sort, that like Places or Degrees in every Number, may stand in the same Rank, that is, Units above Units, Tens above Tens, Hundreds above Hundreds, &c. So these Numbers 1213 and 462, being given to be added together, you are to order them as appears in the Margin.

*Addition of
Numbers of one
Denomination*

1213
462

V. Having thus placed the Numbers, and drawn a Line under them, add them together, beginning with the Units first, and saying thus, 2 and 3 make 5, which write under the Rank of Units; then proceed to the second Rank, and say, 6 and 1 make 7, which write under the second Rank (being the Place of Tens); again 4 and 2 make 6, which write under the third Rank. Lastly, write down 1, being all that stands in the fourth Rank; so the Sum of the said given Numbers is found to be 1675, and the Operation will appear as in the Margin.

1213
462
——
1675

In like manner, the Numbers 2315, 7423, and 141, being given to be added together, their Sum will be found to be 9879. and the Operation will stand as you see in the Example.

2315
7423
141
——
9879

VI. When the Sum of the Figures of any of the Ranks amounts to Ten, or any Number of Tens without any Excess, write down a Cypher under that Rank; but when the Sum of any Rank exceeds Ten, or any Number of Tens, set down the Excess under such Rank, and for every Ten contained in the Sum of any Rank, reserve an Unit or 1 in your Mind, and add such Unit or Units to the Figures of the next Rank towards the Left-hand; so the Numbers 4937, 9878, and 394, being given to be added together, the Operation will be thus, viz. Beginning with the Rank of Units, I say, 4, 8 and 7 make 19,

4937
9878
394
——
5209

where-

wherefore I write down 9, the Excess above Ten, and carrying 1 in mind instead of the ten contained in the said 19, I say 1 and 9 (9 being the lowermost Figure of the second Rank) make 10, which added to 7 and 3, the other Figures of the same Rank, the whole Sum of them is 20; therefore setting down a Cypher under the Line in that Rank (because the Excess above the two Tens is nothing) I carry 2 to the third Rank, and say, 2 and 3 (3 being the lowermost Figure of the third Rank) make 5, which being added to 8 and 9 (the other Figures of the same Rank) the Sum of them is 22; therefore writing down 2 (being the Excess above the two Tens) under the Line in the third Rank, I carry 2 in mind (because there were two tens in 22) to the fourth Rank, and say, 2 and 9 make 11, which added to 4 make 15; this 15, because it is the Sum of the last Rank, I write totally down under the Line, on the Left-hand of the Figures before subscribed; so the Sum of the three Numbers given, is found to be 15209, as in the Example.

*Addition of
Numbers of di-
verse Denomi-
nations.*

VII. When Numbers given to be added, express Things of diverse Denominations; first write them down orderly (according to the Examples in *Chap. II.*, then after a Line is drawn under them all, begin to add the Numbers of the least Denomination, and if the Sum of them amounts to one Integer, or many Integers of the next greater Denomination, with some Excess of the less Denomination, set down that Excess, or a Cypher when there is no Excess, under the Line, so as it may stand under the least Denomination, and keep the said Integer or Integers in mind, to be added to those of the next greater Denomination on the Left-hand: But when the Sum of the Numbers of the least Denomination does not amount to one Integer of the next greater Denomination, set down the Sum it self under the Line; then add the Integer or Integers kept in mind (when any happens) to the Numbers of the next greater Denomination on the Left-hand, and proceed to add them, as also those of every greater Denomination, in like manner as above is directed; until you come to the Numbers of the greatest (or highest) Denomination, which are to be added according to the foregoing Rules *V.* and *VI.* of this Chapter. So these several Sums 24*l.* — 13*s.* 5*d.* — 3*f.* Also 12*l.* — 0*s.* — 8*d.* and 5*l.* — 18*s.* — 2*f.* being propos'd to be added, their total Sum is 42*l.* — 12*s.* — 2*d.* — 1*f.* For having written them down orderly according to the Second Rule of the *Second Chapter*,

Chapter, and drawn a Line underneath, I begin with the Farthings first, and say, two Farthings and three Farthings make five Farthings, that is one Penny with a Farthing over and above; wherefore setting down 1 under the Denomination of Farthings, I carry one Penny to the Denomination of Pence; then I say, 1, 8, and 5 Pence make 14 Pence, which contain one Shilling and two Pence; wherefore writing two under the Denomination of Pence, I likewise carry 1 Shilling to the Denomination of Shillings: Then adding the said 1 Shilling to 18 Shillings and 13 Shillings, the Sum will be found 1 Pound and 12 Shillings; wherefore setting down 12 under the Denomination of Shillings, I carry 1 Pound in mind to the Denomination of Pounds, saying, 1 Pound in mind, together with 5, 2, and 4 Pounds which stand in the first Rank of Pounds, make 12 Pounds; wherefore (according to the sixth Rule of this Chapter) I write 2, the Excess above 10, under the said first Rank of Pounds, and carry 1 in mind for the said 10 to the second Rank of Pounds: Then saying in like manner, 1 in mind, together with 1 and 2 which stand in the second Rank of Pounds make 4, which I write under the Line; that done, I find the Total of the three given Sumsto be 42 *l.*—12 *s.*—and 2 *d.*—1 *f.*

In like manner 3 *lb.*—05 *oz.*—19 *pw.*—15 *gr.* Also 2 *lb.*—00 *oz.*—3 *p.w.*—7 *gr.* Also 0 *lb.*—10 *oz.*—6 *p.w.*—0 *gr.* And 0 *lb.*—2 *oz.*—0 *p.w.*—17 *gr.* being given to be added together, their Sum will be found 7 *lb.*—01 *oz.*—9 *p.w.*—15 *gr.* and the Work will stand thus:

<i>lb.</i>	<i>oz.</i>	<i>p.w.</i>	<i>gr.</i>
03	05	19	14
02	00	03	07
00	10	06	00
00	09	00	17
07	01	09	15

Note. In adding together the Numbers in the last Example, it must be remember'd, that twenty-four Grains make one Penny-weight; twenty Penny-weight one Ounce; and twelve Ounces one Pound Troy (as before is declared in the third Rule of the *Second Chapter*;) and then you are to proceed according to *Rule VII.* of this Chapter.

More Examples of *Rule VII.* are these following, which presuppose the Learner to be well exercis'd in the Tables of

Chap.

Chap. II. that he may readily know what Integers are to be carried from every lesser Denomination to the next greater.

Addition of English Money.

lb.	s.	d.	f.	l.	s.	d.
250	17	10	1	0	13	05
175	12	11	3	0	17	08
052	05	06	0	0	00	10
009	00	08	1	0	10	03
506	13	07	2	0	15	06
974	10	00	3	2	17	08

Addition of Troy-weight.

lb.	oz.	p.w.	gr.	oz.	p.w.	gr.
23	07	16	13	536	13	16
17	10	15	07	206	11	10
325	06	19	20	068	00	05
49	11	07	12	099	00	12
417	00	19	04	907	15	19

Addition of Avoirdupois-weight.

C.	q.	lb.	lb.	oz.	dr.
235	3	13	14	13	12
576	1	17	5	10	13
628	2	15	12	00	06
412	0	10	06	09	05
1852	3	27	39	02	05

Addition of Measures of Length.

Yards.	q.	nails	Ells.	q.	n.
26	3	2	15	3	2
13	1	3	16	1	3
12	0	1	09	0	1
29	1	1	12	2	1
81	2	3	53	3	3

Addition

Addition of Superficial Measures of Land.

<i>Acres.</i>	<i>Roods:</i>	<i>Per.</i>	<i>A.</i>	<i>R</i>	<i>P.</i>
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	10
1379	3	09	996	3	36

C H A P. IV.

Subtraction of Whole Numbers.

I. *Subtraction* is that by which one Number is taken out of another, to the end, that the Remainder or Difference between the two Numbers given may be known.

II. The Number out of which the Subtraction is to be made, must be greater, or at least, equal with the other. As you may subtract 4347 out of 9478, so can you not subtract 9478 out of 4347. *Subtraction of Numbers of one Denomination.*

III. In Subtraction rank the two given Numbers one under the other as in Addition, with this Caution, that the Number placed uppermost may exceed, or at least be equal to the other: So if the Number 4347 be given to be subtracted from 9478, I order them as in the Margin: Then proceeding to the Subtraction, I say, 7 taken out of 8, there remains 1, which I place in the same Rank under the Line. In like manner 4 being taken out of 7, the Remainder is 3, which likewise I set under the Line in the next Rank; again, taking 3 from 4, the Remainder is 1, which I likewise place under the third Rank: Lastly, subtracting 4 from 9, there will remain 5, which I subscribe under the fourth Rank; so the whole Operation being finished, I find that if 4347 be taken out of 9478, the Remainder is 5131 or (which is the same) the Difference between the Numbers 9478 and 4347 is 5131, as in the Example.

In like manner if 106 be subtracted from 2856, the Remainder will be found 2750; for after the Numbers are orderly ranked, I begin at the place of Units, and say 6 from 6, there remains nothing; therefore I subscribe 0, (or nothing;) then proceeding to the second Rank, I say, if 0 be taken from 5, there will remain 5, which I also subscribe under the Line; again, 1 from 8, there remains 7: Lastly, 0 from 2, there remains 2. See the Work in the Margin.

IV. When any of the Figures of the Number given to be subtracted is greater than the upper Figure out of which it is to be subtracted, you must borrow 10 of the next Rank towards the Left hand, and add the said 10 to the said upper Figure; then from the Sum of such Addition subtract the lower Figure and set down the Remainder: In this case the Figure of the next Rank which is to be subtracted, must be esteemed an Unit greater than it is; therefore keeping one in your Mind, add it to the next Figure of the Number given to be subtracted, and deducting all out of the Figure above it, proceed in like sort 'till you have finished the whole Operation. *Example*, Let it be required to subtract 374 out of 8023. Having ranked them as before, I say, 4 out of 3, that cannot be, wherefore borrowing 10 of the next Rank, and adding the same to the said 3, I say, 4 out of 13, there remains 9; then writing 9 under the Line, and carrying 1 in my Mind, I say, 1 and 7 make 8, 8 out of 2 that cannot be, but 8 out of 12 (12 because 10 being borrowed, and added to 2, makes 12) there remains 4, which I subscribe under the line; again 1 in Mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6, which I likewise subscribe under the Line; lastly, 1 in mind being taken out of 8, there remains 7. Thus you see that the Remainder after 374 is subtracted from 8023, is 7649. Note diligently, that as often as 10 is borrowed, 1 must be kept in mind to be added to the Figure standing in the next Place of the lower Number, and the Sum of such Addition must be subtracted from the upper Place; but if it happen that there is no figure in the next Place of the lower number, then the 1 in mind must be subtracted from the upper Place; (as in the last Rank of the last *Example*.) *Another Example*, Let it be required to subtract 92 from 62801. Having placed 62801 the greater number uppermost, and the lesser orderly underneath, I begin at the Place of Units, and say, 2 from 1 I cannot take, but borrowing 10, and adding if

it to the said 1, I say, 2 from 11 there remains 9, which I subscribe under the line; then I proceed and say, 1 in mind with 9, makes 10, 10 out of 0 I cannot take, but borrowing 10, I say 10 out of 10, and there remains 0, wherefore I subscribe 0 under the line; again, 1 in mind out of 8, there remains 7; then because there are no Figures in the lower number, I say 0 out of 2 there remains 2; lastly, 0 out of 6 there remains 6; therefore I conclude that 62801 exceeds 92 by 62709.

V. If the Numbers proposed have diverse Denominations, place them as before, and beginning with the least Denomination first, subtract the lower Number from the upper when it may be subtracted, and place the Remainder underneath; but if it happens that the lower Number cannot be taken out of the upper, you must borrow an Integer of the next greater Denomination on the Left-hand; which Integer, after it is converted into the same Denomination with the said upper Number, must be added to it: Then from the Sum of such Addition, you are to subtract the lower Number, and write down the Remainder, keeping 1 (that is the Integer borrowed) in your mind, to be added to the next Place of the Number given to be subtracted, as before: So 90 *l.* — 14 *s.* — 10 *d.* — 3 *f.* being subtracted from 124 *l.* — 11 *s.* — 7 *d.* — 1 *f.* the Remainder is 33 *l.* — 16 *s.* — 8 *d.* — 2 *f.* For beginning with the Farthings, I say 3 Farthings out of 1 Farthing I cannot take, therefore borrowing 1 Penny (that is an Integer of the next greater Denomination) and having converted this Penny into 4 Farthings, I add them to the aforesaid 1 Farthing; so the Sum is 5 Farthings, out of which subtracting 3 Farthings, there remain 2 Farthings, which I place under the Denomination of Farthings; then I proceed to the next Denomination, and say, 1 Penny which I borrow'd and 10 *d.* make 11 *d.* this 11 *d.* out of 7 *d.* I cannot take; therefore borrowing 1 Shilling, or 12 *d.* and adding 12 *d.* to the said 7 *d.* the sum is 19 *d.* from which I subtract the said 11 *d.* so there remains 8 *d.* which I subscribe under the Denomination of Pence: Again, 1 Shilling which I borrowed, being added to 14 *s.* makes 15 *s.* which I cannot subtract out of 11 *s.* and therefore I borrow 1 Pound or 20 *s.* which being added to the said 11 *s.* make 31 from which subtracting 15 *s.* there remains 16 *s.* which I subscribe under the Denomination of Shillings;

Subtraction of Numbers of diverse Denominations.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
124	11	07	1
90	14	10	3
33	16	08	2

then carrying 1 Pound which I borrow'd to the lower Place of Pounds, I say, 1 in mind with 0 make 1, which taken out of 4, there remains 3; again 9, out of 2 I cannot take, but 9 out of 12 (10 being borrow'd and added to the said 2, according to the fourth Rule of this Chapter,) and there remains 3. Lastly, 1 (for the 10 that was borrow'd) being taken out of 1, there remains nothing; and so at last I find, that if *A* being indebted to *B*, in 124*l.*—11*s.*—7*d.*—1*f.* has paid in part thereof 90*l.*—14*s.*—10*d.*—3*f.* there remains yet undischarged 33*l.*—6*s.*—8*d.*—2*f.*

Subtraction of many Numbers from one Number. VI. When many Numbers are given to be subtracted from a Number propounded, you must first add those Numbers together, according to the Rules of the third Chapter, and then the Sum found is to be subtracted from the Number first given. *Example, A* being

indebted to *B*, in 3240 *l.* paid thereof at one time 700 *l.* at a second Payment 1236 *l.* and at a third 305 *l.* the Question is how much of the Debt remain'd undischarged? First, I add together the several Sums paid, and find the Total to be 2241 *l.* this I subtract from 3240 *l.* so there remains 999 *l.* undischarged, as you see by the Operation in the Margin.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
The Debt.	3240	00	00
	700		
	1236		
	305		
	2241		
	999		

	<i>l.</i>	<i>s.</i>	<i>d.</i>
	340	12	6
	13	18	03
	17	16	10
	372	07	07
	127	12	05

5 *d.* unpaid: See the Work in the Margin.

The Proof of Addition and Subtraction. VII. Addition is proved by Subtraction, and Subtraction by Addition. For having added divers Numbers together, if you subtract one of them out of the Sum, the Remainder must

be equal to all the rest, as you may observe by the Example following, viz. supposing these 4 Numbers are given to be added, viz. 236, 452, 29, 17, and that their sum is found to be 934

(by

(by the Rules of the third Chapter;) it is required to prove whether the said Sum be true or not; to perform this, I draw a Line under the uppermost Number 236, to separate it from the rest, and seek the sum of all the Numbers given, except that uppermost, which sum I find to be 698. Then I subtract the said uppermost number 236, from 934, (the total sum of all the Numbers first found) and because the Remainder 698 is the same with the sum of all the Numbers, excluding the uppermost, I conclude, that the sum of all the Numbers first found was truly computed.

In like manuer Subtraction is proved by Addition; for if you add the Remainder and the Number given to be subtracted together, the sum must be equal to the Number out of which the Subtraction is made; so if 4347 be subtracted from 9478, the Remainder is 5131; for if 5131 be added to 4347, the Sum is 9478, which is the same with the Number out of which the Subtraction was made.

	Example 1.	Example 2.
	<i>out of</i> 9478	<i>l. s. d.</i>
	<i>subtract.</i> 4347	24 — 13 — 07
		19 — 15 — 08
	<i>Rest</i>	5131
	<i>proof</i>	9478

Again, If a Servant receive 24*l.* — 13*s.* — 7*d.* and lay out or disburse 19*l.* — 15*s.* — 8*d.* there must remain in his hands 4*l.* — 17*s.* — 11*d.* for this being added to 19*l.* — 15*s.* — 8*d.* which was the Money he expended, the Sum will be equal to 24*l.* — 13*s.* — 7*d.* (being the Money with which he was first charged.)

More Examples of Subtraction are these that follow.

Subtraction of English Money.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
<i>Rec.</i>	3090	— 10	— 07	— 1		24	— 00	— 00	— 0
<i>paid.</i>	0099	— 14	— 08	— 3		05	— 17	— 11	— 3
<i>rest</i>	2990	— 15	— 10	— 2		18	— 02	— 00	— 1
<i>proof</i>	3090	— 19	— 07	— 1		24	— 00	— 00	— 0

Sub-

Subtraction of Troy-weight.

	lb.	oz.	p.w.	gr.	oz.	p.w.	gr.
Bought	352	— 10	— 13	— 19	205	— 13	— 19
Sold	019	— 11	— 16	— 18	118	— 16	— 20
Rest	332	— 10	— 16	— 21	86	— 16	— 23
Proof	351	— 10	— 13	— 12	205	— 13	— 19

Subtraction of Avoirdupois weight.

	C.	q.	lb.	lb.	oz.	dr.
Bought	256	— 2	— 23	25	— 13	— 12
Sold	079	— 3	— 26	00	— 14	— 13
Rest	176	— 2	— 25	24	— 14	— 15
Proof	256	— 2	— 23	25	— 13	— 12

Subtraction of Superficial Measures of Land.

	Acres, Roods, Perch.	A.	R.	P.
Bought	780 — 2 — 35	2040	— 1	— 20
Sold	090 — 3 — 36	919	— 3	— 30
Rest	689 — 2 — 39	1120	—	— 30
Proof	780 — 2 — 35	2040	— 1	— 20

Questions to Exercise Addition and Subtraction.

Quest. 1. Two Persons, *A* and *B*, owe several Debts, the lesser Debt being that of *A*, is 3045 *l.* the Difference of their Debts is 104 *l.* what is the Debt of *B*. *Answer*, 3149 *l.*

Quest. 2. Two Persons, *A* and *B*, are of several Ages, the Age of the elder, being that of *A* is 70, the Difference of their Ages is 19, what is the Age of *B*? *Answer*, 51.

Quest. 3. What Number is that which being added to 168, makes the Sum to be 205? *Answer*, 37.

Quest. 4. The Sum of two Numbers is 517, the lesser is 40, what is the greater? *Answer*, 477.

Quest.

Chap. V. *Multiplication, &c.*

23

Quest. 5. A certain Person born in the Year of our Lord 1616, desired to know his Age in the Year 1676, what was his Age? *Answer,* 60.

Quest. 6. The greater of two Numbers is 130, their Difference is 49, what is the lesser Number? *Answer* 81.

C H A P. V.

Multiplication of whole Numbers.

I. Multiplication teaches how by two Numbers given to find a third, which shall contain either of the Numbers given so many Times as the other contains 1 or Unity.

II. Of the two Numbers given in Multiplication, one (which you will) is called the Multiplicand, and the other the Multiplier, (or both are called Factors)

III. The Number sought, or arising by the Multiplication of the two Numbers given, is called the Product, the Fact, or the Rectangle: So if 5 be given to be multiplied by 3, or 3 by 5, the Product is 15, that is 3 times 5, or 5 times 3 makes 15; and here 5 may be called the Multiplicand, and 3 the Multiplier, or 3 may be called the Multiplicand, and 5 the Multiplier; and as 3 (one of the two Numbers given) contains 1 or Unity thrice, so 15 the Product contains 5 (the other given Number) thrice; likewise as 5 (one of the given Numbers) contains Unity 5 times, so 15 (the product) contains 3 (the other given Number) 5 times.

VI. Multiplication is either single or compound.

V. Single Multiplication, is, when the Multiplicand and Multiplier consist each of them of one only Figure as in the last Example. In like manner if you multiply 9 by 5, the product is 45, this is likewise single Multiplication: Now the several Varieties of single Multiplication are well expressed in the following Table, usually called *Pythagoras's Table*.

The

The Table of Multiplication.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The Use of the Table is this : Having one Figure given to be multiplied by another, to know the Product of them, find the Multiplicand in the Top of the Table, and the Multiplicator in the first Column thereof towards the Left-hand ; that done, in the Angle of Position just against those two Figures you will find the product, So 9 being given to be multiplied by 5, I find 9 in the Top of the Table, and 5 in the first Column towards the Left-hand ; then carrying my Eye from 5 in a right Line equi-distant to the upper Side or Top Line of the Table, until I come to that Square which is directly under 9, I find 45, which is the Product required. The particular Varieties of this Table ought to be learned by Heart, (that is, a Man must be able to give the Product of any single Multiplication, without the least pause or stay) before he can readily work compound Multiplication, as will further appear hereafter.

Compound Mul. VI. Compound Multiplication, is when the Multiplicator and Multiplicand, either one or both consist of more Figures than one.

VII. In Compound Multiplication, when the Numbers given end with significant Figures, place them as in *Addition* and *Subtraction*. So 134 being given to be multiplied by 2,

134 place them thus: Then proceeding to the Multiplication
 2 say thus; 2 times 4 is 8, which set under the Line in
 268 the Rank of your multiplying Figure ; again, say 2
 times 3 is 6, which likewise set under the Line in the
 next Rank: Lastly, 2 times 1 is 2, which being likewise set
 down under the Line in the next Rank, the Product is discovered to be 268, and the Work will stand as in the Margin

VIII. When

VIII. When the Multiplier consists of more Figures than one, as many Figures as it has, so many several Products must

be set down under the Line, which at last being added into one Sum, give you the total Product of all. So 1232 being given to be multiplied by 23, the Operation will stand thus; for 1232 being multiplied by 3. (according to the last Rule) the Product is 3696 Again, 1232 being multiplied by 2, the Product is 2464, which several Products after they are placed in their due Order, (that is, the first Figure arising in every Product under its respective multiplying Figure) and added together, produce 28336, the Product required: In like Manner 1321 being given to be multiplied by 123, the Product is 162483, and the Operation will appear as you see in the Margin.

$$\begin{array}{r} 1232 \\ \times 23 \\ \hline 3696 \\ 2464 \\ \hline 28336 \end{array}$$

$$\begin{array}{r} 1321 \\ \times 123 \\ \hline 3963 \\ 2642 \\ 1321 \\ \hline 162483 \end{array}$$

IX. When the Product of any of the particular Figures exceeds Ten, place the Excess under the Line as before, and for every Ten that it so exceeds, keep one in mind to be added to the next Rank.

Example, 3084 being given to be multiplied by 36, the Work will stand thus; for 6 times 4 being 24. I set 4 under the Line, and reserve 2 in mind for the two Tens; then I say 6 times 8 is 48, to which if I add 2 kept in mind, the whole is 50; therefore I set down 0 in the next Rank under the Line (0 because there is

$$\begin{array}{r} 3084 \\ \times 36 \\ \hline 18504 \\ 9252 \\ \hline 111024 \end{array}$$

no Excess of 50 above 5 Tens) and keep 5 in my mind for the 5 Tens; again, I say 6 times nothing is nothing, to which adding 5 that I kept in mind, the Whole will be but 5, which I likewise set down under the Line in the next Rank; again 6 times 3 is 18, which (in regard 3 is the last Figure of the Multiplier) I set wholly down; so that the particular Product arising from the multiplying Figure 6 is 18504: In like manner proceeding with the multiplying Figure 3, the particular Product arising will be 9252. Lastly, these several Products being placed in due Order, and added together (after the manner of the 8th Rule of this Chapter) will give 111024, which is the total Product arising from the Multiplication of 3084 by 36, and the Operation will stand as in the Margin. After the same manner if 5073 be given to be multiplied by 256, the Product

$$\begin{array}{r} 5073 \\ \times 256 \\ \hline 30438 \\ 25369 \\ 10146 \\ \hline 1298688 \end{array}$$

will be found to be 1298688, and the Operation will stand as you see in the Example.

X. When the two Numbers given to be multiplied, do one or both of them end with a Cypher or Cyphers towards the Right-hand, multiply the significant Figures in both Numbers, one by the other, neglecting such Cyphers, and when the Multiplication of the significant Figures is finished, annex on the Right-hand of the Number produced by the Multiplication the Cypher or Cyphers with which one or both of the Numbers first given did end, so will the whole give you the true Product demanded :

Example, 43100 being given to be multiplied by 15000, the Product will be found 646500000 ; for omitting the Cyphers which stand in the last Places towards the Right-hand, as well in the Multiplicand as the Multiplier, I multiply the significant Figures 431, by the Figures 15 (according to the former Rules,) so there will arise 6465, to which annexing on the Right-hand all the Cyphers before omitted, the true Product will be 646500000. More Examples hereof are these following.

$$\begin{array}{r}
 43100 \\
 15000 \\
 \hline
 2155 \\
 431 \\
 \hline
 646500000
 \end{array}$$

$$\begin{array}{r}
 43125 \\
 1500 \\
 \hline
 215625 \\
 43125 \\
 \hline
 64687500
 \end{array}$$

$$\begin{array}{r}
 5108000 \\
 125 \\
 \hline
 25540 \\
 10216 \\
 \hline
 5108 \\
 \hline
 638500000
 \end{array}$$

XI. When in the Multiplier, Cyphers are included between significant Figures, multiply by the said significant Figures, neglecting such Cyphers or Cypher, but observe diligently to set the particular Products of the significant Figures in their due Places according to the 8th Rule of this Chapter.

So if 56324 be given to be multiplied by 20006, I first multiply the whole Multiplicand 56324 by 6, and place the Product orderly under the Line; then passing over the three Cyphers, I multiply 56324 by 2 and place 8 (which is the first Excess of this particular Product) directly under the multiplying Figure 2, and the Rest in their order; so at last the true product will be found 1126817944, and the Work will stand as you see in the Example.

$$\begin{array}{r}
 56324 \\
 20006 \\
 \hline
 337944 \\
 112618 \\
 \hline
 1126817944
 \end{array}$$

More

More Examples hereof are these that follow :

$$\begin{array}{r} 3094 \\ 104 \\ \hline 12376 \\ 3094 \\ \hline 321776 \end{array}$$

$$\begin{array}{r} 23765 \\ 10302 \\ \hline 47539 \\ 71295 \\ \hline 23765 \\ \hline 244827030 \end{array}$$

Note. That one of the principal Cautions to be observed in Multiplication, is the due Placing of the particular Products arising by each multiplying Figure ; and that may be performed, either by taking care to place the first Figure or Cypher which arises in each Product under the respective multiplying Figure ; or at least the first Place arising in the second Product must stand under the second Place of the first Product, and the first Place of the third particular Product, under the third Place of the first, &c.

XII. When a Number is given to be multiplied by a Number that consists of 1 (or an Unit) in the first Place towards the Left-hand, and a Cypher or Cyphers on the Right-hand of such Unit (as are 10, 100, 1000, 10000, &c.) the Multiplication is performed by annexing the Cypher or Cyphers of the Multiplier at the End (to wit, on the Right-hand) of the Multiplicand ; so if 326 be given to be multiplied by 10, the Product is 3260 ; if by 100, the Product is 32600 ; if by 1000, the Product is 326000 ; in like manner if 170 be multiplied by 10, the Product is 1700 ; if by 100, 17000, &c.

XIII. When more Numbers than two are given to be multiplied one by the other, that Kind of Multiplication is called *Continual*, and is thus performed, *viz.* First multiply any two of the Numbers given one by the other, then multiply the Product by another of the Numbers given, and this Product by the Fourth Number given (if there be so many) and in that order till every one of the given Numbers has been made a Multiplier, so the last Product is the true Product required. *Example.* If 4, 18, and 22 were given to be multiplied continually, first 18 multiplied by 4 produces 72, which multiplied by 22 (the third Number) produces 1584, the last Product or Number required. See the Work in the Margin. The Proof of Multiplication is by Division, as will appear by the next Chapter.

Continual Multiplication.

$$\begin{array}{r} 18 \\ 4 \\ \hline 72 \text{ Prod. 1.} \\ 22 \\ \hline 144 \\ 144 \\ \hline 1584 \text{ Prod. 2.} \end{array}$$

C H A P. VI.

Division of whole Numbers.

I. DIVISION is that by which we discover how often one Number is contained in another ; or (which is the same) it shews how to divide a Number propos'd, into as many equal Parts as you please.

II. In Division there are always Three Remarkable Numbers, which are commonly called by these Names, the *Dividend*, the *Divisor*, and the *Quotient*.

III. The *Dividend* is the Number given to be divided into equal Parts.

IV. The *Divisor* is the Number by which the Dividend is to be divided ; that is, it is the Number which declares into how many equal parts the Dividend is to be divided.

V. The *Quotient* is the Number arising from the Division, and shews one of the equal Parts required ; so if 15 were given to be divided by 5, or into 5 equal Parts, the Number arising, or one of the equal Parts will be 3, for 5 is found three times in 15 : And here 15 is the Dividend, 5 the Divisor, and 3 the Quotient.

Division by a single Figure.

VI. Division being the hardest Lesson in Arithmetick, must be heedfully attended to by the Learner, for whose Ease I shall use my utmost Endeavours to make the way smooth by Rules and Examples, beginning with the easiest first, which will be in that case when the Divisor consists of one Figure only ; for *Example*, Let it be required to divide 192, by 8, or 192 Pounds into 8 equal Parts or Shares ; here 192 is the Dividend, 8 is the Divisor, and the Quotient or one of the equal Parts is sought,

VII. Place a crooked Line at each End of the Dividend, that on the Left-hand serving for the place of the Divisor, and that on the right for the Quotient ; then if the Divisor be a single Figure, subscribe a Point under the first Figure of the Dividend towards the Left-hand, if such first Figure be either equal to, or

What the Dividend is.

8) 192 (

.

greater than the Divisor ; but if such first Figure be less than the Divisor, put a Point under the next Place of the Dividend ; which Number so distinguished by the Point may be called the *Dividual* ; so in the *Example*, given in the 6th Rule, 192 being the Dividend, and 8 the Divisor, I set a Point under 9, not under 1, because it is less than the Divisor. This done, the Dividual, or Number whereof the Question must be ask'd, is 19.

VIII. Ha-

VIII. Having thus prepared the Numbers, ask how often the Divisor is contained in the Dividual, and write the Number which answers the Question in the Quotient; then multiply the Divisor by the Number placed in the Quotient, and subscribe the Product under the Dividual. Lastly, having drawn a Line under the Product, subtract it from the Dividual, and subscribe the Remainder orderly under the Line. So demanding how many times the Divisor 8 is found in the Dividual 19, the Answer is two times, therefore I write 2 in the Quotient; then multiplying the Divisor 8 by 2 (the Number placed in the Quotient) the Product is 16, which I subscribe orderly under the Dividual 19; and after a line is drawn under the Product 16, I subtract it from the Dividual 19, and place the Remainder 3 under the Line.

$$\begin{array}{r} 8) 192 \quad (2 \\ \underline{16} \\ 3 \end{array}$$

XI. Put another Point under the next Place of the Dividend towards the Right-hand, and bring down the Figure or Cypher standing in that Place to the Remainder; that is, set it next after it, so the Whole will be a new Dividual: Thus a Point being placed under 2 which stands in the next place of the Dividend, I write 2 next after (to wit on the Right-hand of) the Remainder 3 so 32 is a new Dividual, or Number whereof the Second Question must be asked, and the Work will stand as you see in the Example.

$$\begin{array}{r} 8) 192 \quad (2 \\ \underline{16} \\ 32 \end{array}$$

X. A new Dividual being set a part, renew the Question, and proceed according to the 8th Rule of this Chapter. Thus demanding how often the Divisor 8 is found in the Dividual 32, the Answer is four times; therefore I write 4 in the Quotient: Then multiplying the Divisor 8 by 4 (the Figure last placed in the Quotient) the Product is 32, which I subscribe under the Dividual 32, and after a line is drawn underneath, I subtract the Product 32 from the Dividual 32, and there being no Remainder, I subscribe 0 under the Line; so the whole Work being finish'd the Quotient is found to be 24. and the Operation stands as you see in the Example; therefore I conclude, if 192 Pounds be equally divided among 8 Persons, the Share of each Person will be 24 Pounds.

$$\begin{array}{r} 8) 192 \quad (24 \\ \underline{16} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

A Second Example. Let it be required to divide 936 Pounds into 9 equal Parts; having distinguished the first Dividual by a Point, (according to the 7th Rule of this Chapter)

I de-

9) 936 (1

$$\begin{array}{r} 9 \\ \hline 0 \end{array}$$

I demand how often the Divisor 9 is found in the Dividual 9, and finding it once contained in it, I write 1 in the Quotient; then multiplying the Divisor 9 by 1, the Product is 9, which I subscribe under the Dividual 9; after this, a Line being drawn under the Product 9, I subtract it from the Dividual 9, and there being no Remainder, I place a 0 under the Line, as you see in the *Example*.

9) 936 (10

$$\begin{array}{r} 9 \\ \hline 03 \end{array}$$

Again, placing a Point under 3 which stands in the next Place of the Dividend, I transcribe the said 3 next after the Remainder 0 for a new Dividual; then asking how often the Divisor 9 is contained in the Dividual 3, and not finding it once contained therein, I write 0 in the Quotient:

And now because the Product which ought to arise from the Multiplication of the Divisor by 0 (the Cypher last placed in the Quotient) amounts to 0, the Dividual 3, out of which that Product should have been subtracted, remains the same without Alteration; therefore after a Point is subscribed under

9) 936 (104

$$\begin{array}{r} 9 \\ \hline 036 \\ 36 \\ \hline 0 \end{array}$$

6, the next Place of the Dividend, I annex 6 to the Dividual 3, so there will be a new Dividual, to wit 36: Then demanding how often the Divisor 9 is found in the Dividual 36, the Answer will be 4 times; therefore I place 4 in the Quotient, and multiplying the Divisor 9 by 4, the Product is 36, which I subscribe underneath, and subtract from the Dividual 36; so the Remainder is 0; thus the whole Work being finished, the Quotient is found to be 104, as you see in the *Example*; wherefore I conclude, if 936 *l.* be equally divided among 9 Persons, the Share of each will be 104 *l.* In like manner, if 296163 be divided by 7, the Quotient will be 42309.

*The Substance of Division
by what Method soever.*

The whole Work of Division is briefly contained in this following Verse,

Dic quot, multiplica, subduc, transferque secundam.

Or thus,

*First you must ask how oft, in Quotient Answer make;
Then Multiply, Subtract, a New Dividual take,*

XI. When

XI. When in the Division the Divisor consists of a single Figure only, the Quotient may be express'd, and all the Operation performed in mind, without writing down any Part thereof; so 82506 being given to be halfed or divided into too equal Parts, the Work will be thus: The Divisor 2 is found in 8 four times; in 2 once; in 5 twice, and there will remain 1, which 1 being supposed to stand before (to wit, on the Left-hand of) the Cypher makes 10: then I say 2 is found in 10, 5 times; and last of all in 6, 3 times; so that the true Quotient or one Half of the given Number 82506 is found to be 41253.

A compendious Way of dividing by a single Figure.

In like manner, if 82506 be given to be divided by 3, or into 3 equal Parts, the Work will be thus: The Divisor 3 is found in 8 twice, and there will remain 2, which 2 being supposed to stand before (to wit, on the Left-hand of) the following 2, makes 22; then I say, 3 is found in 22, 7 times; in 15, 5 times; in 0 not at all, and lastly, in 6 twice; so that the true Quotient or one of the 3 equal Parts required is 27502. After the same manner may Division be work'd by any single Figure, without much Charge to the Memory.

Note. Here the Learner may ask, what shall be done with the last Remainder, if any happen, when the Division is finished? For a full Answer to this, I refer the Reader to the Note in the Fifth Rule of the Seventh Chapter; yet I shall here produce an Example where the said Case happens, viz. Let it be required to divide 351

A Note concerning the Remainder after the Division is ended if any happen.

by 8, or 351 Pounds equally among 8 Persons; now, if the Operation be prosecuted according to the former Rules, the Quotient will be found to be 43, and after the Division is finish'd, there will remain 7, that is, each Person must have 43 Pounds and there will be an Overplus of 7 Pounds, which must be also divided equally among the 8 Persons; but that cannot be done till the 7 Pounds be reduced into Shillings, and then those Shillings must be divided by 8, to give every Person his due Share of the Shillings contained in the said 7 Pounds: Again, if there yet remain any Surplusage of Shillings, they must be reduced to Pence, which also are to be divided by 8, to give every Person his due Share of Pence: So that when this Question is fully answered, each Person's Share will appear to be

8) 351 (43 7

be 43!—17!—6d. But how the before mentioned *Reduction* is performed, will be made manifest in the fifth Rule of the next Chapter.

Division by two or more Figures, the first and easiest Method. XII. When the Divisor consists of two, three, or how many Places soever, the Operation is more difficult than the former; but depends upon the same Grounds; and therefore the Learner being well vers'd in the preceeding Method, of dividing by a single Figure, will the more readily understand these that follow, which are two, of which the first is the easier, but the latter more expeditious, and that which indeed is principally to be aimed at: For an *Example* of the former, let it be required to divide 4112772 by 708, or which is the same to divide 4112772 into 708 equal Parts.

First, a Table is to be made to shew at first sight any *Multiple* or Product of the *Divisor*, it being taken twice, thrice, or any number of Times under Ten; so having first written down the *Divisor* itself 708, and drawn a Line on the Right-hand of it, I place 1 on the Right-hand of the Line directly against the *Divisor*; then under the

Multiples of the Divisor	The Divisor	708	1
		1416	2
		2124	3
		2832	4
		3540	5
		4248	6
		4956	7
		5664	8
		6372	9

Divisor 708, I subscribe the Double thereof, which is 1416, and place the Figure 2 directly against the said Double, to wit, on the other side of the Line. Again, adding 1416, (to wit, the Double of the *Divisor*) to the *Divisor* it self 708, the Sum is 2124 for the Triple of the *Divisor*; this Triple I subscribe under the Double, and place 3 on the other side of the Line right against the Triple. Again, adding 2124 (for the Triple of the *Divisor*) to the *Divisor* 708, I find 2832 for the Quadruple of the *Divisor*, which Quadruple I subscribe under the Triple; and proceeding in like manner, at last the Table is finished, which readily shews the *Divisor*, with the Duple, Triple, Quadruple, Quintuple, Sextuple, Septuple, Octuple, and Nonuple of the *Divisor*.

Now for a Proof of the said Table, adding the last Number thereof, to wit, 6372 (which was found to be nine times the *Divisor*) to the *Divisor* 708, I find the Sum to be 7080, which (by the 12th Rule of the fifth Chap.) is evidently ten times the *Divisor*; therefore I conclude that the Table is true, in regard that the last Number of it is derived from all the superior Numbers.

The

The Table of Multiples or Products of the *Divisor* being thus prepared, set down the *Dividend* on the Right-hand of the *Divisor*; then distinguish by a Point so many of the foremost Places of the *Dividend* towards the Left-hand, as are either equal in Value (being consider'd a-part) to the *Divisor*, or, which being greater, yet come nearest to the Value thereof; thus I subscribe a Point under 2, thereby setting a-part 4112, being the fewest of the foremost Places which contain the *Divisor* 708, so is 4112 the *Dividual* or Number whereof the first Question must be ask'd; then demanding how often the *Divisor* 708 is contained in the *Dividual* 4112, the Answer will be found by the Table to be five Times; for looking in the Table I cannot find the *Dividual* exactly; but I see that 6

708	1)	14112772	(5809
		...	
416	2	3540	
212	4	5727	
283	4	5664	
354	5	06372	
424	6	6372	
495	7		0
566	8		
637	9		

times the *Divisor* is the next greater than the *Dividual* 4112, and five times is the next lesser, wherefore I write 5 in the *Quotient*, and the Number in the Table which stands against 5, to wit, 3540, I subscribe under the *Dividual* 4112. Then having drawn a Line underneath, I subtract 3540 (which is five times the *Divisor*) from the *Dividual* 4112, and subscribe the Remainder 572 under the Line; that done, I put a Point under the next Place of the *Dividend* towards the Right-hand, and because the Figure 7 stands in that Place, I transcribe 7 next after the Remainder 572, so there is 5727 for a new *Dividual*.

Then demanding how often the *Divisor* 708 is contained in the *Dividual* 5727, the Answer will be found by the Table to be 8 times; for looking in the Table, I find, that 9 times the *Divisor* is the next greater, but 8 times is the next lesser than the *Dividual*; therefore I write 8 in the *Quotient*, and the Number in the Table, which stands against 8, to wit, 5664 I subscribe under, and subtract it from the *Dividual* 5727, placing the Remainder 63 under the Line.

Again, I put a Point under the next Place of the *Dividend*, where I find the Figure 7, and therefore transcribing 7 next after the Remainder 63, the new *Dividual* will be 637; then demanding how often the *Divisor* 708 is contained in the *Dividual* 637, and not finding it once contained therein, I write 0 in the *Quotient*, and since in this Case (that is when a Cypher answers the Question,) the *Dividual* remains the same without Alteration, the Figure or Cypher standing in the next place of

The Dividend, is to be transcribed after the Dividual for a new Dividual, so writing 2 next after 937, the new Dividual is 6372: Then demanding how often the Divisor 708 is contained in 6372, I find by the *Table* it is contained in it 9 Times; wherefore writing 9 in the *Quotient*, and placing the Number which stands against 9 in the *Table*, to wit, 6372 under the Dividual 6372, and subtracting it from the Dividual, there

Divisor	188	1)	20304	(108
	376	2		
	564	3	188	

	752	4	1504	
	640	5	1504	

	1128	6	0	
	1316	7		
	1504	8		
	1692	9		

Multiples of the Divisor.

will remain 0. Whence I conclude, if 4112772 be divided by 708, or into 708 equal Parts, the true *Quotient* or one of the equal Parts required is 5809. In like manner, if 20304 be divided by 188, that is, into 188 equal Parts, the *Quotient* arising, or one of those equal Parts will be 108, and the Operation will stand as you see.

The preceeding Method of Division by the Help of a *Table* of the *Multiples* or *Products* of the Divisor, as it is most easy, so in some Cases, namely, where the Divisor is great, and a *Quotient* of many Places is required, (as in calculating *Tables* of *Interest*, *Astronomical Tables*, and such like) it excels all other Ways of Division; both in respect of Certainty and Expedition; but for common practice, it is too tedious, and therefore I shall proceed to the choicest practical Method.

XIII. I now come to the last and principal Method of Division, when the Divisor consists of many Places, which

The latter and choicest practical Method of Division when the Divisor consists of many Places.

to such as have the *Table* of *Multiplication* by Heart, will not be difficult: For Example, Let 56304 be a Number given to be divided by 184, that is, into an 184 equal Parts, and the *Quotient*, or one of the equal Parts is required.

First, Distinguish by a Point (as before) so many of the foremost Places of the Dividend towards the Left-hand, as are either equal in Value (when consider'd apart) to the Divisor, or else, which being greater, yet come nearer to it: Thus I subscribe a Point under the Figure 3, thereby setting a-part 563, being the fewest of the foremost Places, which contain the Divisor; so is 563 the Dividual, or Number whereof the first Question must be asked Having thus prepar'd the Numbers, I demand how often the Divisor

Divisor 184 is contained in the Dividual 563; and since to answer this Question and such like, there is a Necessity of Tryal, it will be requisite to shew how this Tryal may fitly be made: First, therefore, compare the Number of Places in the Dividual, with the Number of Places in the Divisor, and when the Number of Places is the same in both, let it be asked how often the first or extreme Figure of the Divisor towards the Left-hand is contained in the first Figure of the Dividual towards the same Hand; so here demanding how often 1 is contained in 5, the Answer is 5 Times; whence I infer that the Divisor 184 is not contained oftner than 5 times in the Dividual 563 (for 6 Times 184 is manifestly greater than 563,) but whether it be contained 5 Times in it or not, Examination must be made either by multiplying (in some By-place) the Divisor 184 by the said 5, and comparing the Product with the Dividual 563; or else thus, saying 5 Times 1 (to wit the 1 in the Divisor) is contained in 5, to wit, the first Figure of the Divisor 563, 5 Times; but then the 8, the following Figure of the Divisor, cannot be found 5 Times in 6, the following Figure of the Dividend, and consequently the Divisor 184 is not contained 5 Times in the Dividual 563; wherefore I make another Tryal to see whether it may be contained 4 Times in it or not; saying, 4 Times 1 is 4, which is found in 5, and there will remain 1; but then 4 Times 8, which is 32, cannot be had in 16 (for the 1 before remaining being supposed to stand on the Left-hand of 6 makes 16) hence I conclude again, that the Divisor 184 is not contained 4 Times in the Dividual 563; wherefore I make another Tryal to see whether it may be contained 3 Times in it or not; saying, 3 Times 1 is 3, which is found in 5, and there will remain 2; again three Times 8 is 24, which is found in 26 (for the 2 before remaining being supposed to stand before the 6 in the Dividual, makes 26) and there will remain 2. Lastly, 3 Times 4 is 12, which is likewise found in 23, (for 2 remaining in the 26 being supposed to stand before the 3 in the Dividual makes 23) whereby I see that the Divisor 184 is contained 3 Times in the Dividual 563; therefore I write 3 in the Quotient, and proceeding according to the 8th Rule of this Chapter, I multiply the Divisor 184 by 3 (the Figure placed in the Quotient) so the Product is 552, which I subscribe orderly under the Dividual 563; then having drawn a Line under the said Product. I subtract it from the Dividual, and subscribe the Remainder, which is 11 under the Line.

F 2

$$\begin{array}{r}
 184)56304(3 \\
 \underline{552} \\
 11
 \end{array}$$

Again,

Again, according to the 9th Rule of this Chapter, I bring down 0, which stands in the next Place of the Dividend, to the Remainder 11, so there is 10 for a new *Dividual*; then demanding how often the Divisor 184 is found in the *Dividual* 110, and not finding it once contained in it, I write 0 in the Quotient (which is to be done as often as the Question is answered by nothing); now because the Product arising from the Multiplication of the Divisor by 0, (the Cypher last placed in the Quotient,) amounts to 0; the

$$\begin{array}{r}
 184) 56304 \quad (306 \\
 \underline{552} \\
 1104 \\
 \underline{1104} \\
 0
 \end{array}$$

Dividual 110, out of which that Product should be subtracted, remains the same without Alteration; therefore, after a Point is subscribed under 4, the following Place of the Dividend, I annex 4 to the last *Dividual* 110, so there will be a new *Dividual*, to wit, 1104; and here the Question at large is, to

know how often 184 is found in 1104; but to lessen the Tryal, because the *Dividual* consists of one Place more than is in the Divisor, it must be asked how often the first Figure of the Divisor on the Left-hand is contained in the two foremost Places of the *Dividual* towards the Left-hand, viz. I demand how often 1 is contained in 11, and altho' it may be had 11 times, yet I need never begin the Tryal above 9 times; therefore I make Tryal with 9, saying, 9 times 1 is 9, which is found in 11, and there will remain 2; but then 9 times 8, which is 72, cannot be found in 20 (20, because the 2 remaining being supposed to stand before 0 in the *Dividual* makes 20,) therefore I make Tryal with 8, saying, 8 times 1 is 8, which is found in 11, and there will remain 3; but then 8 times 8 cannot be had in 30 (30, because the 3 remaining being supposed to stand before the 0 or Cypher makes 30); therefore I make Tryal with 7, saying, 7 times 1 is 7, which is found in 11, and there will remain 4; but then 7 times 8 cannot be had in 40; therefore I make Tryal with 6, saying, 6 times 1 is 6, which is found in 11, and there will remain 5; then 6 times 8 is 48, which is found in 50, and there will remain 2: Lastly, 6 times 4 is 24, which is found in 50, whereby at length I see that the Divisor 184 is contained 6 times in the *Dividual* 1104: Wherefore I write 6 in the Quotient, and proceeding according to the 8th Rule of this Chapter, I multiply the Divisor 184 by 6 (the Figure last placed in the Quotient) so the Product is 1104, which being subscribed under, and subtracted from the *Dividual* 1104, the Remainder is 0, so at last conclude, that the Quotient sought is 306.

Note,

Note. If the Figure assumed for the *Quotient* holds good upon *Tryal*, as aforesaid, by two or three of the foremost Places of the *Dividual*, it will for the most part hold throughout the *Dividual*; but this must be a perpetual Rule, that whensoever the *Product* of the *Multiplication* of the *Divisor* by the Figure placed in the *Quotient* happens to be greater than the *Dividual*, from which it ought to be subtracted, such *Product* must be struck out of the *Work*, and a lesser Figure is to be placed in the *Quotient*.

For a second *Example*, Let it be required to divide 15114220 by 2987, or into 2987 equal Parts.

First, the *Divisor* 2987 being greater than 1511, (to wit, the four foremost Places of the *Dividend*) I set a Point under 4, thereby setting a part 15114 for a *Dividual*; then, because the *Dividual* consist of one Place more than the *Divisor*, I ask how often 2 (the First Figure of the *Divisor* towards the Left-hand) is contained in 15, (the two foremost Places of the *Dividual*) and finding the Answer to be

$$\begin{array}{r} 2987) 15114220 \quad (5 \\ 14935 \\ \hline 179 \end{array}$$

7 Times, I infer thence that the *Divisor* 2987 cannot be contained more than 7 Times in the *Dividual* 15114; but whether it will be contained 7 Times in it or not, Examination must be made, either by multiplying 2987 by 7 (in some By-place) and comparing the *Product* with the *Dividual* 15114; or else by the Manner of *Tryal* before delivered in the last *Example*: So at length it will be discovered, that the *Divisor* 2987 will not be found above 5 Times in the *Dividual* 15114; wherefore (according to the 8th Rule of this Chapter) writing 6 in the *Quotient*, and multiplying 2987 by 5, I subscribe the *Product* of that *Multiplication*, which is 14935, under the *Dividual* 15114; then drawing a Line under the said *Product*, and subtracting it from the *Dividual* 15114, I subscribe the Remainder 179 under the Line.

Again, (according to the 9th Rule of this Chapter) I bring down 2, the next Place of the *Dividend*, to the said Remainder, 79, so the new *Dividual* will be 1792; that done, asking how often the *Divisor* 2987 is contained in the *Dividual* 1792, and not finding it once contained in it, I write 0 in the *Quotient*; and here, because the Question is answered by 0, the next Place of the *Dividend*, to wit 2, is to be brought down to the *Dividual* 1792, so the new *Dividual*

$$\begin{array}{r} 2987) 15114220 \quad (50 \\ 14935 \\ \hline 1792 \end{array}$$

2987)15114220(5060

14935

17922

17922

00

dual is 17922. Then renewing the Question, and proceeding as before, at length the Division being finish'd, the Quotient will be found 5060 exactly, without any Remainder; but if any Remainder had happen'd after the Subtraction of the last Product, it must have been prosecuted according to the Note before given in the Example at the latter End of the 11th

Rule of this Chapter.

In like manner, if 1208939550 be divided by 19999, or into 19999 equal Parts, the Quotient, or one of those equal Parts, will be found 60450, and the Work will stand as you see here.

19999)1208939550(60450

119994

89995

79996

99995

99995

00

This latter Method of Division is to be prefer'd before any of the common Ways of dividing, by dashing out Figures, where the Steps of the Division are so confounded (besides the Burden upon the Memory, by a promiscuous Multiplication and Division) that if any Error happen, it can hardly be corrected without beginning the Work a-new: But in the way before explained, the particular Multiplications, Subtractions,

and Remainders, which belong to every Figure of the Quotient, are so distinctly and clearly express'd, that if an Error happen, the Work may easily be reformed.

XIV. So often as the Question is repeated in Division, so many Places there must be in the Quotient, (which may be discover'd by the Number of Points placed under the Dividend) and so many times is one and the same Kind of Operation repeated, the Substance whereof is contained in the Verse before-mentioned at the end of the Tenth Rule of this Chapter.

XV. When the Divisor consists of 1 or an Unit in the extreme Place towards the Left-hand, and nothing but Cyphers towards the Right, the Division is perform'd by cutting off with a Line so many Places

A Compendious way of dividing by 10, 100, 1000, &c.

of

of the Dividend towards the Right hand, as the Divisor has Cyphers; so the Figures which stand on the Left-hand of the Line, give the Quotient, and those cut off to the Right (if they be significant Figures) are to be proceeded with as a Surplusage or Overplus remaining, according to the Note at the End of the 11th Rule of this Chapter. So

if 4720 l. were given to be divided equally among 10 Persons, the Share of each would be 472 l. also if the said 4720 l. were to be divided equally among 100

10) 4270 (472
100) 47120 (472
1000) 47200 (472

Persons, the Share of each would be 47 l. and there would be a Surplusage or Remainder of 20 l. to be also subdivided among them, after the said 20 l. are converted into Shillings according to the Fifth Rule of the next Chapter. Lastly, if the said 4720 l. were to be divided among 1000 Persons, the Share of each would be 4 l. and there would be a Remainder of 720 l. to be also divided as aforesaid. See the Form of the Work in the Margin.

XVI. When the Divisor consists of any Significant Figure or Figures in the first or foremost Place or Places towards the left-hand, and nothing but a Cypher or Cyphers towards the Right, cut off by a Line so many Places of the Dividend towards the Right-hand as the Divisor has Cyphers towards the Right; then divide the Figures of the Dividend, which stand on the Left-hand of the Line, by the Figures in the Divisor which remain, when the said Cypher or Cyphers are omitted, rememb'ring after the Division is finished, to write down next after the last Remainder the Places of the Dividend which were first cut off: So if 36732 were given to be divided by 20, the Quotient will be 1836, and there will remain 12, viz. if you cut off one Place from the Dividend towards the Right-hand (because the Divisor ends with one Cypher, (and then divide the rest.) to wit, 3673) by 2 (according to the 11th Rule of this Chapter) there will arise in the Quotient 1836, and the last Remainder after such Division is finish'd, will be 1, to which if 2 (the Figure first cut off from the Dividend) be annexed, the Total Remainder is 12.

Another Compendium in Division.

1
20) 36732 (1836

In like manner if 7456787, were given to be divided by 304000, the Quotient will be 24. and there will remain 160787; viz. If you cut off 3 Places from the Dividend towards

304|000) 7456|787 (24

608

1376

1216

160787

the Total Remainder or Surplusage is 160787, which is to be proceeded with, as is directed in the *Note* at the latter End of the Eleventh Rule of this *Chapter*.

XVII. Division and Multiplication interchangeably prove

*The Proof of
Multiplication
and Division.*

one another; for in Division if you multiply the Divisor by the Quotient, the Product will be equal to the Dividend: So in the *Example* of the 13th Rule of this *Chapter*, if 184 the Divisor be multiplied by 306 the Quotient, the Product is 56304, which is the same with the Dividend; but when after the whole Division is finished, any Figures remain of the last Subtraction, add them likewise to the Product: So in the last *Example* of the 16th Rule of this *Chapter*, the Divisor 304000, being multiplied by the Quotient 24, produces 7296000, unto which if you add the Number remaining, to wit, 160787, the Sum is 7456787, which is the same with the Dividend. Again in Multiplication, if the Product be divided by the Multiplier, the Quotient will give you the Multiplicand, or if the Product be divided by the Multiplicand, the Quotient will give you the Multiplier; So in the first *Example* of the 9th Rule of the last *Chapter*, if the Product 111024 be divided by the Multiplicand 3084, the Quotient gives the Multiplier 36.

There is also a *Common Proof* of Multiplication argued from the Multiplicand, the Multiplier, and the Product, by casting away Nines; but by that way of Proof (though rightly used, a false Product will be affirmed to be true; *Example*, If 3462 be multiplied by 786, the true Product is 2721132; but if I say, 4953132 or 3153132 is the Product (or many others which may be given) the Proof by Nines will confirm them to be true Products, though they are false, as will be evident to such as know the *Rule*, which I mention here only to set a Brand upon it, that it may be avoided by all Lovers of Truth.

C H A P. VII.

Reduction.

I. Forasmuch as in *Money*, there are Diversities of Kinds, viz. in *England*, *Pounds*, *Shillings*, *Pence*, and *Farthings*; also diverse Kinds of *Weights*, *Measures*, &c. as has been fully declared in the Second Chapter; and because it is often required to find how many Pieces of one Kind of *Money* are equal in Value to a given Number of another, (and so likewise of *Weights*, *Measures*, &c.) it is requisite in this Place to shew how that is performed, since thereby the *Rules* of *Multiplication*, and *Division* before delivered will be exercis'd. This Kind of Operation is called *Reduction*.

II. *Reduction* is either descending or ascending.

III. *Reduction* descending is, when some Integers of a Number of a greater Denomination being given, it is required to find how many Integers of a lesser Denomination are equal in Value to that given Number of the greater: As when it is demanded to find how many *Shillings* are contained in 30 *l.* Likewise how many *Pence* in 320 *s.* or how many Hours in 355 Days, &c.

IV. *Reduction* ascending is, when some Integers of a Number of a lesser Denomination being given, it is required to find how many Integers of a greater Denomination are equal in Value to that given Number of the lesser: As when it is proposed to find how many *Pence* are contained in 500 *Farthings*: Likewise how many *Shillings* in 348 *Pence*: Or how many Days in 864 Hours, &c.

V. *Reduction* descending is performed by *Multiplication*; for if the given Number of Integers of a greater Denomination be multiplied by a Number which expresses how many Integers of the lesser are equal to one of the Integers given, the Product is the Number of Integers of the lesser Denomination required.

Reduction descending performed by Multiplication.

So 230 l. of *English Money* will be reduced into 4600 s. for if 230 be multiplied by 20, (the Number of *Shillings* which are equal to 1 *Pound*) the Product is 4600; in like manner 4600 s. will be reduced into 55200 d. for if 4600 be multiplied by 12 (the Number of *Pence* contained in 1 *Shilling*) the Product is 55200. Also 55200 *Pence* being multiplied by 4 (because 4 *Farthings* make a *Penny*) are reduced into 220800 *Farthings*, as by the Operation in the Margin is evident.

230 Pounds.
20
4600 Shillings.
12
9200
46
55200 Pence
4
220800 Farthings.
345 Ounces.
20
6900 Penny W.
24
276
138
165600 Grains.

The like Method is to be observed in *Weights, Measures, &c.* So 345 Ounces-Troy are reduced into 6900 Penny-weights, and 6900 Penny-weights, to 165600 Grains, as by the Operation in the Margin you may see.

Compare this with the Note upon the last Example of the Eleventh Rule of the Sixth Chapter. Note. By this Rule the Learner is furnished with Skill to resolve that Case in Division, when the Dividend is less than the Divisor: Example, Let it be required to divide 7 Pounds of *English Money*, equally among 8 Persons; here it is evident that the Dividend 7 is less than the Divisor 8; that is, the Number of Pounds is less than the Number of Persons, and consequently each Share must be less than a Pound: So that in effect it is required to find how many *Shillings* and *Pence* belong to every Person for his Share: First, therefore reduce the 7 Pounds into *Shillings*, which will be 140, these divided by 8 give 17 *Shillings* to each Person, and there will yet be a Remainder of 4 *Shillings* to be also equally divided into 8 Parts; but these 4 *Shillings* must be first reduced into *Pence*, which will be 48, then dividing 48 by 8, the Quotient will give 6 *Pence* more to every Person: So at last it appears that if 7 Pounds of *English Money* be equally divided into 8 Parts, the entire Quotient (or one of the equal Shares) will be 17 *Shillings* and 6 *Pence*.

In like manner, if 354 Pounds of *English Money* be given to be divided equally amongst 125 Persons, the Share of each will be found to be 2 Pounds, 16 *Shillings*, 7 *Pence*, 2 *Farthings*, and somewhat more, but the Parts of a *Farthing* being of no Moment (and not properly to be handled in this Place) are neglected.

Com-

Compare these two Examples with the last Example of the Eleventh Rule of the Sixth Chapter.

In *Reduction* descending, the Learner may receive Help by the subsequent *Tables*.

1. *English Money.*

Pounds	Multiplied by	20	Produce	Shillings.
Shillings		12		Pence.
Pence		4		Farthings.

2. *Troy-Weight.*

Pounds	Multiplied by	12	Produce	Ounces.
Ounces		20		Penny Weights.
Penny-Weights		24		Grains.

Also in *Apothecaries-Weights.*

Ounces-Troy	Multiplied by	8	Produce	Drams.
Drams		3		Scruples.
Scruples		20		Grains.

3. *Of Avoirdupois-Weights.*

Hundred Weig.	Multiplied by	4	Produce	Quarters.
Quarters		28		Pounds.
Pounds		16		Ounces.
Ounces		16		Drams.

4. *Of Liquid Measures.*

Hogsheads	Multiplied by	63	Produce	Gallons.
Gallons		2		Pottles.
Pottles		2		Quarts.
Quarts		2		Pints.

5. Of Dry Measures.

Quarters.					
Busbels	} Multiplied by	8	} Produce	Busbels.	
Pecks		4		Pecks.	
Gallons		2		Gallons.	
Pottles		2		Pottles.	
Quarts		2		Quarts.	
				Pints.	

6. Of Long Measures.

English Miles	} Multiplied by	8	} Produce	Furlongs.	
Furlongs		220		Yards.	
Yards		3		Feet.	
Feet		12		Inches.	
Inches		3		Barley-Corns.	

Also,

Yards or	} Mul. by	4	} Produce	Quarters.	
Ells		4		Nils.	
Quarters					

7. Of Superficial Measures of Land.

Acres	} Mul. by	4	} Produce	Roods.	
Roods		40		Perches or Poles.	

8. Of Time.

Weeks	} Mul. by	7	} Produce	Days.	
Days		24		Hours.	
Hours		60		Minutes.	

To reduce Integers of diverse Denominations into the lowest of these Denominations.

VI. Integers of several Denominations may be reduced into the last of those Denominations according to the Fifth Rule afore-going, by descending orderly to the next inferior Denomination; and adding to each Product such Integers, (if there be any) as are of the same Name.

So 12 Pounds 13 Shillings, and 10 Pence may be reduced into 3046 Pence, in this manner; viz. 12 Pounds multiplied by 20 (because 20 Shillings, make 1 Pound) produce 240 Shillings, to which adding 13 Shillings, the Sum is 253 Shillings. Again, 253 Shillings multiplied by 12 (because 1 Shilling is equal to 12 Pence,) produce 3036 Pence, to which, if 10 Pence be added, the Sum is 3046 Pence, as by the Operation in the Margin is manifest.

l.	s.	d.
12	13	10
240		
add 13		
253		Shillings.
506		
253		
3036		
add 10		
3046 Pence.		

But after that general Method is well understood, the Work of the last Example, and such like, may be contracted thus; viz. To convert 12 Pounds, 13 Shillings, 10 Pence, all into Pence: First, 12 multiplied by 0 (which stands in the Units Place of 20) produces 0, but instead of 0, I write down 3 under the Line, (to wit, the 3 that stands in the Units Place of the 13 Shillings in the Sum proposed:) Then I proceed to multiply 12 by 2, saying twice 1 is 2, to which, adding 1 (for the Ten in the said 13 Shillings) it makes 5, which I set on the Left-hand of 3 before written: Lastly, twice 1 is 2, which I set on the Left-hand of 5; and so 12 Pounds, 13 Shillings, and 10 Pence are converted into 253 Shillings.

l.	s.	d.
12	13	10
20		
153 Shillings.		
12		
500		
253		
3046 Pence.		

It remains to multiply the said 253 by 12 (because 12 Pence make 1 Shilling) and to add 10 to the Product, which may be done thus: First, twice 3 is 6, to which adding 10 (to wit, 10 Pence in the Sum first propounded,) it makes 16; wherefore (according to the Rule of Multiplication) I set 6 under the Line, and keep 1 in mind: Again, twice 5 with 1 in mind making 11, I write down 1, and keep 1 in mind; likewise twice 2 and 1 in mind making 5, I write down 5: Then 253 multiplied by 1 making 253, which I set orderly under 516; Lastly, those Two Products added together make 3046, which is the Number of Pence contained in 12 l. 13 s. 10 d., as before was found out by the general Method.

So 35 Ounces, 16 Penny-Weights, and 12 Grains Troy will be reduced into 17196 Grains.

VII. Reduction ascending is performed by Division; for if the Number of Integers given, be divided by such a Number of the same Integers, as are equal to one of the Integers required, the Quotient is the Number of Integers sought for.

Reduction ascending performed by Division.

So 220800 *Farthings* being divided by 12 (the Number of *Farthings* in a *Penny*, (give 55200 *Pence*, in the *Quotient*: In like manner, if 55200 *Pence* be divided by 12 (the Number of *Pence* in a *Shilling*.) the *Quotient* is 4600 *Shillings*. Lastly, 4600 *Shillings* being divided by 20 (because 20s. make a *Pound sterling*) the *Quotient* is 230 *Pounds sterling*, which are equal to 220800 *Farthings* first given. The Operation is as follows.

$$\begin{array}{r}
 \begin{array}{ccc}
 & 12) & 20) \\
 4)220800 & (55200 & (4600 & (230\ l, \\
 \dots\dots & \dots\dots & \dots\dots & \dots\dots \\
 20 & 48 & 40 & \\
 \hline
 20 & 72 & 60 & \\
 20 & 72 & 60 & \\
 \hline
 08 & 000 & 00 & \\
 8 & & & \\
 \hline
 000 & & &
 \end{array}
 \end{array}$$

In like manner, 34268 *Grains-Troy* will be reduced to 5 *l.* 11 *Ounces*, 7 *Penny-weights*, and 20 *Grains*. This Kind of *Reduction* may be made the easier to the Learner by the following *Tables*.

1. Of *English Money*.

<i>Farthings</i>	} by	{ 4	} give	<i>Pence</i> .	
<i>Pence</i>				{ 12	<i>Shillings</i> .
<i>Shillings</i>					{ 20

2 Of *Troy-Weight*.

<i>Grains</i>	} by	{ 24	} give	<i>Penny-weights</i> .	
<i>Penny-weights</i>				{ 20	<i>Ounces</i> .
<i>Ounces</i>					{ 12

Also in *Apothecaries-Weights*.

<i>Grains</i>	} by	{ 20	} give	<i>Scruples</i> .	
<i>Scruples</i>				{ 3	<i>Drams</i> .
<i>Drams</i>					{ 8

3. Of

3. Of Avoirdupois-Weights.

Drams	} Divid. by {	16	} give {	Ounces.
Ounces		16		Pounds.
Pounds		28		Quarters of C.
Quarters		4		Hundred Weights.

4. Of Liquid Measures.

Pints	} Divid. by {	2	} give {	Quarts.
Quarts		2		Pottles.
Pottles		2		Gallons.
Gallons		63		Hogsheads.

5. Of Dry Measures.

Pints	} Divided by {	2	} give {	Quarts.
Quarts		2		Pottles.
Pottles		2		Gallons.
Gallons		2		Pecks.
Pecks		4		Busbels.
Busbels		8		Quarters.

6. Of Long Measures.

Barley-Corns	} Divided by {	3	} give {	Inches.
Inches		12		Feet.
Feet		3		Yards.
Yards		20		Furlongs.
Furlongs		8		English Miles.

Also,

Nails	} Div. by {	4	} give {	Quarters of Yards.
Quarters		4		Yards.
Quarters		5		Ells English.

7. Of Superficial Measures of Land.

Perches or Poles	} Div. by {	40	} give {	Roods, or Quarters
Roods		4		of Acres.
				Acres.

8. Of Time.

Minutes	} Div by {	60	} give {	Hours.
Hours		24		Days.
Days		7		Weeks.

Note,

Note, That if after Division is finished in Reduction ascending, there be any Remainder, it is of the same Denomination with the Dividend.

Note, also, That Reduction descending and ascending mutually prove one another, by inverting the Question; for as in 56 Pounds Sterling, there will be found 53760 *arthings* by Reduction descending: So for Proof thereof, 63760 *Farthings* will be reduced to 56 Pounds, by Reduction ascending.

Questions to exercise Reduction.

1. In 257 *l.* How many Shillings? *Answer*, 5140.
2. In 3076 *l.* How many Shillings? *Answer*, 61520.
3. In 902 Shillings, How many Pence? *Answer*, 10824.
4. In 2179 Shillings, How many Farthings? *Answer*, 104592.
5. In 49 *l.* — 13 *s.* — 7 *d.* How many Pence? *Answer*, 11923.
6. In 2053 *l.* — 14 *s.* — 9 *d.* — 2 *f.* How many Farthings? *Answer*, 1971590.
7. In 354 *l.* of Troy-weight, How many Grains (of Goldsmiths Weight?) *Answer*, 2039040.
8. In 300 English Miles, How many Yards? *Answer*, 528000.
9. In 1 English Mile, How many Barly-corns Length? *Answer*, 190080.
10. In 560 Acres, How many Perches? *Answer*, 89600.
11. In 225 Acres, 3 Roods, and 30 Perches, How many Perches? *Answer*, 36150.
12. In 11923 Pence, How many Pounds? *Answer*, 49 *l.* — 13 *s.* — 7 *d.*
13. In 5764684 Farthings, How many Pounds? *Answer*, 6004 *l.* — 17 *s.* — 7 *d.*
14. In 234678 Perches, How many Acres?? *Answer*, 1466 Acres, 2 Roods, 38 Perches.
15. In 525960 Minutes of an Hour, How many Days? *Answer*, 365 Days and 6 Hours, (or one Year very near.)
16. In 10080 Pints, How many Hogsheads? *Answer*, 20.
17. In 34678 Grains of Apothecaries Weight, How many Ounces-Troy? *Answer*, 72 Ounces, 1 Dram, 2 Scruples, and 18 Grains.
18. In 106735 Pints of Wheat, How many Quarters? *Answer*, 208 Quarters, 3 Bushels, 2 Pecks, 1 Gallon, 1 Pottle, 1 Quart, 1 Pint.
19. In 3969301 Barly-corns Length, How many Miles? *Answer*, 20 Miles, 7 Furlongs, 12 Yards, 2 Feet, 4 Inches, and 1 Barly-corn's Length.
20. In 1900800 Barley-corns Length, o w many Miles? *Answer*, 10.

Chap. VIII.

Of the Rule of Three Direct.

I. **T**HE *Rule of Three* is so called, because by three Numbers known or given, it teaches to find a fourth unknown; it is also called the *Golden Rule* for the Excellency thereof: *Lastly*, it is called the *Rule of Proportion*, for the Reason hereafter declared.

II. The *Rule of Three* is either Single, or Compound.

III. The *Single Rule* is, when Three Terms or Numbers are proposed, and a *The Rule of Three*. fourth Proportional to them is demanded.

IV. Four Numbers are said to be Proportionals, when the first contains the second, or is contained by the second, in the same manner as the third contains the fourth, or is contained by the fourth. So these 4 Numbers are said to be Proportionals, 8, 4, 12, 6; for as 8 contains 4 twice, so does 12 contain 6 twice, and therefore is said to have such Proportion to 4 as 12 has to 6 likewise these are Proportionals, 4, 8, 6, 12. For as 4 is the Half of 8, so is 6 the Half of 12; and therefore 4 is said to have such Proportion to 8, as 6 has to 12.

V. The Terms or Numbers of the *Rule of Three* (to wit, the three Numbers given, and the fourth sought) consist of two different Denominations, viz. two of the three given Terms have one Name, and the other given Term with the Term required have another: So *The diverse Denominations of the Terms in the Rule of Three.* this Question being demanded, If four Students spend 19 Pounds in certain Months, How much Money will serve 8 Students for the same Time, and at the same Rate of Expence? Here Students and Pounds are the two Denominations of the Terms in the Question; viz. 4 and 8 (being two of the Terms proposed) have the Denominations of Students, and 19 the other Term given, together with the Term required, have the Denomination of Pounds.

VI. In the *Rule of Three*, two of the three given Terms imply a Supposition, and the third moves a Question: So in the afore-mentioned Question a Supposition is made, that 4 Students

H

students

dents spend 19 Pounds, and a Question is moved with the Number 8, to wit, how many Pounds will 8 Students spend?

VII, In the Rule of Three, the Numbers given must be so ranked, that the known Number or Term upon which the Question is asked, may possess the third Place in the Rule; also of the other two, that which has the same Denomination with the third, must be in the first Place: Lastly, the other known Term, which is of the same Denomination with the fourth Term sought, (or Answer of the Question) must possess the second Place: So in the Question before-mentioned, the Terms, 4, 19, and 8, are to be thus placed, viz, 8 is the Term upon which the Question is moved, and therefore to possess the third Place in the Rule; 4 is of the same Denomination with 8, viz. of Students, and therefore to be in the first Place: Lastly, 19 being of the same Denomination with the Term sought for, viz. of Money, is to be in the second Place: And so they will be placed in the Rule thus:

<i>Students.</i>	<i>Pounds.</i>	<i>Students.</i>
If 4	— 19 —	— 8

That is to say, If 4 Students spend 19 Pounds, what will 8 Students spend? And here, for the better discerning of the Term or Number upon which the Question is asked, you may observe, That for the most part it is the known Number in the Question which immediately follows these or such like Words; viz. *How many? How much? What will? How long? How far? &c.*

VIII. The Rule of Three is either Direct, or Inverse.

IX. The Rule of Three Direct is, when the Sense or Tenour of the Question requires that the fourth Number sought have such Proportion to the second, as the third Number has to the first; so in the afore-mentioned Question, If 4 Students spend 19 Pounds, how many Pounds will 8 Students spend at the same Rate of Expence? It is evident, that the Thing requir'd it to find a Number which may have such Proportion to 19, as 8 has to 4; that is, as 8 is the Double of 4, so ought the fourth Number to be the Double of 19; for if 19 Pounds be required to maintain 4 Students a certain Time, as much more must needs be required for the Maintenance of 8 Students the same Time; and therefore in this Case, we may say

say in a direct Proportion, as 4 is to 8, so is 19 to a Number which ought to be as much more as 19.

X. In the *Direct Rule of Three*, if you multiply the second Term by the Third, (or which is all one) the Third Term by the Second, and then divide the Product by the first, the *Quotient* will give the fourth Term or fourth Proportional required. So in the Question before proposed, if you multiply 19 by 8 the Product is 152, which, if you divide by 4, the *Quotient* will give you 38, the fourth Term demanded, and the Work will stand thus.

How to work this Rule of three Direct, the three given Terms being single Numbers.

Stud. l. Stud. l.
 If 4—19—8—(38
 8
 4) 152 (38 Pounds
 12
 32
 32
 0

A Second Example may be this, if 8 Yards cost 9 Pounds, how much will 3 Yards cost?

Answer, 3 l. ——— 7 s. ——— 6 d.

This Question being stated according to the Seventh Rule of this Chapter, will stand as here you see; then multiplying (as before) the second Term 9 by the third Time 3, the Product is 17, which being divided by the first Term 8, the Quotient is 3 Pounds, and there is a Remainder of three Pounds, which must be reduced into Sixty Shillings, and after those Shillings are divided by 8, and the rest of the Work prosecuted according to the Note at the latter End of the 11th Rule of the 6th Chapter, at length the entire Quotient or Answer of the Question is 3 l. ——— 7 s. ——— 6 d,

y. l. y. l. s. d.
 8—9—3 (3 : 7 : 6,
 3
 8) 37 (3 Pounds.
 24
 3 the Remaind.
 20
 8 (60 (7 shillings.
 56
 4 the Rema.nd.
 12
 8) 48 (6 Pence.

A Third Example, if 51 Ounces of Silver Plate be sold for 13 Pounds sterling, what is the Price of 1 Ounce of that Plate?

Answer, 5 s. ——— 1 d. and somewhat more. The Operation is thus: After the three known Terms of this Question are rightly ordered, they will stand as here you see in the Example; then multiplying the second Time 13 by the third Term 1, the Product will be also 13, (for *Multiplication* by 1 makes no Alteration;) which 13 being divided by 51, after the manner of

oz. l. ex.
 51—13—1
 1
 13
 20
 51) 260 (; Shil.
 255
 5
 12
 51) 60 (1 Pen.
 51
 9

Operation delivered in the *Note* upon the 5th Rule of the 7th Chapter, the entire Quotient or Answer of the Question will at length be found to be 5 s. — 1d. and somewhat more; but the Surplusage being less than a Farthing, is omitted as useless.

Example 4. What must be paid to a Labourer for his Wages for 27 Weeks at the Rate of 4s. for 1 Week? *Answer*, 5 l. — 8s.

After the three given Terms are rightly placed in the Rule, they will stand as you see in the *Example* ; then multiplying the third Term 27 by the second Term 4, the Product is 108, which I should divide by the first Term 1 ; but since Division by 1 makes no Alteration, the Quotient is also 108, so that the Fourth Term sought is 108 Shillings, which being reduced to Pounds, according to the seventh Rule of the seventh Chapter, gives 5 l. — 8 s. for the *Answer* of the *Question*:

<i>Weeks</i>	<i>Shill.</i>	<i>Week.</i>
1	— 4 —	27
	4	
	—	
	108	

XI. In the *Rule of Three*, if after the Question is stated according to the seventh Rule of this Chapter, any of the three given Terms be a compound Term consisting of diverse Denominations ; as Pounds, Shillings, and Pence ; or Weeks, Days, Hours, &c. such compound Term must first be reduced into the lowest of those Denominations (by the 6th Rule of the seventh Chapter) to the end that the three given Terms may be three single Numbers ; also of these three single Numbers, the first and third must always be of one and the same Denomination : For if it happen that they express Things of different Names, that of the two which has the greater Name (or Denomination) is to be reduced into the same Name with the lesser (by the 5th Rule of the seventh Chapter :) These Preparations being observed, the Rest of the Work is to be prosecuted according to the 10th Rule of this Chapter. *Example.* What will 48 Ounces, 17 Penny-weight, and 20 Grains of Silver-Plate amount to at the Rate of 5 s. — 6 d. the Ounce? *Answer*, 13 l. — 8 s. 10 d. — 3 f. very near.

This Question being stated according to the seventh Rule of this Chapter, will stand in the Rule as you see in the *Example*, to wit, If 1 Ounce cost 5 s. — 6 d. what will 48 oz — 16 p. w. — 20 gr. cost? Here, because the third Term is compounded of diverse Denominations, it must be reduced into the

The lowest of those Denominations, to wit, Grains; so, by the sixth Rule of the seventh Chapter, there will be found 23468 Grains for the third Term: Likewise, because the second Term 5 s. — 6 d. is a compound Term,

oz.	s.	d.	oz.	p.w.	gr.
1	5	6	48	17	20
20	12		20		
20	66		977		
24			24		
480			3928		
			1954		
23468 Grains.					

whose lowest Name is Pence, it must be reduced into Pence (by the aforesaid Rule;) So there will be found 66 Pence for the second Term: But rather, because the first Term has the Name Ounce, and the third Term the Name Grain, the first Term 1 Ounce must be converted into 480 Grains (which are equal to 1 Ounce) then will the three Terms or single Numbers stand in the Rule, as here you see, viz. If 480 Grains cost 66 gr. pence. gr. Pence, how many Pence will 23468 Grains cost? Now, proceeding according to the Tenth Rule of this Chapter, there will arise in the Quotient 3226 Pence, besides a Remainder of 408 Pence, which being reduced to 1632 Farthings, and those divided by the first Term 480, the Quotient will be 3 Farthings: So that the entire Quotient is 3226 Pence, 3 Farthings. and somewhat more; (but the Parts of a Farthing being of no Moment, may be neglected. Lastly, the said 3226 Pence being reduced according to the seventh Rule of the seventh Chapter, give 13 l. — 8 s. — 10 d. — 3 f. so that 13 l. — 8 s. 10 d. — 3 f. and somewhat more, will be the Answer of the Question.

XII. For the Proof of the Direct Rule of Three, multiply the Fourth Term by the First; which done, if that Product be equal to the Product of the second Term multiplied by the Third, the Work is right, otherwise it is erroneous: So in the first Example, 38 the fourth Term being multiplied by the first Term 4. the Product is 152, which is also the Product of 19 multiplied by 8. But if it happen that after the fourth Term, or Answer of the Question, is found in the same Denomination with the second Term, there is yet a Remainder, such

The Proof of the Rule of Three Direct.

such Remainder must be added to the Product of the first Term, multiplied by such fourth Term, and then the Sum must be equal to the Product of the second and third Terms, (the second Term consisting of the same Denomination with the fourth :) So in the last Example the fourth Term is 3226, and there happens to be a Remainder of 408, which being added to the Product of the Multiplication of the said 3226 by the first Term 408, gives 1548888, which is the same with the Product of the third Term 23468 multiplied by the second Term 66, as will appear when work'd.

XIII. When the first of the three given Numbers in the Rule of Three Direct, is 1 or Unity, the Question may often be answered more speedily than by the Rule of Three, even by those who have but little Skill in Arithmetick, as will partly appear by the following Examples, viz.

1. At 17 s. — 9 d. the Yard, what will 48 Yards cost? Answer, 74 l. — 11 s. For Reason shews that 84 Yards must (at the said Rate) cost 84 Angels, 84 Crowns, 84 half Crowns, and 84 Three Pences, all which being computed and added together, will give the full Value of 84 Yards, viz.

	l.	s.	d.
84 Angels make —————	42	00	00
84 Crowns —————	21	00	00
84 Half Crowns —————	10	10	00
84 Three Pences —————	1	01	00
Sum 74 —————	11	00	

2. At the Rate of 9 s. the Bushel of Wheat, what will 51 Quarters amount to? Answer 183 l. — 12 s. — 0 d.

(It is evident that the Price of 1 Quarter (which consists of 8 Bushels) will be 8 Angels wanting 8 Shillings: Therefore,

	l.	s.	d.
from 8 Angels, to wit, —————	4	00	00
subtract —————	0	8	00
remains the Price of 1 Quarter —————	3	12	00

Then the Value of 51 Quarters, at the Rate of 3 l. — 12 s. — 0 d. the Quarter, may be found in manner following, viz.

51 times

	<i>l.</i>	<i>s.</i>	<i>d.</i>
51 times 3 <i>l.</i> or three times 51 <i>l.</i> is	51	00	00
51 Angels make	25	10	00
51 Shillings doubled make	5	02	00
the Price of 51 Quarters	183	12	00

3. What is a Chest of Sugar worth, that *Tare is that wherein weighs neat Weight (the Tare being subtracted) 7 C.—3 q.—7 lb. at the Rate of a Bag for Pepper, as 6 *l.*—3 *s.*—4 *d.* for 1 C?* Answer, Chest for Sugar, &c.
48 *l.*—3 *s.*—6 *d.*—2 *f.*

	<i>l.</i>	<i>s.</i>	<i>d.</i>
7 times 6 Pounds make	42	00	00
7 times 3 Shillings	1	01	00
7 Groats	0	02	04
The Half of 6 <i>l.</i> —3 <i>s.</i> —4 <i>d.</i> for 2 <i>qu.</i> is	3	01	08
The Half of 3 <i>l.</i> —1 <i>s.</i> —8 <i>d.</i> for 1 <i>qu.</i> is	1	10	10
The fourth Part of 1 <i>l.</i> —10 <i>s.</i> —10 <i>d.</i> } (because 7 <i>l.</i> is a fourth Part of 28 <i>l.</i> } or of 1 <i>qu.</i>)	0	07	08 ^{<i>f.</i>}
	48	03	06—2

Practical Rules of this Nature cannot be compleatly understood without some Skill in Fractions, as will hereafter appear in the second Chapter of the *Appendix*: And therefore I shall conclude this Chapter with the following Questions, whose *Answers* are annexed to them, and may be found out by the Preceeding Rules; but the Operations are purposely omitted, and left as an Exercise for the Learner,

Questions to exercise the Rule of Three Direct.

1. If 17 Yards of Cloth cost 29 *l.*—2 *s.*—6 *d.* what will 35 Yards cost at that Rate? Answer, 39 *l.*—6 *s.*—6 *d.*
2. If 35 Yards cost 39 *l.*—7 *s.*—6 *d.*, how many Yards may be bought at that Rate for 19 *l.*—2 *s.*—6 *d.*? Answer, 17 Yards.
3. If 35 Yards cost 29 *l.*—7 *s.*—6 *d.*, what are 17 Yards worth at that Rate? Answer 19 *l.*—2 *s.*—6 *d.*
4. If 17 Yards be sold for 19 *l.*—2 *s.*—6 *d.* how many Yards will 39 *l.*—7 *s.*—6 *d.* buy at that Rate? Answer, 35 Yards.
5. What

5. What must I pay for the Carriage of 17 Hundred-weight, 3 Quarters, and 11 Pounds *Averdupois*, at the Rate of 7 Shillings the Hundred-weight? *Ans.* 6 *l.*—4 *s.*—11 *d.*—1 *f.*

6. If 6 *l.*—4 *s.*—11 *d.*—1 *f.* be paid for the Carriage of 17 Hundred-weight, 3 Quarters, and 11 Pounds, what was paid for the Carriage of 1 Pound weight? *Ans.* 3 Farthings.

7. What must I pay for 39 Ounces, 7 Penny-weight, and 18 Grains of white Plate, at the Rate of 5 *s.* and 5 *d.* the Ounce? *Answer,* 10 *l.*—13 *s.*—4 *d.* and three Quarters of a Farthing.

8. What must 1 *l.* (or 20 *s.*) pay towards a Tax, when 326 *l.*—6 *s.*—8 *d.* is assessed at 41 *l.*—16 *s.*—2 *d.*—3 *f.*? *Answer,* 2 *s.*—5 *d.*—2 *f.*

9. What will the Interest of 876 *l.*—17 *s.*—6 *d.* amount to for 1 Year, at the Rate of 6 *l.* for 100 *l.* for the same Time? *Answer,* 57 *l.*—12 *s.*—3 *d.*

10. If 3 Yards in Length of English Measure be equal to 4 Ells Flemish; how many Flemish Ells are contained in 120 Yards English? *Answer,* 160 Flemish Ells.

11. If 4 Flemish Ells in Length, be equal to 3 English Yards; how many English Yards in 300 Flemish Ells? *Answer,* 225 English Yards.

12. If 3 Ells in Length of English Measure, be equal to 5 Flemish Ells, how many Flemish Ells in 120 English Ells? *Answer,* 200 Flemish Ells.

13. If 5 Flemish Ells in Length, be equal to 3 English Ells; how many English Ells in 145 Flemish Ells? *Answer,* 87, English Ells.

14. If 3 Ounces of Silk-weight, be equal to 4 Ounces of *Venice-weight*; how many Ounces *Venice* are equal to 60 Ounces of Silk-weight? *Answer,* 80 Ounces *Venice*.

15. A Merchant deliver'd at *London*, 120 *l.* Sterling to receive 207 *l.* Flemish at *Amsterdam*; what was 1 *l.* Sterling valu'd at in Flemish Money? *Answer,* 1 *l.*—14 *s.*—6 *d.*

16. If a Bill of Exchange be accepted at *London*, for Payment of 400 *l.* Sterling, for the Value deliver'd at *Amsterdam*, at 1 *l.*—13 *s.*—6 *d.* Flemish for 1 *l.* Sterling, how much Flemish Money was deliver'd at *Amsterdam*? *Answer,* 670 *l.* Flemish.

17. When the Exchange from *Antwerp* to *London* is at 1 *l.*—4 *s.*—7 *d.* Flemish for 1 *l.* Sterling; how much Sterling must I pay at *London* to receive 236 *l.* Flemish at *Antwerp*? *Answer,* 192 Sterling.

18. A Merchant delivered at *London* 370 *l.* Sterling by Exchange for *Roan*, at 74 *d.* Sterling for 50 *s* Tournois; how much Tournois ought he to receive at *Roan*? *Answer*, 60000 *s* Tournois.

19. In 370 Ducats, at 4 *s* — 2 *d.* the Ducat; how many French Crowns at 6 *s* — 2 *d*? *Answer*, 250 Crowns? For if 74 *d.* give 1 Crown, 18500 *d.* (or 370 Ducats) will give 250 Crowns.

20 In 516 Dollars, at 4 *s*. — 5 *d.* the Dollar; how many Guineas at 1 *l.* — 15 *s*. — 6 *d.* the Piece? *Answer*, 106 Guineas, For if 258 *d.* give 1 Guinea, 27348 *d.* (or 516 Dollars,) will give 106 Guineas.

C H A P. IX.

Of the Inverse Rule of Three.

I. **T**HE Rule of *Three Inverse* is, when the fourth Term requir'd ought to proceed from the second Term, according to the same Rule or Proportion that the first proceeds from the third: So this Question being propounded, If 8 Horses will be maintained 12 Days with a certain Quantity of Provender, How many Days will the same Quantity maintain 16 Horses? Here, as 8 is half 16, so ought the fourth Term required to be half 12; for if certain Bushels of Provender serve 8 Horses 12 Days, 16 Horses will eat as much Provender in half that time: And therefore you cannot say here in a direct Proportion (as before in the Rule of Three Direct) as 8 to 16, so is 12 to ano- *horses days horses*
 ther Number which ought to be in that Case 8 — 12 — 16
 as great again as 12; but contrariwise by
 an *inverted Proportion*, beginning with the last Term first; as 16 is to 8, so is 12 to another Number, which ought to be in this Case half 12. And by the due Observation of this Definition, together with that of the Rule of Three Direct (laid down in the Ninth Rule of the Eighth Chapter) when any Question belonging to the Single Rule of Three is proposed, you may readily discern by which of those Rules it ought to be resolv'd; for if the Three Terms given look for a fourth in a direct Proportion as they stand ranked in the Rule, you must resolve the Question by the direct Rule; contrariwise when the Proportion is inverted or turned backwards, it ought to be resolved by the *Inverse Rule of Three*, which here follows.

I

II. In

II. In the Inverse Rule of Three, after the Three given Terms are rightly placed in the Rule, and reduced (if there be need) according to the Eleventh Rule of the Eight Chapter, multiply the first Term by the second, (or which is the same) the second Term by the first, and then divide the Product by the third Term, so the Quotient will give you the fourth Term required, or Answer of the Question; thus in the Question premised in the last Rule, if you multiply 12 by 8, the Product is 96, which if you divide by 16, the Quotient gives you 6, the fourth Term required, as by the subsequent Operation is manifest.

$$\begin{array}{ccccccc}
 \text{horses} & & \text{days} & & \text{horses} & & \text{days} \\
 8 & \text{---} & 12 & \text{---} & 16 & \text{---} & 6 \\
 & & 8 & & & & \\
 \hline
 & & 16 &) & 96 & (6 & \\
 & & 96 & & & & \\
 \hline
 & & 0 & & & &
 \end{array}$$

III. For the more ready discovering, whether a Question propounded belongs to the Rule of Three Direct, or to the Rule Inverse, observe the following Directions, viz. 1 By the Sense and Tenour of the Question consider, whether more be required or less; that is, whether the Number sought for must be greater or less than the second Term: Secondly, Esteeming the first and third Terms as Extremes in respect of the second, this will be a general Rule; namely, when more is required, the lesser Extreme is the Divisor; but when less is required, the greater Extreme is the Divisor. Lastly, the Divisor being found out, it will be apparent whether the Rule be Direct or Inverse; for when the Divisor is the first Term, it is a Rule Direct; but when the Divisor is the third Term, the Rule is Inverse. Another Example of the Rule Inverse may be this: If 12 Mowers do mow certain Acres in 4 Days, in what Time will 23 Mowers perform the same Work? Answer, 2 Days, 2 Hours, and somewhat more. Here, the 3 known Terms being rightly placed in the Rule will

$$\begin{array}{r}
 \text{M. D. M.} \\
 12 \text{---} 4 \quad 23 \\
 \quad 4 \\
 23) \quad 48 \text{ (2 Days.} \\
 \quad 46 \\
 \hline
 \quad 2 \\
 \quad 24 \\
 25) \quad 48 \text{ (2 Hours.} \\
 \quad 46 \\
 \hline
 \quad 2
 \end{array}$$

will stand as you see in the *Example*; and since it is evident that 23 Men will require less than 12 Men to finish the same Work, therefore (by the Rule afore-going) the greater of the Two extreme Numbers 23 and 12 must be the Divisor; and because the Divisor 23 stands in the third Place, this Question is to be work'd by the Rule Inverse; wherefore multiplying the first Term 12 by the second Term 4, the Product is 48, which being divided by the first Term 23, the Quotient gives 2 Days, and there is a Remainder of 2 Days, which being reduced to Hours, and those divided by 23, the Quotient will be 2 Hours; and there is yet a Remainder of 2 Hours to be subdivided into 23 Parts if you please; so that the fourth Term sought, or Answer of the Question is, 2 Days, 2 Hours, and somewhat more.

Again, Take this for a third *Example*, If I lend my Friend 356 Pounds for 1 Year and 35 Days, (the Year being supposed to consist of 365 Days,) how long time ought he to lend me 500 Pounds to requite my Courtesy? *Answer*, 284 Days and somewhat more, there being a Remainder, to wit 400, after the Division is finished, as by the subsequent Operation is manifest.

$$\begin{array}{r}
 \begin{array}{cccc}
 l. & y. & d. & l. \\
 356 & \text{---} & 1 : 35 & \text{---} 500
 \end{array} \\
 \hline
 & & 365 & \\
 & & \text{add } 35 & \\
 & & \hline
 \text{multiply } \left\{ \begin{array}{l} 400 \\ 356 \end{array} \right. & & & \\
 \hline
 5100)1424100(284 \text{ Days.}
 \end{array}$$

IV. The Proof of the Inverse Rule of Three is this, Multiply the third Term by the fourth, then if this Product be equal to the Product of the first Term multiplied by the second, the Work is true, otherwise erroneous; so in the *Example* of the second Rule, the Product of 16 and 6 is equal to the Product of 8 and 12. But if it happen that, after the fourth Term, or answer of the Question, is found in the same Denomination with the second Term, there is yet a Remainder, such Remainder must be added to the Product of the third Term, multiplied by the fourth, and then the Sum must be equal to the Product of the first and second Terms (such second Term being of the same particular Denomination with the fourth :) So in the last *Example*, the fourth Term is 284 Days, and there

The Proof of the Rule of Three Inverse.

remains 400 after the Division is finish'd, this 400 being added to the Product of the Multiplication of the the third Term 300 by the fourth Term 284, gives 142400 which is equal to the Product of the first Term 356, multiplied by the second Term 400 Days.

C H A P. X.

The Double Golden Rule Direct, performed by Two Single Rules.

I. **T**HE Compound Golden Rule is, when more than 3 Terms are proposed.

II. Under the Compound Golden Rule, is comprehended the Double Golden Rule, and divers Rules of plural Proportion.

III. The Double Golden Rule is, when Five Terms being given, a Sixth proportional to them is demanded: As in this Question, if 4 Students spend 19 Pounds in 3 Months, how much will serve 8 Students 9 Months? Or this, if 9 Bushels of Provender serve 8 Horses 12 Days how many Days will 24 Bushels last 16 Horses.

IV. The Five Terms given in this Rule consist of two Parts, *viz.* A Supposition expressed in the three first Terms; and a Demand made in the two last: So in the first Example of the last Rule, this Clause (if 4 Students spend 19 Pounds in 3 Months) is the Supposition, and this (how much will serve 8 Students 9 Months?) is the Demand: Likewise in the other Example of the same Rule, this Clause (if 9 Bushels of Provender serve 8 Horses 12 Days) is the Supposition, and this (How long, or how many Days will 24 Bushels last 16 Horses?) is the proposed Demand.

V. Here for ranking the Terms given in their due Order, first observe amongst the Terms of Supposition, which of them has the same Denomination with the Term required: Then reserving that Term for the second Place, write the other two Terms of Supposition one above another in the first Place; and lastly, the Terms of Demand likewise one above another in the third Place of the Rule, in such sort that the uppermost may have the same Denomination with the uppermost of those in the first Place. Example, If 4 Students spend 19 Pounds in 3 Months, how

how much will serve 8 Students 9 Months? Here the three Terms of Supposition are 4, 19 and 3, and of these Terms 19 has the same Denomination with the Term required, *viz.* of Pounds; for you are to enquire how much Money is requisite for the Maintenance of 8 Students 9 Months: Wherefore reserving 19 for the second Place, I write 4 and 3 one above another, thus; then drawing a Line 4—19 upon the Right-hand of 4, I write 19 in the second Place; this done, the work will stand as in the Margin: Last of all, the Terms of Demand being 8 and 9, and 8 having the Denomination of Students, I place it in the same Line with 4 and 19, and write 9 under it; all this performed, the Terms in this Question rank themselves as follows.

Viz, Thus,

$$\begin{array}{r} 4 \text{ — } 19 \text{ — } 8 \\ 3 \qquad \qquad 9 \end{array}$$

Or Thus,

$$\begin{array}{r} 3 \text{ — } 9 \text{ — } 9 \\ 4 \qquad \qquad 8 \end{array}$$

In like manner, if the second Question of the third Rule of this Chapter were propounded; the Terms thereof ought to be disposed

Thus,

$$\begin{array}{r} 8 \text{ — } 12 \text{ — } 16 \\ 9 \qquad \qquad 24 \end{array}$$

Or Thus,

$$\begin{array}{r} 9 \text{ — } 12 \text{ — } 24 \\ 8 \qquad \qquad 16 \end{array}$$

VI. Questions belonging to the double Golden Rule may be resolved by two single Rules of Three, or by the Golden Rule Compound of five Numbers.

VII. When Questions of this Nature are resolved by two single Rules, the Proportions are as follows: *The Proportions of the double Golden Rule, when 'tis performed by two single Rules.*

- I. As the uppermost Term of the first Place, is to the middle Term; so is the uppermost Term of the last Place to a fourth Number.
- II. As the lower Term of the first Place is to that fourth Number; so is the lower Term of the last Place to the Term required.

So

So in this Example before recited, using tacitly the lower Term of the first Place as a common Number in the first Proportion, *say thus*,

I. If 4 Students spend 19 Pounds (in 3 Months) what will serve 8 Students the same time?

Or thus, If 4 Students spend 19 Pounds, what will 8 spend?

Which *Rule of Three* will be discovered to be direct (by the third Rule of the Ninth Chapter;) therefore the fourth Proportional Proceeding from the said three given Numbers 4, 19, and 8 is 38 (by the 10th Rule of the 8th Chapter aforegoing.) Again, to find the Term required, using tacitly the uppermost Term of the third Place as a common Number in this last Proportion, *say as follows*.

II. If in three Months 38 Pounds are spent (by 8 Students) how much will serve them for 9 Months?

Or thus, if 3 give 38, what will 9 yield.

Which *Rule of Three* will likewise be discovered to be direct (by the third Rule of the 9th Chapter;) therefore the fourth Proportional proceeding from the said 3 Numbers, 3, 38, and 9, you'll likewise find (by the 10th Rule of the 8th Chapter before recited) to be 114; for 38 being multiplied by 9 the Product is 342, which divided by 3, yields in the Quotient 114: So that I conclude, if 4 Students spend 19 Pounds in 3 Months, 114 Pounds will serve 8 Students 9 Months, as you may further observe in the Work following:

$\begin{array}{ccc} 4 & 19 & 8 \\ 3 & & 9 \end{array} \quad (114$	
$\begin{array}{ccc} 4 & 19 & 8 \\ & 8 & \end{array} \quad (38$	$\begin{array}{ccc} 3 & 38 & 9 \\ & 9 & \end{array} \quad (114$
$\begin{array}{r} 4 \overline{) 152} \quad (38 \\ \underline{12} \\ 32 \\ \underline{32} \\ 0 \end{array}$	$\begin{array}{r} 3 \overline{) 342} \quad (114 \\ \underline{3} \\ 04 \\ \underline{3} \\ 12 \\ \underline{12} \\ 0 \end{array}$

In

In like manner, if two single Rules of Three be formed (according to the preceeding 7th Rule) out of the five Numbers given in the last mentioned Question, the same being ranked according to the latter Manner of ordering the said Numbers in the fifth Rule, each of the said two Rules of Three will be a Rule Direct, and the same Answer of the Question, to wit, 114 Pounds will be discovered, as may be seen by the subsequent Operation.

3 — 19 — 9	4 — 57 — 8
4 — 114	8 — 114
3 — 19 — 9 — (57	4 — 57 — 8 — (114
9	8
3) 171 (57	4) 456 (114
15	4
21	05
2	4
0	16
	16
	0

VIII. The Double Golden Rule is either *Direct*, or *Inverse*.

IX. The Double Golden Rule Direct, is, when both the Single Rules do each of them look for a fourth Term in a Direct Proportion: As in the Example of the Seventh Rule, where each of the two Single Rules of Three is a Rule Direct.

The double Golden Rule Direct.

For another Example take this, If the Carriage of 8 C. weight 128 Miles, cost 48 Shillings, for how much may I have 4 C. weight carried 32 Miles after the same Rate? The Terms of this Question, according to the fifth Rule of this Chapter, rank themselves in this Order.

128 — 48 — 52
8 — 4

Now taking tacitly the lower Term of the first Place as a common Number, I form the first Rule of Three according to the seventh Rule; saying.

I. If

I. If the Carriage of a certain Weight (to wit, 8 C.) 128 Miles, costs 48 Shillings, what will the Carriage of the same Weight 32 Miles cost?

Here it is easy to discern, that the fewer Miles any Weight is carried, the less Money will pay for the Carriage of that Weight; therefore the fourth Number sought by the said Rule of Three must be less than the second Number 48: And in regard that by the third Rule of the Ninth Chapter, when less is required, the greater Extreme (whether it be the first or third Number) must be the Divisor; therefore the first Number 128 is the Divisor, and consequently the Rule of Three above proposed, is a Rule Direct; then finding out the fourth Number by the tenth Rule of the Eighth Chapter, to be 12 Shillings, I proceed to the second Proportion, and say,

II. If the Carriage of 8 C. (32 Miles) cost 12 Shillings, how much must I give to have 4 C. carried the same Distance?

And here likewise finding a fourth Number to be looked for in a direct Proportion, I discover that fourth, by the said tenth Rule of the Eighth Chapter, to be 6s. which is the Term demanded, and the Answer to the Question propounded: So that at last I conclude, if the Carriage of 8 C. 128 Miles cost 48s. the Carriage of 4 C. 32 Miles will cost 6s. according to the same Rate: See the whole Work.

$$\begin{array}{r}
 128 \text{ ——— } 48 \text{ ——— } 32 \\
 8 \qquad \qquad \qquad 4 \text{ — } (6 \\
 \hline
 128 \text{ ——— } 48 \text{ ——— } 32 \text{ — } (12 \\
 \qquad \qquad \qquad 32 \\
 \qquad \qquad \qquad \hline
 \qquad \qquad \qquad 96 \\
 \qquad \qquad \qquad 144 \\
 \qquad \qquad \qquad \hline
 128) 1536 (12 \\
 \qquad \qquad 128 \\
 \qquad \qquad \hline
 \qquad \qquad 256 \\
 \qquad \qquad 256 \\
 \qquad \qquad \hline
 \qquad \qquad (0 \\
 \hline
 8 \text{ ——— } 12 \text{ ——— } 4 \text{ ——— } (6 \\
 \qquad \qquad \qquad 4 \\
 \qquad \qquad \qquad \hline
 8) 48 (6 \\
 \qquad \qquad 48 \\
 \qquad \qquad \hline
 \qquad \qquad 0
 \end{array}$$

Chap. XI.

The double Golden Rule Inverse, performed by two Single Rules.

THE *Double Golden Rule Inverse*, is, when one of the Single Rules looks for a fourth Term in an inverted Proportion: As in the last *Example* proposed in the fifth Rule of the last Chapter. For if you rank the Terms of that Question, according to the said fifth Rule, thus:

The double Golden Rule Inverse.

8 ——— 12 ——— 16
9 24

And then work by two Single Rules of Three, formed according to the seventh Rule of the last Chapter, you'll find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be Inverse, and the latter Direct. For saying first, if 8 Horses be maintained 12 Days (by 9 Bushels of Provender,) how many Days will 16 Horses be kept by so much Provender? Here the Answer 6 Days will be found out by the Rule of Three Inverse: *Secondly*, Saying, If 9 Bushels of Provender be eaten up (by 16 Horses) in 6 Days, in how many Days will 24 Bushels be spent? Here the Answer 16 Days will be found out by the *Rule of Three Direct*.

But if you order the given Terms of the same Question, Thus:

9 ——— 12 ——— 24
8 16

And then work by two *Single Rules of Three*, formed according to the seventh Rule of the last Chapter; you'll find by the third Rule of the ninth Chapter, that the first of the said two Rules of Three will be Direct, and the latter Inverse. For saying, first, if 9 Bushels of Provender last 12 Days (to maintain 8 Horses) how many Days will 24 Bushels serve the same Number of Horses? The Answer 32 Days will be found out by the Rule of Three Direct. *Secondly*, saying, If 8 Horses are maintained 32 Days, (by 24 Bushels of Provender,) how long will 16 Horses be kept by the same Quantity of Provender? Here the Answer 16 Days will be found out by the Rule of Three Inverse.

K

Therefore,

Therefore, whensoever a Question belonging to the *Double Rule of Three* is separated into two Single Rules of Three, (according to the preceeding Rules) if one happens to be a Rule Inverse, that Double Rule is called the *Double Rule Inverse*.

Now the Resolution of the Question propos'd being ranked after the first Manner, is as follows.

$$\begin{array}{r} 8 \text{ --- } 12 \text{ --- } 16 \\ 9 \text{ --- } 24 \text{ --- } (16 \end{array}$$

$$\begin{array}{r} 8 \text{ --- } 12 \text{ --- } 16 \text{ --- } (6 \\ \quad \quad \quad 8 \\ \hline 16) \ 96 \ (6 \\ \quad \quad 96 \\ \hline \quad \quad 0 \end{array}$$

$$\begin{array}{r} 9 \text{ --- } 6 \text{ --- } 24 \text{ --- } (16 \\ \quad \quad \quad 6 \\ \hline 9) \ 144 \ (16 \\ \quad \quad \quad 9 \\ \hline \quad \quad 54 \\ \quad \quad 54 \\ \hline \quad \quad 0 \end{array}$$

Again, The Resolution of the same Question, being ranked after the last Manner; is this;

$$9 \text{ --- }$$

$$\begin{array}{r} 9 \text{ --- } 12 \text{ --- } 24 \\ 8 \qquad \qquad 16 \text{ --- } (16 \end{array}$$

$$\begin{array}{r} 9 \text{ --- } 12 \text{ --- } 24 \text{ --- } (32 \\ 12 \end{array}$$

$$\begin{array}{r} 48 \\ 24 \\ \hline 9) 288 (32 \\ \cdot \cdot \end{array}$$

$$27$$

$$\begin{array}{r} 18 \\ 18 \\ \hline \end{array}$$

$$0$$

$$\begin{array}{r} 8 \text{ --- } 32 \text{ --- } 16 \text{ --- } (16 \\ 8 \end{array}$$

$$\begin{array}{r} 16) 256 (16 \\ 16 \end{array}$$

$$\begin{array}{r} 96 \\ 96 \\ \hline \end{array}$$

$$0$$

So that at last I say, If 9 Bushels of Provender serve 8 Horses 12 Days, 24 Bushels will last 16 Horses 16 Days, which is the Resolution of the Question propounded.

CHAP. XII.

The Golden Rule Compound of Five Numbers.

1. **T**HE Golden Rule compound of Five Numbers, is, when the Terms being ranked, as before, instead of the Double Terms, we use their Products, and then proceed to find the Term required by one Single Rule of Three.

K 2

II. Her

II. Here, when the Question propos'd ought to be performed by the Double Rule Direct, multiplying the

The Golden Rule Compound Terms of the first Place, the one by the other, take their Product for the first Term, the middle Number for the second, and the Product of the

two last Terms for the third Term; that done, having found by the Rule of Three Direct, a fourth Proportional to those three, that fourth Term so found is the Number you look for; so this Question being again propounded, If 4 Students spend 19*l.* in 3 Months, how much will serve 8 Students 9 Months? And the Terms thereof being ranked as before, viz. thus;

$$\begin{array}{ccc} 4 & \text{---} & 19 & \text{---} & 8 \\ 3 & & & & 9 \end{array}$$

The Product of 4, multiplied by 3, is 12, and the Product of 8 multiplied by 9 is 72; wherefore I say, As 12 to 19, so is 72 to the Term required, which I find by the Single Rule of Three Direct to be 114. So that if 4 Students spend 19*l.* in 3 Months, 114*l.* will be requisite for the Maintenance of 8 Students 9 Months; see the whole Operation, as follows.

$$\begin{array}{r} \begin{array}{ccc} 4 & \text{---} & 19 & \text{---} & 8 \\ 3 & & & & 9 \end{array} \\ \hline 12 & & & & 9 \text{---} (114 \\ & & & & \hline & & & & 72 \\ & & & & 19 \\ & & & & \hline & & & & 648 \\ & & & & 72 \\ & & & & \hline 12) & 1368 & (114 \\ & \cdot \cdot \cdot & \\ & 12 & \\ & \hline & 16 & \\ & 12 & \\ & \hline & 48 & \\ & 48 & \\ & \hline & 0 & \end{array}$$

In like manner this being the Question as before, (in the last Rule of the Tenth Chapter,) if the Carriage of 8 C. 128 Miles, cost 48 s. What will the Carriage of 4 C. 32 Miles stand me in? the Answer thereto will be 6 s. as appears by the Work.

$$\begin{array}{r}
 128 \quad \text{---} \quad 48 \quad \text{---} \quad 32 \\
 8 \quad \quad \quad 128 \quad \quad \quad 4 \\
 \hline
 1024 \quad \quad 384 \quad \quad 128 \\
 \quad \quad \quad 96 \\
 \quad \quad \quad 48 \\
 \hline
 1024) \quad 6144 \quad (6 \text{ Shillings.} \\
 \quad \quad 6144 \\
 \hline
 \quad \quad \quad 0
 \end{array}$$

III. When the Question proposed ought to be resolv'd by the double Rule Inverse, having multiplied the double Terms a-cross, that is, the uppermost Term of the first Place by the lower of the last, and the uppermost of the last Place by the lower of the first; write each Product under the lower Term by which it is produced: And then if the Inverse Proportion be found in the uppermost Line, using those Products as single Terms proceed to find the Term required by the Single Rule of Three Direct: But in case you find the Inverse Proportion in the lower Line, perform the Work by the Single Rule of Three Inverse.

The Golden Rule compound of Five Numbers perform'd by one single Rule Direct or Inverse.

So in the Example above-mentioned, if 9 Bushels of Provender serve 8 Horses 12 Days, how long will 24 Bushels last 16 Horses? Here if you rank the Terms *thus*, you'll find the Inverse Proportion in the first Line, as is observed in the last Chapter: And therefore having subscribed the Products according to the Direction given in this Rule, I proceed to satisfy the Demand of this Question by the single Rule of Three Direct, as appears by the following Work.

$$\begin{array}{r}
 8 \text{ --- } 12 \text{ --- } 16 \\
 9 \text{ --- } 24 \text{ --- } 16 \\
 \hline
 144 \qquad 192 \\
 \qquad 12 \\
 \hline
 \qquad 384 \\
 \qquad 192 \\
 \hline
 144) 2304 (16 \\
 \qquad 144 \\
 \hline
 \qquad 864 \\
 \qquad 864 \\
 \hline
 \qquad 0
 \end{array}$$

But the Terms of this Question being ranked *thus*; the Inverse Proportion is found in the lower Line, as
 9—12—24 you may observe likewise by the last Chapter:
 8 16 Whereupon in this Case, to resolve the Question,
 I proceed by the Single Rule of Three Inverse,
 as appears by the Work hereto annexed: Howsoever therefore
 you work the Question, you'll find the Term required to be
 16; so that at last I conclude, as before in the last Chapter, If
 9 Bushels of Provender serve 8 Horses 12 Days, 24 Bushels will
 last 16 Horses 16 Days.

$$\begin{array}{r}
 9 \text{ --- } 12 \text{ --- } 24 \\
 8 \text{ --- } 16 \text{ --- } 16 \\
 \hline
 192 \qquad 144 \\
 \qquad 12 \\
 \hline
 \qquad 384 \\
 \qquad 192 \\
 \hline
 144) 2304 (16 \\
 \qquad 144 \\
 \hline
 \qquad 864 \\
 \qquad 864 \\
 \hline
 \qquad 0
 \end{array}$$

C H A P. XIII.

The Rule of Fellowship.

I. **T**HE Rules of plural Proportion are those, by which we resolve Questions that are discoverable by more Golden Rules than one, and yet cannot be performed by the double Golden Rule mentioned before in the Three last Chapters. Of these Rules there are diverse Kinds and Varieties according to the Nature of the Question proposed; for here the Terms given are sometimes Four, sometimes Five, sometimes more, and the Terms required sometimes more than one, &c.

*Rules of plural
Proportion.*

II. Two particular Rules of plural Proportion are these, the Rule of Fellowship, and the Rule of Alligation.

III. The Rule of Fellowship is that, by which in Accounts among divers Men (their several Stocks together with the whole Gain or Loss being given) the Gain or Loss of each particular Man may be discovered: As in this Example, *A* and *B* were Sharers in a Parcel of Merchandize, in the Purchase of which, *A* laid out 7*l.* and *B* 11*l.* and they having sold this Commodity, find that their clear Gains amount to 54*s.* Now here the Question to be resolved by this Rule is, What Part of that 54*s.* belongs to *A*, and what to *B*, according to the Rate of the several Sums and Stocks which they adventur'd? Again, *A*, *B*, and *C*, freight a Ship from the *Canaries* for *England* with 108 Tuns of Wine, of which *A* had 48, *B* 36, and *C* 24, the Mariners meeting with a Storm at Sea, were constrained for the Safety of their Lives, to cast 45 Tuns thereof over-board; here the Question to be resolved is, How many of the 45 Tuns every particular Merchant has lost, according to the Rate of his Adventure.

*The Rule of
Fellowship.*

IV. The Rule of Fellowship is either Single, or Double.

V. The single Rule is, when the Stocks proposed continue in the Adventure (or common Bank) equal Times, to wit, one stock as long Time as another.

VI. In

How to work the single Rule. VI. In the single Rule of Fellowship, take the Total of all the Stocks for the first Term, the whole Gain or Loss, for the second, and the particular Stocks for the third Term; that done, repeating the Rule of Three so often, as there are particular Stocks in the Question, the fourth Terms produced upon those several Operations, are the respective Gains or Losses of those particular Stocks propounded: So in the first Example above-mentioned 7 l. and 11 l. are the Stocks proposed, whose total is 18 l. which I take for the first Term: Again, 54 s. the common Gain, is the second Term, and 7 l. the first particular Stock, is the third Term of the first Proportion; whereupon I say, as 18 l. to 54 s. so 7 l. to another Number, which by the Direct Rule of Three I find to be 21 s. viz. the Part of the Gain due to A, who expended the 7 l. Stock. Then for the second Proportion, I say, as 18 l. to 54 s. so 11 l. to another Number, which I likewise find by the Rule of three Direct to be 33 s. viz. the Part of the Gain due to B, for his 11 l. Stock.

$$\begin{array}{r} 7 \\ 11 \end{array} \left. \vphantom{\begin{array}{r} 7 \\ 11 \end{array}} \right\} 18 \text{ ————— } 54 \left\{ \begin{array}{r} 7 \text{ ————— } 21 \\ 11 \text{ ————— } 33 \end{array} \right.$$

Again, in the other premised Example, the particular Loss that happens to A, is 20 Tuns, to B, 15, and to C, 10 Tuns.

$$\begin{array}{r} 48 \\ 36 \\ 24 \end{array} \left. \vphantom{\begin{array}{r} 48 \\ 36 \\ 24 \end{array}} \right\} 108 \text{ — } 46 \text{ — } \left\{ \begin{array}{r} 48 \text{ — } 20 \\ 36 \text{ — } 15 \\ 24 \text{ — } 10 \end{array} \right.$$

VII. The Double Rule of Fellowship is, when
2. *Double.* the Stocks proposed are double Numbers, viz. when each Stock has Relation to a particular Time:
Example, A, B, and C, hold a Pasture in common, for which they pay 45 l. per. Annum. In this Pasture A had 24 Oxen during 32 Days; B. had 12 there 48 Days, and C. fed 16 Oxen there 24 Days: Now the Question to be resolved by this Rule is, what Part each of these Tenants ought to pay of the 45 l. Rent? And here you may observe, that the Stocks propounded are double Numbers, viz. each Stock of Oxen has Reference to a particular Time; for the respective Stock of A. is 24 Oxen, and its particular Time is 32 Days; again, the Stock of B, is 12 Oxen, and the respective Time is 48 Days: And lastly, the Stock of C. is 16 Oxen, and its peculiar Time is 24 Days, which as you see are double Numbers.

VIII. In

VIII. In the Double Rule of Fellowship, multiply each particular Stock by its respective Time, and take the Total of their Products for the first Term, the whole Gain or Loss for the second, and the said particular Products of the Double Numbers for the third Term: This done, repeating, as before, the *Rule of Three*, so often as there are Products of the Double Numbers; the fourth Terms produced upon those several Operations, are the Numbers you look for: So in the Example of the last Rule, the Product of 24 and 32 is 768, the Product of 12 and 48 is 576, and the Product of 16 and 24 is 384, the Sum of these Products is 1728, which is the first Term in the Question; then 45 *l.* the Rent, is the second Term, and 768 the first Product, is the third Term of the first proportion: Wherefore I say, as 1728 to 45 *l.* so 768 to another Number, which I find by the *Direct Rule of Three* to be 20 *l.* viz the Part of the Rent that A ought to pay: Then for the second Proportion I say, as 1728 to 45 *l.* so 576 to 15 *l.* which is the Part that B. ought to pay: And lastly, as 1728 to 45 *l.* so is 384 to 10 *l.* viz. the Part that C. must pay.

How to work the Double Rule.

$$\begin{array}{r} 768 \\ 576 \\ 384 \end{array} \left. \vphantom{\begin{array}{r} 768 \\ 576 \\ 384 \end{array}} \right\} 1728 \text{ ——— } 45 \text{ ——— } \left. \vphantom{\begin{array}{r} 768 \\ 576 \\ 384 \end{array}} \right\} \begin{array}{r} 768 - 20 \\ 576 - 15 \\ 384 - 10 \end{array}$$

A second Example of the Eighth Rule: Three Merchants A. B. and C. enter Partnership, and agree to continue in a joint Adventure 16 Months; A puts into the common Stock at the Beginning of the said Term 100 Pounds, at 8 Months End he takes out 40 Pounds, and 4 Months after such taking out he puts in 140 Pounds. B, puts in at first 200 Pounds, at 6 Months End he puts in 50 Pounds more, and 4 Months after the putting in of the 50 Pounds, he takes out an 100 Pounds. C. puts in at first 150 Pounds, at 4 Months End he takes out 50 Pounds, and 8 Months after such taking out puts in 100 Pounds. Now at the End of the said 16 Months, they had gained 357 Pounds, the Question is, How much of the said Gains belongs to every Merchant for his Share?

In Questions of this Nature, Two Things are principally to be observed: 1. The whole Time of Partnership. 2. The respective Time belonging to each Man's particular Stock; so here, it is evident that the whole Time is 16 Months, and the particular Stocks and Times belonging to every Merchant will be as follows. viz.

L

A had

A. Had 100 *l.* in the common Stock for 8 Months, }
 therefore 100 multiplied by 8 produces _____ } 800
 Also 60 *l.* for 4 Months, therefore 60 multiplied by }
 4 produces _____ } 240
 Also 200 *l.* for 4 Months, therefore 200 multiplied }
 by 4 produces _____ } 800
 The Total of the Products of Money and Time for
 A. is _____ } 1840

B. Had 200 *l.* in the common Stock for 6 Months, }
 therefore 200 multiplied by 6 produces _____ } 1200
 Also 250 *l.* for 4 Months, therefore 250 multiplied }
 by 4 produces _____ } 1000
 Also 150 *l.* for 6 Months, therefore 150 multiplied }
 by 6 produces _____ } 900
 The Total of the Products of Money and Time for
 B. is _____ } 3100

C. Had 150 *l.* in the common Stock for 4 Months, }
 therefore 150 multiplied by 4 produces _____ } 600
 Also 100 *l.* for 8 Months, therefore 100 multiplied }
 by 8 produces _____ } 800
 Also 200 *l.* for 4 Months, therefore 200 multiplied }
 by 4 produces _____ } 800
 The Total of the Products of Money and Time for
 C. is _____ } 2200

Then adding the said *Three Totals* together, to wit, 1840, 3100, and 2200, the Sum is 7140; wherefore proceeding as in the last Example, I say, by the *Rule of Three Direct*, as 7140 is to the total Gain 357 Pounds; so is 1840 to 92 Pounds the Gain of A: Again, As 7140 is to 357; so is 3100 to 155 the Gain of B: Lastly, as 7140 is to 357; so is 2200 to 110 the Gain of C.

IX. The *Rule of Fellowship* is proved by Addition of the Terms required, whose Sum ought to be equal to the second Term in the Question, otherwise the whole Work is erroneous: So in the First Example of the Sixth Rule afore-going, 21*s.* and 33*s.* being added together, are equal to 54*s.* the second Term in that Question. Likewise in the last Example of the same Rule, as also in the first Example of the last Rule, the Sum of 20, 15, and 10, the Terms required, are equal to 45, the second Term propounded.

C H A P. XIV.

Of the Rule of Alligation.

I. **T**HE Rule of *Alligation* is that, by which we resolve Questions, that concern the mixing of diverse Simples together.

II. *Alligation* is either Medial, or Alternate.

III. *Alligation Medial* is, when having the several Quantities and Rates of diverse Simples proposed, we discover the mean Rate of a Mixture compounded of those Simples. So 10 Bushels of Wheat at 4 s. or (which is all one) 48 d. the Bushel; 40 Bushels of Rye at 3 s. or 36 d. the Bushel; and 50 Bushels of Barley at 2 s. or 24 d. the Bushel; being mixed with 20 Bushels of Oats at 12 d. the Bushel, the Rule of *Alligation Medial* shews you the mean Price of that Mixture.

IV. In *Alligation Medial*, First sum up the given Quantities, then find the total Value of all the Simples: That done, the Proportion will be as follows.

Alligation Medial.

The Operations and Proportions of the same Rule.

As the Sum of the Quantities is to the total Value of the Simples:

So is any Part of the Mixture proposed, to the required mean Rate or Price of that Part.

Repeating again the premised Example of the third Rule, I demand how much one Bushel of that Mixture is worth? Now the Sum of 10, 40, 50, 20, (the given Quantities) is 120 Bushels, and the Value of the 10 Bushels of Wheat at 48 d. the Bushel, amounts to 480 d. for 48 being multiplied by 10, the Product is 480: Again the Value of the 40 Bushels of Rye at 36 d. the Bushel, is 1440 d. The Value of the 50 Bushels of Barley at 24 d. the Bushel, is 1200 d. And the Value of 20 Bushels of Oats at 12 d. the Bushel, is 240 d. All these Values being added together, their Total is 3360 d. I say then, by the *Rule of Three Direct*, If a 120 Bushels give 3360 d. what will 1 Bushel yield? The Rule presently answers me 28 d. Whereupon I conclude, that a Bushel of that Mixture may be afforded for 28 d. that is, 2 s. 4 d. which is the Resolution of the *Question proposed*.

$$120 \text{ ————— } 3360 \text{ ————— } 1 \text{ ————— } 28.$$

In like manner, if it be demanded, what 8 Bushels or a Quarter of that Mixture is worth, the *Answer* will be 224 *d.* which being divided by 12, and by that Means reduced into *Shillings* is 18 *s.* 8 *d.*

$$120 \text{ ————— } 3360 \text{ ————— } 8 \text{ ————— } 224$$

V. In *Alligation Medial*, the Tryal of the Work is by comparing the total Value of the several Simples with
The Proof. the Value of the whole Mixture: For when those Sums accord, the Operation is perfect: So in the first Example of the last Rule:

		<i>l.</i>	<i>s.</i>	<i>d.</i>
The Value of	10 Bushels of Wheat at 4 <i>s.</i> the Bushel, is	2	0	0
	40 Bushels of Rye at 3 <i>s.</i> the Bushel, is	6	0	0
	50 Bushels of Barley at 2 <i>s.</i> the Bushel, is	5	0	0
	And 20 Bushels of Oats at 12 <i>d.</i> the Bushel, is	1	0	0

All which amount to ————— 14 — 0 — 0
 which is likewise the Value of 120 Bushels at 28 *d.* or 2 *s.* 4 *d.* the Bushel, for that also amounts to 14 *l.*

VI. *Alligation Alternate* is, when having the several Rates of diverse Simples given, we discover such Quantities of them, as are necessary to make a Mixture, which may bear a certain Rate propounded.

Example: A Man being determined to mix 10 Bushels of Wheat at 4 *s.* or 48 *d.* the Bushel, with Rye of 3 *s.* or 36 *d.* the Bushel, with Barley of 2 *s.* or 24 *d.* the Bushel, and with Oats of 1 *s.* or 12 *d.* the Bushel; the Rule of *Alligation Alternate* will discover to you how much Rye, how much Barley, and how much Oats he ought to add to the 10 Bushels of Wheat, in such sort, that the Mixture of them all together may bear a certain Rate or Price proposed.

VII. In Questions of *Alligation Alternate*, you must rank the Terms after such a Manner, that the given Rate of the Mixture may represent the Root, and the several Rates of the Simples may stand as Branches issuing from that Root: So the
Example of the last Rule being laid down, I demand how much Rye, Barley, and Oats, ought to be added to the 10 Bushels of Wheat, that the Mixture of all together may bear the Rate or Price of 28 *d.* or 2 *s.* 4 *d.* the Bushel: And therefore drawing a Line of Connexion, I place 28 *d.* the given

given Rate of the Mixture, upon the left Hand thereof, by it self, representing the *Root*, and likewise write the other Rates proposed, viz: 48 d. 36 d. 24 d. and 12 d. one above another upon the Right-hand of that Line of Connexion, which Rates are conceived to issue from 28 d, as Branches from the *Root*, the Fabrick whereof appears plainly in the Margin.

$$28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$$

VIII. Having ranked the Terms in their due Order, link the Branches together by certain Arches, in such sort, that one that is greater than the *Root* or Rate of the Mixture, may always be coupled with another that is less than the same: So in the premised Example, 48 may be linked with 12, and 36 with 24, or otherwise 48 may be coupled with 24, and 36 with 12, and then the work will stand

How to couple the Terms.

Thus, 28 $\left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$

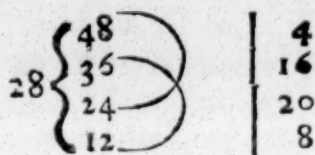
Or thus, 28 $\left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$

IX. Having alligated the Branches, and found the Differences betwixt them and the Root, write the Differences of each Branch just against its respective Yoke-fellow. So the Branches of the Example afore-going being linked after the first Manner, and the Difference between 28 and 48 (by the third or fourth Rule of the fourth Chapter of this Book) being 20, I place 20 just against 12, the respective Yoke-fellow of 48. Again, 16, being the Difference between 28 and 12, I write it just against 48. In like manner 8 being the Difference between 28 and 36, I place it right against 24. And lastly, 4, the Difference between 28 and 24, I write just against 36: In the end the whole *Fabrick* of the Work (as the Branches are thus linked) will stand as in this Example.

How to order the Differences.

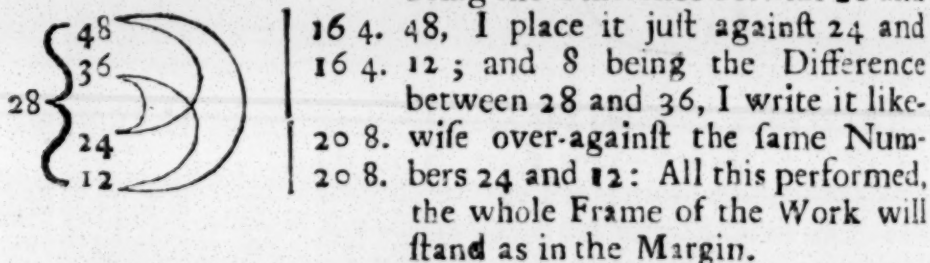
28 $\left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right.$ | $\begin{array}{l} 16 \\ 4 \\ 8 \\ 20 \end{array}$

But the Branches being linked after the other Manner, the Work will be thus disposed:



For in this Case 48 has 24 for its Yoke-fellow, and the respective Comrade of 36 is 12; and here the interchangeable placing of the Differences (as in the premised Examples) is that which is more particularly termed *Alternation*.

X. When one Branch is linked to diverse other Branches, and not to one alone, the Differences ought to be as often transcribed, as it is so diversely linked. So in the premised Example, you may (if you please) conceive 12 to be coupled both with 48 and 36; likewise 24 may be conceived to be linked with the same 48 and 36; wherefore the Difference between 28 and 12 being 16, I write it both just against 48 and 36. In like Manner the Difference between 28 and 24 being 4, I write it likewise over against the same Numbers 48 and 36. Again, 20 being the Difference betwixt 28 and



16 4. 48, I place it just against 24 and
16 4. 12; and 8 being the Difference
between 28 and 36, I write it like-
wise over-against the same Num-
bers 24 and 12: All this performed,
the whole Frame of the Work will
stand as in the Margin.

2. Take this for another Example: It is required to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats of 12 *d.* the Bushel; and the Question now is, How much Rye, Barley, and Oats ought to be added to the 10 Bushels of Wheat, that the entire Mixture may be afforded at 16 *d.* the Bushel? Here the Branches of this Question (according to the Eight Rule of this Chapter) ought to be liked *thus*.



And as for the *Alternation* of the Differences, it is evident (by the present Rule.) that the Difference between 16 and 12 being 4, ought to be thrice transcribed, *viz.* first, just against 48, then

48, then against 36, and last of all against 24. Again, 32 the Difference between 16 and 48, as also 20 the Difference between 16 and 36; and lastly, 8 the Difference betwixt 16 and 24, ought all to be placed just against 12.

$$\begin{array}{r|l} \begin{array}{l} 48 \\ 36 \\ 24 \\ 2 \end{array} & \begin{array}{l} 4 \\ 4 \\ 4 \\ 32.20.8. \end{array} \end{array}$$

3. I determining to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, desire to know how much of each I ought to take, that I may afford the whole Mixture at 40 *d.* the Bushel: Here the whole Work being ordered according to the Rules fore-going, it will stand as follows.

$$\begin{array}{r|l} \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} & \begin{array}{l} 4. 16. 28. \\ 8 \\ 8 \\ 8 \end{array} \end{array}$$

4. A Man intending to mix 10 Bushels of Wheat at 48 *d.* the Bushel, with Rye of 36 *d.* the Bushel, with Barley of 24 *d.* the Bushel, with Pease of 16 *d.* the Bushel, and with Oats of 12 *d.* the Bushel, desires to know how much Rye, Barley, Pease, and Oats he ought to add to the 10 Bushels of Wheat, that the whole Mass of Corn so mixed may be afforded at 20 *d.* the Bushel. This Question being thus proposed, the Terms of it (by the Rules fore going) may be *alligated*, and the Differences of the Terms *alternated*. as follows.

$$\begin{array}{r|l} \begin{array}{l} 48 \\ 36 \\ 24 \\ 16 \\ 12 \end{array} & \begin{array}{l} 4 \\ 4 \\ 4. 8. \\ 28. 16. 4. \\ 4 \end{array} \end{array}$$

5. Lastly, A Goldsmith has some Gold at 24 *Carats*, other of 21 *Carats*, and other some of 19 *Carats* fine; which he would

would so mix with *Alloy*, that 192 Ounces of the entire *Mixture* may bear 17 *Carets fine*; Now, the *Question* is, How much of every Sort, as also how much *Alloy* he must take to accomplish his Desire? Before you can well understand *What a Caret* this *Question*, it will be necessary to explain *fine*, and *what* what a *Caret fine*, and what *Alloy* is: The *Alloy* is. Mint-Masters and Goldsmiths to distinguish the different *Fineness* of Gold, esteem an entire Ounce to contain 24 *Carets*, and one Ounce of Gold, that being tried in the Fire, loses nothing of the Weight, is said to be 24 *Carets fine*: Again the Ounce that being tried, loses one four and twentieth Part of the Weight, is said to be 23 *Carets fine*: In like manner, that which loseth two four and twentieth Parts of the Ounce, is esteemed to be 22 *Carets fine*, and so consequently of the Rest: And as for the *Alloy*, it is Silver, Copper, or some other baser Metal, with which the Goldsmiths use to mix their Gold, to the intent they may moderate, or abate the *Fineness* of it, Here you may also observe, that as the *Fineness* of Gold is measured by *Carets*, so is the *Fineness* of Silver estimated by *Ounces*: In such sort, that a Pound of Silver, which, being try'd a certain Time in the Fire, loses Nothing of the Weight, is said to be 12 Ounces *fine*: But a Pound, that being try'd, loses somewhat of the Weight, is said to be the *Remainder* of the *Weight fine*. Example, A Pound of Silver, that loses in the Fire one Ounce 8 p. w. is estimated to be 10 Ounces 12 p. w. *fine*, and that which loses 2 Ounces 8 p. w. 10 Grains, is said to be 9 Ounces, 11 p. 14 Grains *fine*, &c. Now, to rank the Terms of the last mentioned *Question*, as also the Differences of the Terms in their due Order, because the three given Branches (*viz.* 24 *Carets*, 21 *Carets*, and 19 *Carets*) are all greater than 17 *Carets* the *Root* or *Rate* of the *Mixture*: I add 0 as another Branch, which I conceive to be less than the *Root*, and then proceed as in the former Operations; the wole *Frame* of the Work is expressed here, as follows.

$$\begin{array}{r|l}
 \begin{array}{c} 24 \\ 21 \\ 19 \\ 0 \end{array} \left. \vphantom{\begin{array}{c} 24 \\ 21 \\ 19 \\ 0 \end{array}} \right\} 17 & \begin{array}{l} 17 \\ 17 \\ 17. \\ 7. 4. 2. \end{array}
 \end{array}$$

XIth When

XI. When in one and the same Line there are found more Differences than one, add them together, and write the Sum just against the same Differences before a straight Line drawn towards the right hand of the Work.

How to add the Differences.

So the first Example of the last Rule being proposed, the Sum of 16 and 4 (the Differences placed just against the first Branch) being 20, I write it over-against the same Differences, before the new Line drawn upon the Right-hand of the Work, and so consequently the Rest in their due Order, as appears by the Example hereunto annexed.

28 {	48		16 4.		20.
	36		16 4.		20.
	24		20 8.		28.
	12		20 8.		28.

In like manner the last Example of the last Rule being offered, the whole Fabrick of the Work will stand as follows :

17 {	24		17		17
	21		17		17
	19		17		17
	0		7. 4. 2		13

XII. Alligation Alternate, is, either Partial, or Total.

XIII. Alternation Partial, is, When having the several Rates of diverse Simples, and the Quantity of one of them given, we discover the several Quantities of the Rest, in such sort, that a Mixture of those Simples being made according to the Quantity given, and the Quantities so found, that Mixture may bear a certain Rate proposed : Of this Kind is the Example of the sixth Rule, as also all the Examples of the tenth Rule except the last.

Alternation Partial.

XIV. In Questions of Alternation Partial, the Proportion is as follows.

The Proportions used in this Rule.

As the Difference annexed to the first Branch is to the several Differences of the rest :

So is the Quantity proposed to the several Quantities required.

So the Example of the sixth and seventh Rules of this Chapter being again repeated, and the Terms of it ; as also the Dif-

M

ferences

ferences of Terms being ordered after the first manner (shew'd

The first
Case.



you in the Ninth Rule
afore-going) it is evident
that for every 16 Bush-
els of Wheat, that I
take in the Mixture, I
ought to take 4 Bushels

of Rye, 8 Bushels of Barley, and 20 Bushels of Oats; and
therefore I say,

- I. As 16 the Difference annexed to the first Branch (being the Rate of the Wheat) is to 4, the Difference join'd to the next, being the Rate of the Rye; so is 10 the given Quantity of the Wheat to another Number, which being found, by the Rule of *Three Direct*, to be two Bushels, and an half (or two Pecks) is the Quantity of Rye necessary in the *Mixture*.
- II. As 16 to 8, so is 10 to another Number, which being likewise found by the Rule of *Three* to be five Bushels, is the Quantity of Barley necessary in the *Mixture*.
- III. As 16 to 20, so is 10 to another Number, which being in like sort found by the Rule of *Three* to be 12 Bushels and half of a Bushel, is the Quantity of Oats requisite in the *Mixture*.

So that at last I conclude, a Heap of Corn composed of 10 Bushels of Wheat, 2 Bushels and a half of Rye, 5 Bushels of Barley, and 12 Bushels and a half of Oats (when those several Grains bear the *Prices* aforesaid) may be afforded at 2 s. 4 d. the Bushel.

The same Example being ordered after the
2d Case. second manner (expressed likewise in the 9th Rule of this present Chapter) I say,

- I. As 4 the Difference annexed to the Rate of the Wheat, is to 16 the Difference set to the Rate of the Rye; so is 10 the given Quantity of the Wheat, to 40 Bushels the required Quantity of the Rye.
- II. As 4 to 20, so is 10 to 50 Bushels the requisite Quantity of the Barley.
- III. As 4 to 8, so is 10 to 20 Bushels the Quantity of the Oats necessary in the *Mixture*.

$$\begin{array}{r|l}
 28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right. & \begin{array}{l} 4 \\ 16 \\ 20 \\ 8 \end{array}
 \end{array}$$

So that I conclude again, a Mals of Corn being compounded of 10 Bushels of Wheat, 40 Bushels of Rye, 50 Bushels of Barley, and 20 Bushels of Oats, (when those Grains bear the Prices proposed in this Example) may be afforded at 2 s. 4 d. the Bushel as before.

3. That Example being disposed after the third manner (express'd in the tenth and eleventh Rules of this Chapter) *I say*, 3d Case.

I. As 20 the Sum of the Differences joyn'd to the Rate of the Wheat, is to 20 the Sum of the Differences annexed to the Rate of the Rye; so is 10 the given Quantity of the Wheat, to 10 Bushels the required Quantity of the Rye.

II. As 20 to 28, so is 10 to 14 Bushels the requisite Quantity of the Barley.

III. As 20 to 28, so is 10 to 14 Bushels, the Quantity of Oats demanded in the Mixture.

$$\begin{array}{r|l|l}
 28 \left\{ \begin{array}{l} 48 \\ 36 \\ 24 \\ 12 \end{array} \right. & \begin{array}{l} 16.4 \\ 16.4 \\ 20.8 \\ 20.8 \end{array} & \begin{array}{l} 20 \\ 20 \\ 28 \\ 28 \end{array}
 \end{array}$$

Whereupon this third time likewise I conclude, that (those Grains still retaining the given Rates) 10 Bushels of Wheat, 10 Bushels of Rye, 14 Bushels of Barley, and 14 Bushels of Oats, being all mixt together, will constitute a Mals of Corn, that may be afforded at 28 d. or 2 s. 4 d. the Bushel.

By this Example thus diversified, it plainly appears, that the Quantities requir'd may be altered as often as the Question given will allow diverse Alligations, and yet the Mixture produced will still hold the given Rate; but when the Question propounded will admit but one only Way of Alligation, the Quantities required to make the Mixture, cannot be varied; so the second Example of the Tenth Rule of this Chapter, being again produced, and ordered according to the Direction of the Eleventh Rule afore-going, *I say*,

- I. As 4 to 4, so 10 to 10 Bushels of Rye.
 II. As 4 to 4, so 10 to 10 Bushels of Barley.
 III. As 4 to 60, so 10 to 150 Bushels of Oats.

$$\begin{array}{r|l|l}
 48 & 4 & 4 \\
 16 \left\{ \begin{array}{l} 36 \\ 24 \\ 12 \end{array} \right. & 4 & 4 \\
 & 4 & 4 \\
 & 32, 20, 8. & 60
 \end{array}$$

So that for this Question I conclude, to 10 Bushels of Wheat you ought to add 10 Bushels of Rye, 10 Bushels of Barley, and 150 of Oats, to the end that a Mixture of Corn might be made, which may be sold at 16 *d.* the Bushel: And here the Quantities found (*viz.* 10, 10, and 150) cannot be altered, because the Terms of this Question will not admit any other Variety of Alligation.

XV. In Alternation Partial, the Proof is likewise by comparing the total Value of the several Simples, with the Value of the whole Mixture: *The Proof.* So in the second Example of the last Rule, the total Value of the 10 Bushels of Wheat, 40 Bushels of Rye, 50 Bushels of Barley, and 20 Bushels of Oats, amounts to 14*l.* which is also the Value of the whole Mixture at 2*s.* 4*d.* the Bushel, as appears by the Example of the fifth Rule of this present Chapter.

XVI. Alternation Total is, when having the total Quantity of all the Simples, together with their several Rates, we produce their several Quantities, in such Manner, that a Mixture of them being made according to the Quantities so found, that Mixture may bear a certain Rate proposed: Of this Sort is the last Example of the Tenth Rule foregoing; as also *this*: A Goldsmith having diverse Sorts of Gold, *viz.* some of 24 Carects, other of 22 Carects, some of 18 Carects, and other some of 16 Carects *fine*, is desirous to melt of all these Sorts so much together, as may make a Mass containing 60 Ounces of 21 Carects *fine*: Now this Rule of *Alternation Total* shews how much you are to take of every Sort, to the end the whole Mass may contain just 60 Ounces of 21 Carects, the *Fineness* proposed.

XVII. In Questions of Alternation Total, *The Proportions.* the Proportion is as follows.

As

As the Sum of all the Differences is to the Total Quantity of all the Simples: So is the Correspondent Difference of every Rate to the respective Quantity of the same Rate.

So the last Example of the last Rule being proposed, I say.

- I. As 12 the Sum of the Differences is to 60 Ounces the Total Quantity of all the Simples: So is 5 the Correspondent Difference of 24 Carets the first Rate, to 25 Ounces, viz. the required Quantity of the Gold of the same Rate, which may be taken to make the Mixture proposed.
- II. As 12 to 60, so is 3 the Correspondent Difference of 22 Carets the second Rate, to 15 Ounces, viz. the Quantity of the Gold of 22 Carets, that ought to be used in the Mixture.
- III. As 12 to 60: so is 1 to 5 Ounces of the Gold of 18 Carets fine.
- IV. As 12 to 60: so is 3 to 15 Ounces of the Gold of 16 Carets fine, which are requisite to be taken for the proposed Mixture.

$$\begin{array}{r|l}
 \begin{array}{l} 24 \\ 22 \\ 18 \\ 16 \end{array} & \begin{array}{l} 5 \\ 3 \\ 1 \\ 3 \end{array} \\
 \hline
 12
 \end{array}$$

Whereupon I conclude, that 25 Ounces of 24 Carets fine, 15 Ounces of 22 Carets, 5 Ounces of 18 Carets, and 15 Ounces of 16 Carets fine, being all melted together, will produce a Mass of Gold containing 60 Ounces of 21 Carets fine, which is the Resolution of the Question proposed.

Again, The last Example of the Tenth Rule being here repeated, and ordered according to the Direction of the eleventh Rule, I say,

- I. As 64 to 192, so is 17 to 51 Ounces of 24 Carets fine.
- II. As 64 to 192, so is 17 to 51 Ounces of 21 Carets fine.
- III. As 64 to 192, so is 17 to 51 Ounces of 19 Carets fine.
- IV. As 64 to 192, so is 13 to 39 Ounces of Alloy.

$$\begin{array}{r|l}
 \begin{array}{l} 24 \\ 21 \\ 19 \\ 0 \end{array} & \begin{array}{l} 17 \\ 17 \\ 17 \\ 7. 4. 2. \end{array} \\
 \hline
 64
 \end{array}$$

And

And therefore for Conclusion I say, that 51 Ounces of Gold, 24 Carats fine, 51 Ounces of 21 Carats fine, 51 Ounces of 19 Carats fine, and 39 Ounces of Alloy being all mingled together, will produce a Mass containing 192 Ounces of Gold, 17 Carats fine, which is the Satisfaction of the Question promised.

And here observe (as before in the Exposition of the Fourteenth Rule of this Chapter) that the Operations of the first of these Examples may be varied according to the Diversity of the Alligations which it will admit; whereas the last Example is not subject to any variety, the Alligations thereof remaining always the same.

XVIII. Here the Operation is perfect, when the Sum of the Quantities found agrees with the total Quantity given: So in the first Example of the last Rule, 25, 15, 5, and 15 (the Quantities found) being all added together amount to 60, which is the total Quantity proposed.

CHAP. XV.

The Rule of False.

I. THE Rule of False is always performed by false and supposititious Numbers taken at Pleasure after the Proposition is made, and the Question stated: For things are said to be found out by the Rule of False, when by false Terms supposed, we discover the true terms required.

II. The Rule of False, is either of single, or double Position.

The Rule of
single Position.

III. The Rule of Single Position is, when at once, viz. by one false Position we have Means to discover the true Resolution of the proposed Question.

For Example: A, B, and C, determining to buy together a certain Quantity of Timber, that should cost them 36*l*. agree among themselves that B shall pay of that Sum a third Part more than A, and that C shall pay a fourth more than B. Now the Question is, What particular Sum each of these Parties ought to pay of the 36*l*. To resolve this Question; First, put the Case that A ought to pay 6*l*. of the 36*l*. and then B must pay 8*l*. because he pays one third Part more than A. And lastly, C ought to pay 10*l*. because he is to lay out one fourth Part more

more than *B*. This done, although by Addition of these Three Sums, viz. 6, 8, and 10, I find that I have made a wrong *Position* (their Total amounting only to 24 *l*. which should have been 36 *l*.) nevertheless by those supposititious Numbers, I have means to discover the true Sums which the several Parties ought to pay: For I say by the Rule of *Three Direct*,

I. As 24 to 36, so is 6 to 9 *l*. the Part that *A* must pay.

II. As 24 to 36, so is 8 to 12 *l*. the Part that *B* ought to pay.

III. As 24 to 36, so is 10 to 15 *l*. the Part of the 36 *l*, that *C* must pay.

IV. Here for Tryal of this Rule, the Total of the Sums, found ought to agree with the Sum given: *The Proof.*
So! in the Example of the last Rule, 9, 12, and 15 being all added together, amount to 36, the Sum proposed.

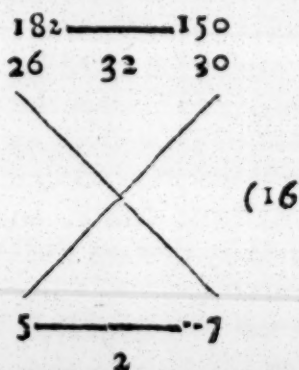
V. The Rule of *Double Position* is, when two false *Positions* are supposed for the Resolution of the Question propounded. As in this, A Workman *The Rule of*
having thresh'd out 40 Quarters of Grain *double Position.*
(Part of it Wheat, and the Rest Barley) receiv'd for his Labour 28 *s*. being paid after the Rate of 12 *d*. for every Quarter of Wheat, and 6 *d*. for each Quarter of Barley: Now here the Question is, How many of those 40 Quarters were Wheat, and how many Barley? Here therefore I first suppose at Random, That there were 26 quarters of Wheat, and 14 of Barley, and then to discover whether I have guess'd right or wrong, I find how much Money is due to the Workman at the Rate of 12 *d*. the Quarter of Wheat, and 6 *d*. the Quarter of Barley; which I find to be 33 *s*. (viz. 26 *s*. for the 26 Quarters of Wheat, and 7 *s*. for the 14 Quartets of Barley,) which he ought to have received, if my Supposition were right; but because it differs from 28 *s*. the true Sum that he received, I perceive I have miss'd the Mark, and therefore discovering how much I have err'd by finding the Difference between 28 *s*. and 33 *s*. I keep in mind 5 their Difference, which is called the *first Error*, or the *Error of the first Position*: Again, I propound for the *second Position*, that there were 30 Quarters of Wheat, and 10 Quarters of Barley; and then the *second Error* I find to be 7; for there is then due to the Workman for the 30 quarters of Wheat 30 *s*. and for the 10 quarters of Barley 5 *s*. in all 35 *s*. which differs from 28 *s*. the true Sum that he receiv'd, by 7 *s*. and here by these two false *Positions*, together with their *Errors*,
you

you may discover how many Quarters of Wheat, and how many of Barley the Workman thresh'd, as shall be further explain'd by the following Rule.

VI. In the Rule of double Position, having drawn two Lines a-cross, and placed the Terms of the false Position, (viz. those that have the same Denomination) at the uppermost End of that Cross, as also each Error under its respective Position at the lower End of the same Cross, multiply each Error by the contrary Position; that is to say, the second Error by the first Position, and the first Error by the second Position; this done, when both the Errors are of one and the same Kind (viz. both Excesses or both Defects) subtract the less Product out of the greater, and then the Remainder is your Dividend: But if the Errors be of different Kinds, (viz. one of them an Excess; and the other a Defect) add those Products together, and the Sum will be your Dividend, which if you divide by the Difference of the Errors, (when they are of one and the same Kind) or by their Sum (when they are of different Kinds) the Quotient will give you the Number you look for, having the same Denomination with the false Positions placed at the upper End of the Cross.

Example 1. The Question of the last Rule being again proposed, I place these Terms, viz. 26 (having the Denomination of the Quarters of Wheat in the first Position) and 30 (having the same Denomination in the second Position) at the upper End of the Cross: As also 5 and 7 the two Errors respectively under them at the lower End of the same Cross, as you may see it exemplified by the following Pattern,

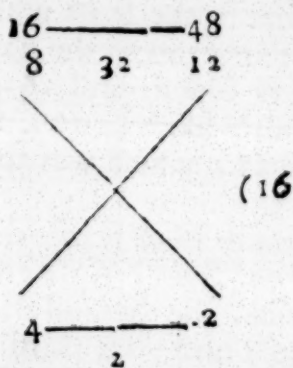
Note, That this Character $\times$ signifies that the lesser of the Numbers, between which it is found, ought to be subtracted from the greater.



This done, having multiplied 26 by 7, the Product is 182, and likewise 30 by 5, the Product is 150, which being deducted out of 182 (because the Errors here are both of the same Kind, that is, are each of them an excess above 28), the Sum that the

the Workman receiv'd,) the Remainder is 32, which being divided by 2 (the Difference between 5 and 6 the two Errours,) leaves in the Quotient 16, for the Quarters of Wheat that the Workman thresh'd, whole Complement to 40, viz. 24, are the Quarters of Barley, that he likewise thresh'd; so at last I conclude, the Workman receiving 28 s. for his Wages in threshing out 40 Quarters of Grain (being Part Wheat, Part Barley) at 12 d. the Quarter of Wheat, and 6 d. the Quarter of Barley; thresh'd in all 16 Quarters of Wheat, and 24 Quarters of Barley.

Example 2. The same Question being again propounded, I suppose for my *first Position* that there are 8 Quarters of Wheat, and 32 Quarters of Barley, and then the first *Error* will be 4 s. for 8 s. being accounted for the 8 Quarters of Wheat, and 16 s. for the 32 Quarters of Barley, make in all 24 s. which want 4 s. of 28 s. the Sum received: Again, supposing that there are 12 Quarters of Wheat, and 28 Quarters of Barley, the *second Error* will be 2 s.; for 12 s. being allowed for the 12 Quarters of Wheat, and 14 s. for the 28 Quarters of Barley, the sum is 26 s. which comes 2 s. short of 28 s. the right Sum: Now then, 8 being multiplied by 2, the Product is 16; likewise 12 by 4 produces 28, out of which if you deduct 26 (because the Errours in this Case happen to be both Defects under 28 s. the Sum received,) the Remainder is 32, which being divided by 2 (the Difference of the Errours) gives you in the Quotient 16; viz. the Quarters of Wheat, as before.



Example 3. The same Demand being the third Time produced, I take for my *first Position* 10 Quarters of Wheat, and 30 Quarters of Barley, and then proceeding as before, the first Error will prove 3 s. which upon that *Position* I want of 28 s. the right Sum: Again, here for the second *Position* I take 26 Quarters of Wheat. and 14 Quarters of Barley, and then the

N

second

second Errorr will be 5 s. by which upon that *Position* I have exceeded 28 s. the true Sum: Now then multiplying 10 by 5, the Product is 50, and 26 by 3, the Product is 78. And here (because the Errorrs are of different Kinds, one of them being a *Defect*, and the other an *Excess* of 28 s. the true Sum) you are to add 50 and 78 the two Products together, whose Sum is 128, which being divided by 8, the Sum of 3 and 5 the two Errorrs, gives you in the Quotient 16 for the Quarters of Wheat, as before in the former Resolutions: So that what Positions soever you take in this *Question*, you will always find, that the Workman thresh'd 16 Quarters of Wheat, and 24 Quarters of Barley, which is the Resolution of the *Question* propounded.

Note, That this Character $+$ intimates that the Numbers, between which it is found, ought to be added together.

$$\begin{array}{ccc}
 50 & + & 78 \\
 10 & 128 & 29 \\
 & \times & \\
 3 & + & 5 \\
 & 8 &
 \end{array}
 \quad (16)$$

VII. Here the Trial is the same with that which is used in finding out of the Errorrs: So in the *Example* premised, 16 and 24 being the Numbers found, and 16 s. being allowed for the 16 Quarters of Wheat, likewise 12 s. for the 24 Quarters of Barley, their Sum is 28 s. which was the Sum received by the Workman.

Example 4. A certain Man being demanded what was the Age of each of his Four Sons? Answered, That his eldest Son was 4 Years elder than the Second; his second Son was 4 Years elder than the Third; his third Son was 4 Years elder than the Fourth or youngest; and his Fourth or youngest was Half the Age of the Eldest; the *Question* is What was the Age of every Son? Here I guess the Age of the eldest Son to be 16, then it may be inferr'd from the *Question*, that the Age of the second Son was 12, the Age of the Third 8, and the Age of the Fourth or youngest 4, this 4 should be Half 16 (for the *Question* says, That the Age of the youngest was Half

Half the Age of the eldest) but it wants 4 of what it ought to be; wherefore I make a second Position, and take 20 for the Age of the eldest, then the Age of the second must necessarily be 16, the Age of the third 12, and the Age of the fourth 8, which should be half 20, but it wants 2: Now (according to the Rule) multiplying 16 (the first Position) by 2, (the second Error) the Product is 32: Also multiplying

$$\begin{array}{r}
 32 \quad \text{---} \quad 80 \\
 16 \quad \quad 38 \quad 20 \\
 \diagdown \quad \quad \diagup \\
 4 \quad \text{---} \quad 2 \\
 \quad \quad 2 \\
 2) \quad 48 \quad (24
 \end{array}$$

20 (the second Position) by 4 (the first Error) the Product is 80, and because the Errors are both of one Kind, to wit, both defective; I subtract the lesser Product from the greater, so the Remainder is 48 for a Dividend; also subtracting the lesser Error from the greater, the Remainder is 2 for a Divisor: *Lastly*, dividing 48 by 2, the Quotient is 24, and such was the Age of the eldest Son; therefore the Age of the second was 20, the Age of the third 16, and the Age of the fourth 12; which is half the Age of the eldest, as was declared by the Question.

The Doctrine of Vulgar Fractions.

CHAP. XVI.

Notation of Vulgar Fractions.

I. **T**HUS far of *Arithmetick* in *Whole Numbers*, only the Doctrine of *Fractions* ensues, which depends upon this Supposition, That Unity, or at least one whole Thing whatsoever it be, may in mind be conceived divisible into any Number of equal Parts. Some will not allow 1 or Unity to be a Number, when it is considered in the Abstract, and separated from Matter: But since that Prince of Arithmeticians, *Diophantus* of *Alexandria*, in several of his subtil Problems mentions Unity as a Number, and propounds it to be divided into Numbers; I shall take the like Liberty to esteem 1 or Unity as a Number, and likewise suppose it divisible into any Number of equal Parts.

II. A broken Number, otherwise called a *Fraction*. Fraction, is only Part of an Integer or whole Thing, as if you would express in Figures the Length of a Piece of Cloth, that contains three Fourths, or (which is all one) three Quarters of a Yard, you are to write it thus $\frac{3}{4}$; that is, an entire Yard being suppos'd to be divided into Four equal Parts, the Length of the Piece proposed is Three of those Four Parts: In like manner (a Foot being divided into 12 Inches) you are to write 6 Inches thus $\frac{6}{12}$ that is, six twelfth parts of a Foot; or if the Foot be divided into One hundred equal Parts, to express Twenty-five of those Parts, set them down thus $\frac{25}{100}$, that is, Twenty five hundredth Parts of a Foot.

III. A Fraction consists of two Parts, the *Numerator* and the *Denominator*, which are placed one above the other, and separated by a little Line.

IV. The *Numerator* is the Number set above the Line, and the *Denominator* is the Number placed underneath: So in the aforementioned Fraction $\frac{3}{4}$, the Number 3 placed above the Line is the *Numerator*, and the Number 4 set under it is the *Denominator*. Also in this Fraction $\frac{6}{12}$, the *Numerator* is 6, and

6, and the *Denominator* is 12. The *Denominator* is so named, because it denominates or declares into how many equal Parts the Integer or whole Thing is suppos'd to be divided; and the *Numerator* is so called, because it numbers or expresses how many of these equal Parts of the Integer are signified by the Fraction.

V. A Fraction is either Proper, or Improper.

VI. A proper Fraction is that whose Numerator is less than the Denominator; such are the Fractions before mentioned $\frac{3}{4}$, $\frac{6}{12}$, $\frac{25}{100}$, and the like. *A proper Fraction.*

VII. A proper Fraction is either Single, or Compound.

VIII. A single Fraction is that which consists of one Numerator, and one Denominator; such are $\frac{3}{4}$, $\frac{6}{12}$, $\frac{25}{100}$, and the like. *A single Fraction.*

IX. A single Fraction often arises in Division of whole Numbers, for when the Division is finish'd, if any Number remain, it is to be esteem'd as the Numerator of a Fraction, which has the Divisor for a Denominator, and is to be annexed to the Integer or Integers in the Quotient, as Part of the Quotient; which Fraction always expresses certain Parts (or at least a Part) of an Integer or entire Unity, which has the same Denomination with one of the Integers in the Quotient: So if 17 Pounds be given to be equally divided among 5 Persons, there will arise 3 entire Pounds in the Quotient, and there will be a Remainder or Surplusage of 2 Pounds, which 2 is to be placed, as the Numerator of a Fraction, over the Divisor 5 as a Denominator; so will the Fraction be $\frac{2}{5}$ and the compleat Quotient will be $3\frac{2}{5}$, that is 3 Pounds, and 2 fifth Parts of a Pound for every Person's Share.

A single Fraction does likewise arise, when a lesser whole Number is given to be divided by a greater; for in such Case the Dividend is to be made the Numerator of a Fraction, and the Divisor the Denominator; which Fraction is the true Quotient, and always expresses certain Parts (or at least a Part) of an Integer, which has the same Name with the *Dividend*. So if 3 Pounds Sterling be given to be divided equally among 4 Persons, the Share of each, that is, the Quotient will be $\frac{3}{4}$, to wit, 3 fourth Parts of a Pound.

In like manner, if 5 be given to be divided by 8 the Quotient is $\frac{5}{8}$; so that the Numerator of a Fraction is always a Dividend, the *Denominator* is a Divisor, and the *Fraction* it self is the *Quotient*.

A Compound Fraction.

X. A Compound Fraction (otherwise called a Fraction of a Fraction) is that which has more Numerators and Denominators than one, and may be discovered by the Word (*of*) which is interpos'd between the Parts of such compound Fraction: So $\frac{2}{3}$ of $\frac{3}{4}$, is a Fraction of a Fraction, or *compound Fraction*, and expresses two thirds of three fourths of an Integer; viz. a Pound Sterling being supposed the Integer, and first divided into four Parts, three of those four Parts are equal to 15 s. Again, if the said 15 s. be divided into three Parts, two of those three Parts are equal to 10 s. therefore the *compound Fraction* $\frac{2}{3}$ of $\frac{3}{4}$ of a Pound Sterling does express 10 s. In like manner, the compound Fraction $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a Pound Sterling, that is, one fourth of three fourths of four fifths of a Pound Sterling expresses 3 s. as will be farther manifested by the Sixteenth and Ninth Rules of the Seventeenth Chapter.

An improper Fraction.

XI. An improper Fraction is that, whose Numerator is either greater, or at least equal to the Denominator: So this Fraction $\frac{16}{4}$, that is 16 fourths, is called an *Improper Fraction*, and so is this $\frac{4}{4}$; for indeed a Fraction of this Kind may well be surnamed *Improper*, because it will not admit the Definition of a true Fraction; since it is always greater than an entire Unity, or at least equal to it; so Sixteen Farthings, or $\frac{16}{4}$ of a Penny are equal to 4 entire Pence; and 4 Farthings, or $\frac{4}{4}$ of a Penny are equal to 1 Penny: Therefore when the Numerator is greater than the Denominator, such *improper Fraction* signifies more than 1 or an Integer; but when the Numerator is equal to the Denominator, (be it what Number soever) such *improper Fraction* is always equal to Unity, or 1 Integer.

A mixt Number.

XII. A mixt Number consists of entire Unities (or Integers) or at least of Unity (or 1 Integer) and a Fraction annexed: Thus $5\frac{1}{2}$, $1\frac{3}{4}$ and such like, are called mixt Numbers: So that if a Piece of Timber be Five Feet and Eleven Inches in length, you are to write that length thus, $5\frac{11}{12}$; in like manner, one Mile and three Quarters or Fourths of a Mile are to be express'd thus $1\frac{3}{4}$.

C H A P. XVII.

Reduction of Vulgar Fractions.

I. **T**HE same Parts of Numeration, as have been work'd in *Whole Numers* in the preceeding Chapters, are likewise to be performed in *Fractions*: But first of all *Reduction* of *Fractions* in diverse Kinds must be known, which being the principal Skill in the *Doctrine of Fractions*, ought to be diligently observed by the Learner.

II. A Number is said to be a common Measure or Divisor to two or more Numbers given, when it will measure or divide every one of the Numbers given, and leave no Remainder; so 4 is a common Measure to the Numbers 12 and 20; for if 12 be divided by 4, the *Quotient* will be exactly 3, without any Remainder or Surplusage; also if 20 be divided by the same Divisor 4, the *Quotient* will be precisely 5 without any Remainder; in like manner 5 is a common Divisor to these three Numbers 10, 25, and 40.

III. Two Numbers being given, their greatest common Divisor, that is, the greatest Number which will measure or divide each of the Numbers given without leaving any Remainder, may be found out in this manner; viz. Divide the greater Number by the less, then divide the Divisor by the Remainder (if there be any) and so continue dividing the last Divisors by the Remainders, untill there be no Remainder (neglecting the *Quotients*;) so is the last Divisor the greatest common Divisor to the Numbers given.

To find the greatest common Measure unto any two Numbers.

Thus, If the greatest common Divisor to the Numbers 91 and 117 be sought, divide the greater Number 117 by 91, the Remainder is 26, by which dividing 91, the Remainder is 13; by which dividing 26 the Remainder is 0, so is 13 the greatest common Divisor to the Numbers 117 and 91, as is manifest in dividing each of them by 13; for 13 is found in 91 precisely 7 times, and in 17 precisely 9 times. In like manner, 29 will be found a common Divisor to 116 and 245; and 51 a common Divisor to 561 and 612.

$$\begin{array}{r}
 91 \overline{) 117} \quad (1 \\
 \underline{91} \\
 26 \\
 26 \overline{) 91} \quad (3 \\
 \underline{78} \\
 13 \\
 13 \overline{) 26} \quad (2 \\
 \underline{26} \\
 0
 \end{array}$$

IX. A

To reduce a Fraction into the least Terms, viz. 1. By a general Rule.

IV. A single Fraction may be reduced into the least Terms, by dividing the Numerator and Denominator by their greatest common Measure (or Divisor;) for the Quotients will be the Numerator and Denominator of a Fraction equal to the former, and in the least Terms.

So if the Fraction $\frac{91}{117}$ be given to be reduced into the least Terms, search out the greatest common Divisor to 91 and 117 by the last Rule, which will be found 13, and then dividing 91 by 13, the Quotient will be 7 for a new Numerator; also dividing 117 by 13, the Quotient will be 9 for a new Denominator: So the Fraction $\frac{91}{117}$ is reduced into the least Terms, viz. into the Fraction $\frac{7}{9}$. In like manner $\frac{116}{145}$ will be reduced to $\frac{4}{5}$; And $\frac{61}{122}$ to $\frac{1}{2}$: But here you are to observe, That if the greatest common Divisor to the Numerator and Denominator be 1, such Fraction is in its least Terms already: So the Fraction $\frac{13}{13}$ cannot be reduced into lower Terms, because the greatest common Divisor will be found 1, (by the third Rule of this Chapter;) the like may happen to infinite others: And though the last be a general Rule for Reduction of Fractions into their least Terms, yet there are other practical Rules, which in some Cases will be more ready; (especially to Beginners,) viz.

2. By particular Rules.

V. When the Numerator and Denominator are even Numbers, they may be measured or divided by 2. Therefore in such Case you may (as is taught in the Rules of the 6th Chapter) take the Half of the Numerator for a new Numerator, also the Half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the Line which separates the Numerator from the Denominator, and cross the same with a down-right Stroke near the Fraction,

$$\begin{array}{r|l} 16 & 8 \mid 4 \mid 2 \mid 1 \\ \hline 64 & 32 \mid 16 \mid 8 \mid 4 \end{array}$$

as you may see in the Margin; then take the Half of 16, which is 8, for a new Numerator, and the Half of 64, which is 32, for a new Denominator: Again, the Half of 8 is 4, for a new Numerator, also the Half of 32 is 16, for a new Denominator; and proceeding in the like manner, there will be found $\frac{1}{4}$ Equivalent to $\frac{16}{64}$.

VI. When the Numerator and Denominator each of them end with 5, or one of them ends with 5, and the other with a Cypher; they may be both measured or divided by 5. So $\frac{225}{475}$ will be reduced into $\frac{9}{19}$, and $\frac{50}{425}$ into $\frac{2}{17}$, as by the Operation in the Margin is manifest.

$$\begin{array}{r|l} 225 & 45 \mid 9 \\ \hline 475 & 95 \mid 19 \\ \hline 50 & 10 \mid 2 \\ \hline 425 & 85 \mid 17 \end{array}$$

VII. When

VII. Whensoever you can espy any other Number, which will exactly divide the Numerator and Denominator (though it be not the greatest common Divisor) you may divide the Numerator and Denominator by such Number as before: So $\frac{28}{84}$ may be first reduced to $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced to $\frac{1}{3}$ by 7, as by the Operation is manifest.

$$\begin{array}{r} 28 \overline{) 7} \quad 1 \\ 84 \overline{) 21} \quad 3 \end{array}$$

VIII. When the Numerator and Denominator, each of them end with a Cypher or Cyphers; cut off equal Cyphers in both, and the Fraction will be reduced to lesser Terms: So $\frac{400}{500}$ is reduced to $\frac{4}{5}$, and $\frac{700}{900}$ to $\frac{7}{9}$.

$$\begin{array}{r} 4 \overline{) 00} \\ 5 \overline{) 00} \\ 7 \overline{) 00} \\ 9 \overline{) 00} \end{array}$$

IX. The Value of a single Fraction in the known parts of the Integer, may be found out in this manner, viz. multiply the Numerator of the Fraction propos'd, by the Number of known Parts of the next inferiour Denomination which are equal to the Integer, and divide that Product by the Denominator, so is the Quotient the Value of the Fraction in that inferiour Denomination: and if there happen to be any Fraction in the Quotient, you may find the Value of it in the next inferiour Denomination, by the same Rule, and so proceed till you come to the least known Parts.

To find the Value of a single Fraction, in the known Parts of the Integer.

So the Value of $\frac{9}{16}$ of a Pound Sterling will be found 11 s. 3 d. viz. multiply the Numerator 9 by 20 (the Number of Shillings which are equal to 1 pound Sterling) the Product is 180, which being divided by the Denominator 16, the Quotient is 11 $\frac{4}{16}$ Shillings. In like manner, the Value of $\frac{4}{16}$ of a Shilling will be found 3 Pence; for multiplying the Numerator 4 by 12 (the Number of Pence in a Shilling) the Product is 48, which being divided by the Denominator 16, the Quotient is 3 Pence. Also the Value of $\frac{7}{16}$ of a pound Sterling, will be found 10 s. 9 $\frac{1}{2}$ d. And $\frac{3}{4}$ of a pound Troy will be found Equivalent to 3 Ounces 17 Penny-weight and 12 Grains.

$$\begin{array}{r} 9 \\ 20 \\ \hline 16 \overline{) 180} \quad (11 \frac{4}{16}) \\ 16 \\ \hline 20 \\ 16 \\ \hline 4 \\ 12 \\ \hline 16 \overline{) 48} \quad (3) \\ 48 \\ \hline 0 \end{array}$$

X. A mixt Number may be reduced to an improper Fraction Equivalent to the mixt Number, in this

To reduce a mixt Number to an improper Fraction.

manner, viz. Multiply the Integer or Integers in the mixt Number, by the Denominator of the Fraction annexed to the Integer or Integers, and unto the Product add the Numerator of the said Fraction; so is the Sum the Numerator of an improper Fraction, whose Denominator is the same with that of the said Fraction annexed.

So $4\frac{1}{2}$ will be reduced to the improper Fraction $\frac{9}{2}$, for 4 being multiplied by 2, the Product is 8, to which adding the Numerator 1, the Sum is 9 for a new Numerator, which being placed over the Denominator 2, gives the improper Fraction $\frac{9}{2}$; which is equivalent to $4\frac{1}{2}$ (as will appear by the 13th Rule of this Chapter.) In like manner $7\frac{1}{2}$ will be reduced to $\frac{15}{2}$.

XI. A whole Number is reduced to

To reduce a whole Number to an improper Fraction.

an improper Fraction, by placing the whole Number given as a Numerator, and 1 as a Denominator.

So 14 Integers will be reduced to the improper Fraction $\frac{14}{1}$, and one Integer to the improper Fraction $\frac{1}{1}$.

XII. A whole Number is reduced to an improper Fraction, which shall have any Denominator assigned, in multiplying the whole Number given by the Denominator assigned, and placing the Product as a Numerator over the said Denominator.

As if 13 be given to be reduced to an improper Fraction, whose Denominator shall be 4, multiply 13 by 4, the Product is 52, which being placed over 4, gives the improper Fraction $\frac{52}{4}$ equivalent to 13 (as will appear by the next Rule.) In like manner 13 may be reduced to $\frac{91}{7}$.

XIII. An improper Fraction may be reduced to its Equivalent whole Number or mixt Number in

To reduce an improper Fraction to its Equivalent whole or mixt Number.

this manner, viz. Divide the Numerator by the Denominator, and the Quotient will give the whole Number or mixt Number sought: So the improper Fraction $\frac{9}{2}$ will be reduced to this mixt Number $4\frac{1}{2}$; for if 9 be divided by 2, the Quotient is $4\frac{1}{2}$. Also this improper Fraction will be reduced to the whole Number 3.

XIV. Fractions

XIV. Fractions having unequal Denominators, may be reduced to Fractions of the same Value, which shall have equal Denominators, by this Rule and the next following, viz. when two Fractions having unequal Denominators, are propos'd to be reduced into two other Fractions of the same Value, which shall have a common Denominator; multiply the Numerator of the first Fraction (that is either of them) by the Denominator of the second, and the Product shall be a new Numerator (correspondent to the Numerator of that first Fraction;) also multiplying the Numerator of the second Fraction by the Denominator of the first, the Product is a new Numerator (correspondent to the Numerator of the second Fraction;) lastly, multiply the Denominators one by the other, and the Product is a common Denominator to both the new Numerators.

To reduce Fractions to a common Denominator, viz. 1. When two Fractions are propounded.

Thus, If the Fractions $\frac{2}{3}$ and $\frac{4}{5}$ be propos'd, multiply 2 by 5, the Product 10 is a new Numerator correspondent unto 2: Also multiply 4 by 3, the Product 12 is a new Numerator correspondent to 4: Lastly, multiply 3 by 5, and the Product 15 will be a common Denominator to the new Numerators: So the Fractions $\frac{10}{15}$ and $\frac{12}{15}$ are found out, which have equal Denominators, and each of these new Fractions is equal to its correspondent Fraction first given, viz. $\frac{10}{15}$ is equal to $\frac{2}{3}$, and $\frac{12}{15}$ is equal to $\frac{4}{5}$; as is manifest by the fourth Rule of this Chapter.

$$\begin{array}{r} 2 \quad \times \quad 5 \\ \hline 10 \\ 15 \end{array} \quad \begin{array}{r} 4 \\ \hline 12 \\ 15 \end{array}$$

XV. When three or more Fractions having unequal Denominators, are given to be reduced to other Fractions of the same Value with those given, but such as shall have one common Denominator; multiply continually (according to the Thirteenth Rule of the Fifth Chapter) the Numerator of the first Fraction into all the Denominators, except its own, i. e. the Denominator of that Fraction; and reserve the last Product for a new Numerator, instead of that first Numerator: In like manner, multiply continually the Numerator of the second Fraction into all the Denominators, except the Denominator of the second Fraction; and reserve the last Product for a new Numerator, instead of the second Numerator; Proceed in like manner to find

2 When three or more Fractions are to be reduced to others that shall have a Common Denominator.

find out new Numerators for the rest of the given Fractions: Lastly, Multiply continually all the Denominators one into another, and the last Product will be a common Denominator to all the new Numerators.

As for *Example*, If these three Fractions, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{7}$, having unequal (or different) Denominators, be given to be reduced to three other Fractions of the same Value, which shall have equal Denominators, (or one common Denominator.) First,

$$\frac{3}{8} \quad \frac{2}{5} \quad \frac{5}{7}$$

I multiply continually the first Numerator 3 into the second and third Denominators 5 and 7; saying 3 times 5 makes 15, which multiplied by 7 produces 105, for

a new Numerator instead of the first Numerator 3: Secondly, I multiply continually the second Numerator 2 into the first and third Denominators 8 and 7; saying, twice 8 is 16, which multiplied by 7 produces 112, for a new Numerator instead of the second Numerator 2: Thirdly I multiply continually the third Numerator 5 into the first and second Denominators 8 and 5; saying, 8 times 5 makes 40, which multiplied by 5 produces 200, for a new Numerator instead of the third Numerator 5: Fourthly and lastly, I multiply continually all the Denominators 8, 5, and 7 one into another; saying, 8 times 5 makes 40; which multiplied by 7 produces 280 for a Denominator to each of the three new Numerators 105, 112, and 200 before found out: And so these three Fractions $\frac{105}{280}$, $\frac{112}{280}$, and $\frac{200}{280}$, are discovered, which have one common Denominator 280; and every one of them is equal in Value to its Correspondent Fraction first given, viz $\frac{105}{280}$ is equal to $\frac{3}{8}$: Also $\frac{112}{280}$ is equal to $\frac{2}{5}$; and $\frac{200}{280}$ is equal to $\frac{5}{7}$; as may easily be proved by the fourth Rule of this Chapter.

After the same manner, these four Fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, are reducible to these $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$ and $\frac{300}{360}$, which have 360 for a common Denominator; and are equal in Value respectively to the four Fractions given to be reduced.

Note, Although by the afore-going fourteenth and fifteenth Rules, any Multitude of Fractions may be reduced to a common Denominator; yet because Fractions in their least Terms are fittest for Use, I shall shew how lesser Denominators than those that are discovered by the said Rules, may frequently be found out;

I. When the unequal Denominators of two Fractions have a common Divisor greater than 1, divide the Denominators severally by their greatest common Divisor (found out by the fore-going third Rule of this Chapter ;) and then multiply cross-wise in this manner, *viz.* the Numerator of the first Fraction by the latter Quotient, and the Numerator of the latter Fraction by the first Quotient, and reserve the Products for new Numerators: Lastly, multiply the Denominator of the first Fraction by the latter Quotient, (or the Denominator of the latter Fraction by the first Quotient,) so shall the Product be a common Denominator to the said new Numerators: As for Example, If $\frac{5}{12}$ and $\frac{7}{18}$ be proposed to be reduced to a common Denominator, I divide each of the Denominators, 12 and 18, by their greatest common Divisor 6, and the Quotients are 2 and 3; then I multiply 5 the Numerator of the first Fraction by 3, the latter Quotient,

$$\begin{array}{r} 5 \quad \times \quad 7 \\ 6 \overline{) 12} \quad \times \quad 18 \\ \hline 2 \quad \quad 3 \\ \hline 15 \quad 14 \\ \hline 36 \quad 36 \end{array}$$

also 7 the Numerator of the latter Fraction by 2 the first Quotient, and the Products 15 and 14 I reserve for new Numerators instead of 5 and 7: Lastly, I multiply 12 the Denominator of the first Fraction, by 3 the latter Quotient, (or 18 the Denominator of the latter Fraction by 2 the first Quotient,) and the Product 36 is a Denominator to each of the new Numerators 15 and 14: So $\frac{5}{12}$ and $\frac{7}{18}$ are found out, which have the least common Denominator unto which the given Fractions $\frac{5}{12}$ and $\frac{7}{18}$ can be reduced: Also $\frac{5}{12}$ is equal to $\frac{15}{36}$, and $\frac{7}{18}$ to $\frac{14}{36}$.

II. Whensoever the Denominator of a Fraction can be divided by the Denominator of a second Fraction, without any Remainder; then if by the Quotient you multiply severally the Numerator and Denominator of such second Fraction, a third will arise, having the same Value with the second, and the same Denominator with the first Fraction: By this Rule Three or more Fractions may often be reduced to a lesser common Denominator, than that which may be discovered by the foregoing Rule XV. As for Example: Let these six following Fractions be given to be reduced to a common Denominator, *viz.*

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}.$$

Because 36 the Denominator of the first Fraction, being divided by the five other Denominators severally, will give these Quotients 2, 3, 4, 6 and 12, without any Remainder; I mul.

I multiply the Numerator and Denominator of each of the five latter Fractions, by its correspondent Quotient; viz. 11 and 18 by 2 the first Quotient: Also 7 and 12 by 3 the second Quotient, and in like manner the Rest: so instead of those five latter Fractions, five others (hereunder placed after the first of those Six,) are produced, viz.

$$\frac{1}{3} \frac{1}{2}, \frac{2}{3} \frac{1}{2}, \frac{2}{3} \frac{1}{2}, \frac{1}{3} \frac{1}{2}, \frac{1}{3} \frac{1}{2}, \frac{2}{3} \frac{1}{2}.$$

All which Fractions last express'd have a common Denominator 36, and are equal in Value respectively to those given to be reduced.

To reduce a compound Fraction to a single Fraction. See continual Multiplication in the last Rule of the 5th Chapter.

XVI. A compound Fraction (otherwise called a Fraction of a Fraction) may be reduced to a single Fraction in this manner, viz. Multiply all the Numerators continually, and take the Product for a new Numerator; also multiply all the Denominators continually, and the Product will be a new Denominator.

Thus, if the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ be given to be reduced to a single Fraction; multiply the Numerators 2 and 3, one by the other, so is the Product 6 a new Numerator. Also mul-

tiplying the Denominators 3 and 4 one by the other, the Product 12 is a new Denominator so, $\frac{6}{12}$ or $\frac{1}{2}$ is the single Fraction sought, being Equivalent to $\frac{2}{3}$ of $\frac{3}{4}$, the compound Fraction given to be reduced. In

like manner, this compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will be reduced to $\frac{24}{60}$, or $\frac{2}{5}$; for the Numerators 2, 3, 4, being multiplied continually, produce the new Numerator 24; and the Denominators, 3, 4, 5, multiplied continually, produce the new Denominator 60: Lastly, the new Fraction $\frac{24}{60}$ (by the fourth Rule of this Chapter) will be reduced to $\frac{2}{5}$, which is equal to $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{4}{5}$: But to make the Meaning hereof more evident, suppose the Integer to be 1 Pound of English Money; then

$\frac{4}{5}$ of 1 l. (viz. of 20 s.) is ——— 16 s.

$\frac{3}{4}$ of those (viz. of 16 s.) is ——— 12 s.

$\frac{2}{3}$ of those $\frac{3}{4}$ (viz. of 12 s.) is ——— 8 s. or $\frac{2}{5}$ l.

whereby 'tis manifest that $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ l. is equal to $\frac{2}{5}$ l.

By this Rule a Fraction or mix'd Number of a lesser Name may be reduced to a Fraction of a greater Name. As if $3 \frac{1}{2}$ Pence be propos'd to be reduced to an improper Fraction of a Pound Sterling, the Operation will be in this manner, viz. $3 \frac{1}{2}$ or $\frac{7}{2}$ of a Penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound Sterling, which Compound Fraction

Fraction will, by the aforesaid Rule, be reduced to $\frac{7}{8}l$. In like manner, $42\frac{2}{3}$ Minutes of an Hour are equal to $\frac{4}{5}$ of an Hour; for $67\frac{5}{8}$ (that is $42\frac{1}{2}$) of $\frac{1}{2}$ are equal to $\frac{67}{2}$ (or in its least Terms) $\frac{4}{5}$.

Here you may also observe, that when a compound Fraction is one of the given Terms in any Question, it is first of all to be reduced to a single Fraction by the aforesaid sixteenth Rule.

XVII. Two or more Fractions being given, there may be whole Numbers found, which shall have the same Reason or Proportion as the Fractions given, viz. When the Fractions given have unequal Denominators, reduce them to equivalent Fractions, which have a common Denominator (by the 14 or 15th Rule of this Chapter;) then rejecting the common Denominator, the Numerators will have the same Reason or Proportion as the Fractions first proposed.

To find whole Numbers, which shall have the same Reason, as any Fractions or mixt Numbers given.

So $\frac{3}{4}$ and $\frac{5}{8}$ being given, will first of all be reduced to their equivalent Fractions $\frac{3}{4}$ and $\frac{5}{8}$; then rejecting the common Denominator 40, the Numerators 24 and 25 have the same Reason with $\frac{3}{4}$ and $\frac{5}{8}$, viz. As $\frac{3}{4}$ is to $\frac{5}{8}$, so is 24 to 25: Also if the Fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ were proposed, there will be found 8, 16 and 32, which are in the same Proportion one to the other as those Fractions given: In like manner, if mixt Numbers be given, there may be whole Numbers found which shall have the same Reason or Proportion, as the mixt Numbers; so $5\frac{1}{2}$ and $3\frac{1}{3}$ being given, will be first reduced to the improper Fractions $\frac{11}{2}$ and $\frac{10}{3}$ (by the Tenth Rule of this Chapter): Also the said $\frac{11}{2}$ and $\frac{10}{3}$ will be reduced to $\frac{33}{6}$ and $\frac{20}{6}$; then rejecting the common Denominator 6, the Numerators 33 and 20 will have the same Reason as $5\frac{1}{2}$ and $3\frac{1}{3}$, viz. As 33 is to 20, so is $5\frac{1}{2}$ to $3\frac{1}{3}$. Also $16\frac{1}{2}$ and 18 being given, there will be found 33 and 36, which being divided by their common Divisor 3, (found out by the third Rule of this Chapter) will give 11 and 12, which have the same Reason as $16\frac{1}{2}$ and 18.

C H A P. XVIII.

Addition of Vulgar Fractions and mixt Numbers.

I. WHEN the Numbers given to be added are single Fractions, and have equal Denominators, add all the Numerators together; so is the Sum the Numerator of a Fraction, whose Denominator is the same with the common Denominator, which new Fraction is the Sum of the Fractions given to be added.

To add Single Fractions, viz.
 1. *When they have equal Denominators.*

So $\frac{3}{9}$ and $\frac{2}{9}$ being propos'd to be added, their Sum will be found $\frac{5}{9}$, viz. the Sum of the Numerators, 3 and 2 is 5, which being placed over the common Denominator 9, gives $\frac{5}{9}$: In like Manner the Sum of these Fractions $\frac{7}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, and $\frac{2}{8}$, will be found $1\frac{7}{8}$, which (by the 13th Rule of the 17th Chapter) will be found Equivalent to $2\frac{1}{8}$; so that $2\frac{1}{8}$ is the Sum of the Fractions given to be added.

II. When the Fractions propos'd to be added have unequal Denominators, they are first to be reduced to Fractions of the same Value, which shall have a common Denominator (by the Fourteenth or Fifteenth Rule of the Seventeenth Chapter;) and then they may be added by the first Rule of this Chapter.

So if $\frac{2}{3}$ and $\frac{2}{5}$ were given to be added, their Sum will be found $1\frac{4}{15}$; for (by the Fourteenth Rule of the Seventeenth Chapter)

$$\begin{array}{r} \overset{2}{-}\overset{3}{X}- \\ \hline \overset{3}{10} \quad \overset{5}{9} \\ \hline 15 \quad 15 \end{array}$$

$\frac{2}{3}$, that is $1\frac{4}{15}$

$\frac{2}{3}$ and $\frac{2}{5}$ will be reduced to their equivalent Fractions $\frac{10}{15}$ and $\frac{4}{15}$, which having equal Denominators may be added according to the first Rule of this Chapter, and so the Sum will be found $1\frac{4}{15}$: In like manner, the Sum of these Fractions $\frac{13}{12}$ and $\frac{3}{4}$ will be found $1\frac{5}{6}$. Also the Sum of these six Fractions $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{2}{3}$, after they are reduced to a common Denominator (according to the latter Example in the Note at the End of the Fifteenth Rule of the Seventeenth Chapter) will be found $1\frac{2}{3}$, that is, $3\frac{1}{2}$.

III. When any of the Fractions given to be added is a compound Fraction, such compound Fraction is first of all to be reduced to a single Fraction (by the Sixteenth Rule of the Seventeenth Chapter) and then you may proceed as before.

So

So $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{1}{4}$ being given to be added, their Sum will be found $\frac{5}{6}$; for the compound Fraction $\frac{2}{3}$ of $\frac{1}{4}$ will (by the 16th Rule of the 17th Chapter) be reduced to $\frac{1}{6}$ (or in its least Terms) $\frac{1}{6}$; which added to the single Fraction $\frac{1}{2}$ (according to the second Rule of this Chapter) gives $\frac{5}{6}$. Here you may observe, That the Fractions given to be added in all the former Cases, are supposed to be Fractions of Integers, which have one and the same particular Denomination, viz. If one of the Fractions, given to be added, be a Fraction of a Pound Sterling, all the rest ought to be Fractions of a Pound Sterling, and the like is to be understood of other Denominations.

By Denomination, is meant the Name of any Integer or Thing.

IV. When Fractions of Integers, of different Denominations are given to be added, they are first of all to be reduced to Fractions of Integers that have one and the same particular Denomination, (by the sixteenth Rule of the seventeenth Chapter;) and then they may be added by the first or second Rule of this Chapter.

To add Fractions of Integers which have different Denominations.

So if $\frac{2}{3}$ of a Pound Sterling, $\frac{1}{4}$ of a Shilling, and $\frac{5}{8}$ of a Penny were given to be added, reduce the two latter into Fractions of a Pound Sterling (by the Sixteenth Rule of the Seventeenth Chapter,) viz. $\frac{1}{4}$ of a Shilling is $\frac{1}{20}$ of a Pound Sterling, which compound Fraction being reduced to a single Fraction gives $\frac{1}{40}$ l. Likewise $\frac{5}{8}$ of a Penny is $\frac{5}{16}$ of $\frac{1}{20}$ of a Pound Sterling, which compound Fraction being reduced, gives $\frac{1}{64}$ l. Lastly, $\frac{2}{3}$ l. $\frac{1}{20}$ l. and $\frac{1}{64}$ l. being added according to the second Rule of this Chapter, their Sum will be found $\frac{280068}{345600}$, or in its least Terms $\frac{2339}{2880}$ l.

V. When mixt Numbers are given to be added, find first of all the Sum of the Fractions (by the first and the second Rule of this Chapter;) then add the Integer or Integers (if there be any found) in the Sum of the Fractions, to the whole Numbers, and collect the Sum of them, as you were taught by the Rules of the third Chapter.

To add mixt Numbers.

So if $3\frac{1}{2}$, $4\frac{1}{3}$, and $16\frac{5}{8}$ were given to be added, their Sum will be found $14\frac{11}{24}$, viz. the Sum of the Fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{8}$, will be found (by the second Rule of this Chapter) to be $\frac{11}{24}$, and the Sum of the whole Numbers, 3, 4, and 16, is 23, to which adding 1 (the Integer found in the Sum of the

Fractions) the Sum is 24; so that $24\frac{1}{4}$ is the Sum of the mixt Numbers given to be added.

C H A P. XIX.

Subtraction of Vulgar Fractions and mixt Numbers.

I. WHEN the Numbers given are both single Fractions, and have equal Denominators: subtract the lesser

The Subtraction of single Fractions, v z.

1. When they have a common Denominator.

the Remainder over the common Denominator, so is such new Fraction the Difference between the Fractions given. Thus the Difference betwixt the Fractions $\frac{9}{11}$ and $\frac{7}{11}$ is $\frac{2}{11}$, which is found by subtracting the lesser Numerator 7, from the Greater Numerator 9, and placing the Remainder 2 over the common Denominator 11: Also the Difference between the Fractions $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{1}{6}$; that is, the Fraction $\frac{1}{2}$ exceeds $\frac{1}{3}$ by $\frac{1}{6}$.

II. When the Numbers given are both single Fractions, and have not a common Denominator, reduce them to Fractions of the same Value which have a common Denominator (by the fourteenth or fifteenth Rule of the seventeenth Chapter) and then find their Difference by the last Rule.

So the Difference between the Fractions $\frac{6}{7}$ and $\frac{2}{3}$, will be found $\frac{1}{21}$, viz. reducing the Fractions given to their equivalent Fractions $\frac{18}{21}$ and $\frac{14}{21}$, which have a common Denominator, the Difference sought will be found $\frac{4}{21}$, by the First Rule of this Chapter. Likewise $\frac{1}{2}$ being subtracted from $\frac{1}{3}$, there remains $\frac{1}{6}$.

III. When one of the Numbers given is a whole Number or a mixt Number, also when both of them are mixt Numbers; reduce such whole, or mixt Numbers to an improper Fraction or Fractions, by the tenth or eleventh Rule of the seventeenth Chapter, and then the Operation will be according to the first or second Rule of this Chapter.

The Subtraction of mixt Numbers, viz. 1 By a general Rule.

So $7\frac{3}{5}$ being given to be subtracted from 12, the Remainder will be found $4\frac{2}{5}$; viz. First $7\frac{3}{5}$ will be reduced to the improper Fraction $\frac{38}{5}$, also 12 will be reduced to $\frac{12}{1}$, then these two improper Fractions $\frac{38}{5}$, and $\frac{12}{1}$, will be reduced to their equivalent Fractions $\frac{38}{5}$ and $\frac{60}{5}$, (which have a common Denominator.) Lastly, the Difference between $\frac{38}{5}$ and $\frac{60}{5}$, is $\frac{22}{5}$, or $4\frac{2}{5}$; In like manner, $9\frac{1}{2}$ being given to be subtracted from $12\frac{1}{5}$, the Remainder will be found $2\frac{7}{10}$, as by the subsequent Operation is manifest.

12	$7\frac{3}{5}$	$12\frac{1}{5}$	$9\frac{1}{2}$
$\frac{12}{1}$	$\frac{38}{5}$	$\frac{61}{5}$	$\frac{19}{2}$
60		122	
38		95	
$\frac{22}{5}$, that is, $4\frac{2}{5}$		$\frac{27}{10}$, that is, $2\frac{7}{10}$	

Though the Three last Rules are sufficient for all Cases in *Subtraction of Fractions*, mixt Numbers, or whole and mixt; nevertheless, the following Rules will be more expeditious in the Subtraction of mixt Numbers, or and whole mixt, especially when the Integers consist of many Places, as is manifest by the Operation, viz.

IV. When a whole Number is given to be subtracted from a mixt Number, subtract the said whole Number from the Integer or Integers of the mixt Number (as is taught by the Rules of the 4th Chapter,) and unto the Remainder annex the fractional Part of the mixt Number given; so is the mixt Number thus found, the Remainder or Difference sought.

As if 7 be given to be subtracted from $24\frac{5}{8}$, the Remainder will be $17\frac{5}{8}$, as appears from the Operation.

2 By particular Rules, viz.
1. A whole Number from a mixt Number.

$$\begin{array}{r} 24\frac{5}{8} \\ 7 \\ \hline 17\frac{5}{8} \end{array}$$

V. When a Fraction is given to be subtracted from an Integer, subtract the Numerator from the Denominator, and place that which remains over the Denominator, which new Fraction thus found, is the Remainder or Difference sought.

2. A Fraction from an Integer.

So $\frac{3}{5}$ being subtracted from an Integer, or 1, the Remainder is $\frac{2}{5}$: Also $\frac{1}{2}$ being subtracted from 1, the Remainder is $\frac{1}{2}$.

VI. When a Fraction is given to be subtracted from a whole Number greater than 1, subtract the said Fraction from one of the Integers (given by the last Rule;) so the Remaining Fraction being annexed to the Number of Integers less'n'd by Unity or 1, gives the Remainder or Difference sought.

Thus $\frac{5}{7}$ being subtracted from 17, the Remainder is $16\frac{2}{7}$; also $\frac{7}{12}$ being subtracted from 39, the Remainder is $38\frac{5}{12}$.

VII. When a mixt Number is given to be subtracted from a whole Number, subtract first of all (by the Fifth Rule of this Chapter) the fractional Part of the mixt Number from an Integer borrowed from the whole Number given, and set down the remaining Fraction; then adding the Integer borrowed to the Integer or Integers of the mixt Number, subtract the said Sum from the whole Number given (as is taught in Subtraction of whole Numbers;) so that which remains, together with the remaining Fraction before found, is the Remainder or Difference sought.

$$\begin{array}{r} 50 \\ 9\frac{7}{12} \\ \hline 40\frac{5}{12} \end{array}$$

So if $9\frac{7}{12}$ be subtracted from 50, the Remainder is $40\frac{5}{12}$, as by the Operation is manifest.

VIII. When a Fraction is given to be subtracted from a mixt Number, and the said Fraction is less than the fractional Part of the mixt Number; subtract the lesser Fraction from the greater by the first or second Rule of this Chapter; then the remaining Fraction being annexed to the Integer or Integers of the mixt Number, gives the Remainder or Difference sought for.

5. A Fraction from a mixt Number by this and the next Rule.

$$\begin{array}{r} 12\frac{7}{8} \\ 0\frac{5}{8} \\ \hline 12\frac{2}{8} \end{array}$$

So $\frac{5}{8}$ being subtracted from $12\frac{7}{8}$, the Remainder is $12\frac{2}{8}$, as by the Operation is manifest.

IX. When a Fraction is given to be subtracted from a mixt Number, and the said Fraction is greater than the Fractional Part of the mixt Number, subtract the said greater Fraction from an Integer borrowed from the mixt Number (by the fifth Rule of this Chapter,) and add the remaining Fraction to the fractional Part of the mixt Number, (by the first or second Rule of the XVIIIth Chapter;) so the Fraction found by that Addition, being annexed to the Integers of the mixt Number lessened by an Integer, or 1, gives the Remainder or Difference sought.

Thus

Thus $\frac{5}{8}$ being subtracted from $13\frac{3}{8}$; the Remainder is $12\frac{5}{8}$, viz. subtracting $\frac{5}{8}$ from 1, the Remainder is $\frac{3}{8}$, which added to $\frac{3}{8}$ gives $\frac{6}{8}$, which being annexed to 12, (the Number of Integers in the mixt Number lessened by 1 or Unity) gives $12\frac{5}{8}$, the Remainder sought.

$$\begin{array}{r} 13\frac{3}{8} \\ - \frac{5}{8} \\ \hline 12\frac{5}{8} \end{array}$$

X. When a mixt Number is given to be subtracted from a mixt Number, and the fractional Part of the mixt Number to be subtracted, is less than the fractional Part of the mixt Number from which you are to subtract, subtract the said lesser Fraction from the greater (by the first or second Rule of this Chapter) and set down the remaining Fraction: Also subtract the Integers of the lesser mixt Number from the Integers of the greater (as in Subtraction of whole Numbers;) so is the mixt Number thus found, the Remainder or Difference sought for.

6. A mixt Number from a Mixt Number; by this and the next Rule.

So if $17\frac{3}{8}$ be given to be subtracted from $20\frac{5}{8}$; the Remainder will be found $3\frac{1}{8}$, viz. subtracting $\frac{3}{8}$ from $\frac{5}{8}$, the Remainder is $\frac{2}{8}$, also subtracting 17 from 20, the Remainder is 3.

$$\begin{array}{r} 20\frac{5}{8} \\ - 17\frac{3}{8} \\ \hline 3\frac{1}{8} \end{array}$$

XI. When a mixt Number is given to be subtracted from a mixt Number, and the fractional Part of the mixt Number to be subtracted, is greater than the fractional Part of the mixt Number from which you are to subtract; subtract the said greater Fraction from an Integer borrowed from the greater mixt Number (by the fifth Rule of this Chapter,) and add the remaining Fraction to the fractional Part of the greater mixt Number (by the first or second Rule of the eighteenth Chapter;) so is the Sum to be reserved as the fractional Part of the Remainder sought: Then add the Integer borrowed to the Integer or Integers of the lesser mixt Number, and subtract the Sum from the Integers of the greater mixt Number (as in Subtraction of whole Numbers;) so that which remains, together with the Fraction before reserved, is the Remainder or Difference sought for.

Thus

Thus if $20\frac{7}{8}$ be given to be subtracted from $35\frac{1}{2}$, the Remainder will be found $14\frac{2}{4}$, viz. Subtracting $\frac{7}{8}$ from an Integer, or 1, the Remainder is $\frac{1}{8}$, which added to $\frac{1}{2}$, gives $\frac{3}{4}$; then adding the Integer borrowed to 20, it will be 21, which subtracted from 35, the Remainder is 14, so the Remainder or Difference sought, is $14\frac{2}{4}$.

When you cannot clearly discern which is the greater of two Fractions, having unequal Denominations, reduce them to Fractions of the same Value which have a common Denominator, (by the fourteenth Rule of the seventeenth Chapter,) and then it will be apparent which of the two Fractions is the greater. As if it be desired to know which of these two Fractions $\frac{6}{7}$ and $\frac{11}{13}$ is the greater; after they are reduced to $\frac{78}{91}$ and $\frac{77}{91}$, it is evident that the former exceeds the latter by $\frac{1}{91}$.

CHAP. XX.

Multiplication of Vulgar Fractions, and Mixt Numbers.

I. **W**HEN the Numbers given to be multiplied, are both single Fractions, multiply the Numerators one by the other, and take the Product for a new Numerator; also multiply the Denominators one by the other, and the Product will be a new Denominator, which new Fraction is the Product sought for.

So $\frac{1}{2}$ and $\frac{5}{8}$ being given to be multiplied, the Product will be found $\frac{5}{16}$; for 7 multiplied by 5 produces 35 for a new Numerator; and 12 multiplied by 8 gives 96 for a new Denominator; also $\frac{5}{7}$ and $\frac{3}{7}$ being multiplied one by the other, the Product will be found $\frac{15}{49}$. Here you may observe that in the Multiplication of proper Fractions, the Product is always less than either of the Terms given; for in Multiplication such

Pro-

Proportion as Unity or 1 has to either of the Terms given, the same Proportion has the other Term to the Product.

II. When one of the Numbers given is a whole Number or a mixt Number; also when both of them are mixt Numbers, reduce such whole Number, or mixt Number or Numbers to an improper Fraction or Fractions, by the 10th or 11th Rules of the 17th Chapter, and then the Operation will be the same as in the last Rule.

To multiply mixt Numbers.

So $8\frac{2}{3}$ being given to be multiplied by 5, the Product will be found $43\frac{1}{3}$; viz. $8\frac{2}{3}$ being reduced to the improper Fraction $\frac{26}{3}$, also 5 into $\frac{5}{1}$, multiply 26 by 5, the Product is 130 for a new Numerator: Also multiplying 3 by 1, the Product is 3, for a new Denominator, which new Fraction $\frac{130}{3}$ being reduced (according to the thirteenth Rule of the seventeenth Chapter,) will be $43\frac{1}{3}$ the Product sought. In like manner, $7\frac{1}{2}$ being multiplied by $\frac{3}{4}$, the Product will be found 42. Here observe, That when either of the Terms given is a compound Fraction, it must first of all be reduced to a single Fraction, and then the Operation is as before.

Note 1. Sometimes the Work of Multiplication in Fractions may be very usefully contracted by the following Rule, viz.

When two Fractions proposed to be multiplied (whether they be proper or improper) are such that the Numerator of the one, and the Denominator of the other may be severally divided by some common Divisor without a Remainder, you may take the Quotients instead of the said Numerator and Denominator, and then multiply as before in the first Rule of this Chapter: As for Example, if $\frac{6}{7}$ be to be multiplied by $1\frac{1}{2}$ because 6, the Numerator of the first, and 12, the Denominator of the latter Fraction, being severally divided by their common Divisor 6, give the Quotients 1 and 2, I set these (or imagine them to be set) in the Places of 6 and 12; by which Exchange there arise $\frac{1}{7}$ and $\frac{2}{3}$, these multiplied one by the other (according to the first Rule of this Chapter) produce $\frac{2}{21}$, the required Product of $\frac{6}{7}$ into $1\frac{1}{2}$ in the smallest Terms.

Again, To multiply $\frac{1}{4}\frac{6}{8}$ by $1\frac{3}{4}$; because the Numerator of the first Fraction and the Denominator of the latter, being each divided by 16, give the Quotients 1 and 1, I set 1 and 1 in

in the Places of 16 and 16 ; likewise because 48 the Denominator of the first, and 3 the Numerator of the latter Fraction, being each divided by their common Divisor 3, give 16 and 1, I take 16 and 1 instead of 48 and 3 ; so by those Exchanges there arise $\frac{1}{16}$ and 1, which multiplied one by the other, produce $\frac{1}{16}$, which is the Product in the smallest Terms made by the Multiplication of $\frac{1}{48}$ into (or by) $\frac{1}{3}$.

2. To take any Part or Parts of a Number proposed, is nothing else but to multiply the said Number by the Fraction, which declares what Part is to be taken : So if you desire to know what is $\frac{5}{8}$ of 320, multiply 320 by $\frac{5}{8}$ or $\frac{40}{8}$ by $\frac{5}{1}$, and the Product will be 200. In like manner, $\frac{2}{5}$ of $45\frac{1}{8}$ is $30\frac{1}{4}$. Also $\frac{1}{4}$ of 120 is 30.

3. Sometimes the Work of Multiplication in mixt Numbers may be compendiously performed after the manner of these following Examples; *viz.* If it be required to multiply $120\frac{1}{4}$ by $48\frac{1}{2}$, first multiply the whole Numbers mutually, to wit, 120 by 48, and place the particular Products orderly one under the

$$\begin{array}{r} 120\frac{1}{4} \\ 48\frac{1}{2} \\ \hline 960 \\ 480 \\ 12 \\ 60 \\ \hline 5832\frac{1}{8} \end{array}$$

other, as in Multiplication of whole Numbers ; then multiply the said whole Numbers first given by the Fractions alternately, *viz.* Take $\frac{1}{4}$ of 48, which is 12, also take $\frac{1}{2}$ of 120, which is 60, and place the said 12 and 60 orderly to be added to the former particular Products : Lastly, add all together, and to the Sum an-

next the Product of the two Fractions ; to wit, in this Example, the Product of the Multiplication of $\frac{1}{4}$ by $\frac{1}{2}$, which is $\frac{1}{8}$, so the total Product required will be $5832\frac{1}{8}$, as you see by the Example in the Margin. In like manner, if $18\frac{1}{2}$ be multiplied by $40\frac{1}{3}$, the Product will be $746\frac{1}{6}$; and if $29\frac{1}{2}$ be multiplied by 50, the Product is 1475, as you see by the Examples following :

$$\begin{array}{r|l} 18\frac{1}{2} & 29\frac{1}{2} \\ 40\frac{1}{3} & 50 \\ \hline 720 & 1450 \\ 20 & 25 \\ 6 & \\ \hline 746\frac{1}{6} & 1475 \end{array}$$

4. When

4. When a Fraction is to be multiplied by a Number which happens to be the same with the Denominator, take the Numerator for the Product; so if this Fraction $\frac{3}{4}$ be proposed to be multiplied by the Denominator 4, the Product will be $\frac{12}{4}$, that is, 3, which is the same with the Numerator 3. In like manner, if $\frac{5}{8}$ be multiplied by the Denominator 8, the Product is equal to 5 the Numerator of the said $\frac{5}{8}$.

CHAP. XXI.

Division of Vulgar Fractions and Mixt Numbers.

I. When the Numbers given are both single Fractions, multiply the Denominator of the Divisor by the Numerator of the Dividend; and take the Product for a new Numerator: Also multiply the Numerator of the Divisor by the Denominator of the Dividend, and the Product is a new Denominator; which new Fraction is the Quotient sought.

So if $\frac{4}{5}$ be given to be divided by $\frac{3}{7}$, the Quotient will be found $\frac{28}{15}$; viz. multiplying 5 by 4 the Product is 20 for a new Numerator; also multiplying 3 by 9 the Product is 27 for a new Denominator, so is $\frac{20}{27}$ the Quotient sought. In like manner, if $\frac{5}{8}$ be given to be divided by $\frac{2}{7}$, the Quotient will be found $\frac{35}{16}$ that is, $2\frac{3}{16}$, as you see in the Example: Here you may observe that in Division, by proper Fractions, the Quotient is always greater than either of the Fractions given; for in Division, as the Divisor is in Proportion to 1 or Unity, so is the Dividend to the Quotient.

II. When one of the Numbers given is a whole Number or a mixt Number; also when both are mixt Numbers, reduce such whole Number or mixt Number or Numbers to an improper Fraction or Fractions, by the Tenth or Eleventh Rule of the 7th Chapter, and then the Operation will be the same as in the last Rule.

So if 42 be divided by $7\frac{1}{2}$, the Quotient will be found $5\frac{3}{5}$, for $7\frac{1}{2}$ and 42 will be reduced to these improper Fractions $\frac{15}{2}$ and $\frac{84}{2}$, then multiplying 42 by 2, the Product 84 for a new Numerator, also

$$\begin{array}{r} 7\frac{1}{2}) \quad 42 \quad (\\ \underline{15) \quad 84} \quad (5\frac{3}{5} \end{array}$$

Q

multi.

multiplying 15 by 1, the Product is fifteen for a new Denominator; so is $\frac{3}{4}$. the Quotient sought, which is equal to $5\frac{3}{4}$ (as is evident by the 13th Rule of the 17th Chapter). In like manner) if $6\frac{1}{2}$ be divided by $3\frac{2}{3}$, the Quotient will be $1\frac{3}{4}$. Also if $5\frac{1}{2}$ be divided by $12\frac{1}{2}$, the Quotient will be $\frac{3}{7}$.

Note. Sometimes the Work of Division in Fractions may be very usefully contracted by this following Rule, *viz.* When either the two Numerators, or the two Denominators of the Fractions propos'd, can be divided severally by some common Divisor without a Remainder; you may take the Quotients instead of the said Numerators or Denominators, and then divide by the first Rule of this Chapter: As for Example, If $\frac{12}{17}$ be to be divided by $\frac{8}{7}$, because the Numerators 12 and 8 being each divided by their common Divisor 4, will give the Quotients 3 and 2; I take these instead of 12 and 8, by which Exchange there arise $\frac{3}{7}$ and $\frac{2}{7}$. the former of which being divided by the latter, (according to the first Rule of this Chapter) gives $\frac{3}{2}$, which is the Quotient in the least Terms that arises by dividing $\frac{12}{17}$ by $\frac{8}{7}$.

Again, To divide $2\frac{5}{8}$ by $1\frac{5}{8}$; because the Numerators 25 and 15 being severally divided by their common Divisor 5, give the Quotients 5 and 3; likewise because the Denominators 8 and 8 being each divided by 8, give the Quotient 1 and 1, I set 5 and three in the Places of the Numerators 25 and 15, also 1 and 1 in the Places of the Denominators 8 and 8, whence arise 5 and 3: Lastly, dividing $\frac{5}{1}$ by $\frac{3}{1}$, that is 5 by 3, there arises $\frac{5}{3}$ that is, $1\frac{2}{3}$, which is the desired Quotient of $2\frac{5}{8}$ divided by $1\frac{5}{8}$.

Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. THE Difference of two Numbers is $1\frac{1}{4}$, the lesser Number is $2\frac{1}{8}$, what is the greater? *Ans.* $3\frac{3}{8}$, (found by *Addition*.)

Quest. 2. What Number is that, which if added to $3\frac{5}{8}$ gives the Sum $8\frac{3}{8}$? *Ans.* $4\frac{1}{4}$ (found by *Subtraction*.)

Quest. 3. There is in three Bags the Sum of $121\frac{2}{4}$ l. *viz.* in the first Bag $50\frac{5}{8}$ l. in the second $40\frac{4}{8}$ l. what is in the third Bag? *Ans.* $20\frac{1}{4}$ l. (found by *Addition* and *Subtraction*.)

Quest. 4. Two Merchants A and B having certain Shares in a Ship, the Share of A is $\frac{7}{13}$ of the Ship, that of B $\frac{2}{13}$, what is the Difference between their Parts? *Ans.* The Share of A exceeds the Share of B by $\frac{5}{13}$ (found by *Subtraction*) *Quest.*

Quest. 5. What is $\frac{5}{8}$ of $130\frac{1}{2}$? *Answ.* 81^2 (found by *Multi- plication.*)

Quest. 6. What Number is that, which being multiplied by $\frac{3}{4}$, produces $25\frac{2}{3}$? *Answ.* $42\frac{1}{2}$ (found by *Division.*)

Now follows the Doctrine of *Decimal Fractions.*

The Doctrine of Decimal Fractions.

C H A P. XXII.

Notation of Decimal Fractions.

IT is hard to determine, who was the first that brought *Decimal Arithmetick* to light, though it be a late Invention; but without doubt it has received much Improvement within the Compass of a few Years by the Industry of *Artists*, and now seems to be arrived at Perfection. The Excellency thereof is best known to such as can apply it to the practical Part of the *Mathematicks*, and to the Construction of *Tables*, which depend upon standing or constant Proportions; such are *Trigonometrical Canons, Tables* for the computing of *Compound Interest, &c.* in which Cases *Decimal Operations* afford so great Help, that, in my Opinion, many Ages have not produced a more useful Invention. But it may be objected, that *Decimal Arithmetick* for the most part gives an Imperfect Solution to a Question. This I grant, yet the Answer so given may be as useful as that which is exactly true; for in common Affairs, the Loss of $\frac{1}{1000}$ Part of a Grain or of an *Inch, &c.* to wit, any Quantity which cannot be seen, is inconsiderable: But I would not be mistaken; for in extolling *Decimals*, I do not cry down *Vulgar Fractions*; since Experience shews, that *Decimal Fractions* are commonly abused, by being applied to all manner of Questions about *Money, Weights, &c.* when indeed many Questions may be resolved with more Facility by *Vulgar Arithmetick*, as may partly appear by this Example, viz. At 9 l. — 6 s. — 8 d. the Hundred weight of Tobacco, what will 987 Hundred weight cost? *Answ.* 9212 l. which by the common *Rule of Practice* by *Aliquot Parts* is found out in a Quarter of the Time, that will

The proper Use of Decimal Arithmetick.

Decimal Fractions sometimes abused.

necessarily be required to work it by *Decimals*, which at last will give an imperfect Answer: I might instance the like Inconvenience diverse ways, were it not for Loss of Time; so that the right Use of *Decimals* depends upon the Discretion of the Artist.

II. When a single Fraction has for its Denominator, a Number consisting of 1 or Unity in the extreme Place towards the Left Hand, and nothing but a Cypher or Cyphers towards the Right, it is more particularly called a *Decimal*: Of this kind are these that follow, viz. $\frac{1}{10}$ that is, five Tenths; $\frac{1}{100}$ five hundredth parts; likewise these are Decimal Fractions, $\frac{34}{1000}$, $\frac{205}{10000}$, $\frac{1023}{100000}$, &c.

III. A Decimal Fraction may be express'd without the Denominator, by prefixing a Point or Comma before (to wit, on the Left-hand of) the Numerator; so $\frac{1}{10}$ may be writ thus .5 or thus ,5 and $\frac{25}{100}$ thus, .25 or thus ,25.

IV. In Decimals, when the Numerator does not consist of so many Places as the Denominator has Cyphers; fill up the void Places in the Numerator with Cyphers prefixed on the Left-hand: So $\frac{1}{100}$ is set down thus, .05; likewise $\frac{5}{1000}$, thus .050; and $\frac{205}{10000}$, thus, .0205, likewise $\frac{6}{100000}$, thus, .006.

V. In Decimals thus express'd, the Denominator is discoverable by the Places of the Numerator: For if the Numerator consists of one Place, the Denominator consists of 1 or Unity with one Cypher; if of two Places, the Denominator consists of 1 with two Cyphers annexed; if of three, the Denominator consists of 1 or Unity, with three Cyphers annexed: So the Denominator of .25 is 100 the Denominator of .050 is 1000, and the Denominator of .006 is 1000.

VI. Cyphers at the End of a Decimal do neither augment nor diminish the Value of it: So .2 .20, .200, .2000 are *Decimals*, which have one and the same Value; for $\frac{2}{10}$ being abbreviated by the eighth Rule of the 17th Chapter, will be made $\frac{2}{10}$, and so will $\frac{2000}{10000}$ or $\frac{20000}{100000}$

VII. Therefore Decimal Fractions are easily reduced to a common Denominator, (which is a troublesome Work in Vulgar Fractions;) for if all the Numerators of as many Decimal Fractions as are given, be made to consist of the same Number of Places, by annexing a Cypher or Cyphers at the End. (that is on the Right-hand) of such Numerators as are defective, they will all be reduced to a common Denominator: So these Decimals, .2, .03, .027, (which signifie $\frac{2}{10}$ $\frac{3}{100}$ $\frac{27}{1000}$) may

may be reduced to these, .200, .030, .027, which have 1000 for a common Denominator.

VIII. The Order of Places in any Decimal proceeds from the Left-hand to the Right, contrary to the Order of Places in Integers, which is from the Right-hand to the Left: So in this Decimal 247 the Figure 2 stands in the first Place, (being the outermost towards the Left-hand, and next to the Point,) the Figure 4 stands in the second Place, and 7 in the third. Also in this Decimal .0245 a Cypher stands in the first Place, 2 in the second, 4 in the third, and 5 in the fourth.

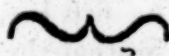
IX. Every Place in the Numerator of a Decimal Fraction has a peculiar Denominator, or proper Value, viz the Denominator of the first Place is 10, of the second 100, of the third 1000, &c. So that the first Place of a Decimal, signifies tenth Parts of an Unite or Integer; the second Place, hundredth Parts of an Integer; the third Place, thousandth Parts of an Integer, &c. Hence it is manifest, that this Decimal .3254 (every Place thereof being considered a-part by it self) consists of .3 .02, .005, .0004, (viz. $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$, which being reduced to a common Denominator (by the seventh Rule of this Chapter) will give these 3000, .0200, .0050, .0004, viz. $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$, all which collectively make .3254 (or $\frac{3254}{10000}$)

X. In whole Numbers, the first place above (that is on the Left-hand of) the place of Units signifies Tens of Units, but in Fractions the first Place beneath (that is, on the Right hand of the Place of Units) denotes tenth Parts of 1 or Unity, and is called the first Place of Decimal Parts, or place of Primes; likewise the second Place above the Place of Units signifies hundreds of Units; but the second Place beneath the place of Units, expresses hundredth parts of 1 or Unity, and is called the second Place of Decimals, or Place of Seconds: So that as the Values of the places in Integers, ascend in a decuple Proportion from the place of Units towards the Left-hand; so the Values of the places of Decimals descend in a subdecuple Proportion beneath the place of Units towards the Right-hand; viz. among the places of Integers, every following place towards the Left-hand, is ten times the Value of the next preceeding place: But among the Places of Decimal Parts, every following place towards the Right-Hand is one tenth Part of the Value of the next preceeding place: All which will be evident by the following TABLE.

A T A-

A TABLE for the NOTATION of Integers and Decimals.

I N T E G E R S.

 of Units.					 of 1 or Unity.				
&c. Ten Thousands Thousands Hundreds Tens Units (under 10)					Tentb Parts Hundredtb Parts Thousandtb Parts Ten thousandtb Parts &c.				
7 3 2 8 5					. 8 2 3 7				
&c. Fifth Place Fourth Place Third Place Second Place First Place					First Place Second Place Third Place Fourth Place &c.				

D E C I M A L P A R T S.

In the fore-going Table you may observe, That the Places of Integers, or whole Numbers are separated from the Places of *Decimal Parts* of 1, or Unity, by a Point; so the Number on the Left-hand of the Point expresses 73285 Integers or Units: But the Number on the Right-hand of the Point only shews 8237 Parts of 1 (or an Integer) suppos'd to be divided into 10000 equal Parts. In like manner, this Number 5.8 signifies 5 Integers and eight tenth Parts of an Integer, and this Number 285.82 denotes 285 Integers (or Units) and $\frac{82}{100}$ Parts of an Integer.

C H A P

C H A P. XXIII.

Concerning the Reduction of Vulgar Fractions to Decimal Fractions.

I. IF the greatest Integer of Money, as also of Weight, Measure, &c. were subdivided decimally, to wit, a Pound of English Money into ten equal Pieces of Coin, and every one of these into ten other equal Pieces, &c. and Weights, Measures, &c. after the same manner: The Doctrine of Arithmetick would be taught, with much more Ease and Expedition than now it is; but it being improbable that such a Reformation will ever be brought to pass. I shall proceed in directing a Course to the Studious, for obtaining the frugal Use of such Decimal Fractions as are in their Power.

II. Since in Arithmetical Questions, some of the given Numbers for the most part happen to be Fractions, a Way must be shew'd how to reduce a Vulgar Fraction to a Decimal Fraction; yet in some Cases there is no need of this Reduction: For Example, a Foot in length is commonly subdivided into 12 Inches, an Inch into 4 Quarters, and every Quarter into 2 half Quarters; but a Foot may as easily, and a great deal more commodiously be divided, first into ten equal Parts, and then every of those into ten other equal Parts; and each of these into ten other equal Parts; (or at least such Division may be supposed or imagined when it cannot actually be made.) This Foot in Length so divided, being applied to the Sides of Superficial Figures or Solids, will at first sight give the Quantities of Lines in Feet and Decimal Parts of a Foot (as readily as a Foot vulgarly divided will shew you how many Feet, Inches, Quarters, and half Quarters are contained in any Line) from whence the Superficial or solid Content may be found in Feet by Multiplication only; and how much this excels the vulgar Way, I shall partly manifest in the Fifth Rule of the twenty sixth Chapter. The like Subdivision I would have to be made of a Yard, Pearch, &c.

III. A single Fraction, which is no Decimal Fraction, may be reduced to a Decimal of the same Value, or infinitely near (for all vulgar Fractions cannot be exactly

How to reduce a vulgar Fraction to a Decimal Fraction.

reduced

reduced to Decimals) by the Rule of Three Direct; for as the Denominator of any single Fraction whatsoever, is to the Numerator thereof, so is any other proposed Denominator to its correspondent Numerator: *Example*, Let it be required to reduce $\frac{5}{8}$ to a Decimal, whose Denominator is assigned to be 1000: Say by the Rule of Three, if the Denominator 8 has 5 for a Numerator, what will the Denominator 1000 require for a Numerator? Multiply and divide as the Rule of Three Direct requires, so will the Fourth Proportional be found to be 625, which is the Numerator sought; therefore $\frac{625}{1000}$, or .625, is a Decimal Fraction equal in Value to $\frac{5}{8}$. Another *Example*, Let it be required to reduce $\frac{7}{24}$ to a Decimal Fraction, whose Denominator shall be 100000; say by the Rule of Three, If 240 the Denominator give 7 for Numerator, what will the Denominator 100000 require for a Numerator? *Ans.* 2916 and somewhat more; but that which the said 2916 wants of being a true Numerator is less than $\frac{1}{1000}$ Part of an Integer; therefore the Decimal Fraction $\frac{2916}{100000}$ or .02916 is almost equal to $\frac{7}{240}$ which $\frac{7}{24}$ cannot be exactly reduced to a Decimal Fraction. The like will happen in the Reduction of most vulgar Fractions to Decimals; in which Case, the Denominator of the Decimal must be assigned to be so great, that what is wanting in the Numerator may be of an inconsiderable Value.

IV. Upon the aforesaid Ground, the known or accustomed Parts of *Money, Weight, Measure, Time, &c.* may be reduced to Decimals: For if you desire to know what decimal Fraction of a Pound Sterling is equal in Value to one Shilling, consider first that a Pound is the Integer, and that 20 Shilling are equal to that Integer, therefore 1 Shilling is $\frac{1}{20}$ of a Pound. Now, if we conceive one Pound to be divided into 100000 parts, *viz.* If we assign 100000 for the Denominator of a decimal Fraction, the Numerator will be found by the last Rule to be 5000; so that $\frac{5000}{100000}$ or .05000 or .05 (for Cyphers at the End of a Decimal are of no use, as has been shewn in the Sixth Rule of the twenty second Chapter) is a decimal Fraction of a pound, and is exactly equal to 1 s. or $\frac{1}{20}$ part of a pound Sterling.

In like manner, forasmuch as 240 pence are equal to a pound of *English Money*, 7 pence are $\frac{7}{240}$ parts of a pound, which Fraction will be reduced to this Decimal .02916 *l.* which is very near equal to $\frac{7}{240}$ for it does not want $\frac{1}{100000}$ part of a pound. And, since 960 Farthings are equal to a pound *English*,

English, one Farthing is $\frac{1}{4}$ Part of a Pound, which will be reduced to this *Decimal* .001041. very near; but if you please to proceed nearer to the Truth, you will find this *Decimal* .00104166, &c. to answer a Farthing, and so by augmenting the Denominator with Cyphers, you may proceed infinitely near, when you cannot attain to the Truth it self. After the same Method, may the vulgar *Saxagenary Fractions* used in *Astronomy* be reduced to *Decimals*: For since a *Degree* is usually subdivided into Sixty Parts, called *Minutes* or *Primes*; a *Prime* or *Minute* into Sixty Parts, called *Seconds*; a *Second* into Sixty *Thirds*; a *Third* into Sixty *Fourths*, &c. and consequently a *Degree* is equal to 60 *Minutes* (or *Primes*) or to 3600 *Seconds*, or 216000 *Thirds*, or 12960000 *Fourths*, &c. It is evident that 7 *Minutes* (or *Primes*,) are $\frac{7}{60}$ Parts of a *Degree*, which by the third Rule of this Chapter may be reduced to the *Decimal* .1166, &c. Also 29 *Thirds* are $\frac{29}{216000}$ Parts of a *Degree*, which may be reduced to the *Decimal* .000134. &c.

And farther, 58:33:14:12, that is, 58 *Primes*, 33 *Seconds*, 14 *Thirds*, and 12 *Fourths* may be reduced to a *Decimal* in this manner, viz. reduce them all to *Fourths* (according to the Sixth Rule of the Seventh Chapter) so will you find 12647652 *Fourths*, which are $\frac{12647652}{12960000}$ Parts of a *Degree*, which vulgar *Fraction* may be reduced to this *Decimal* of a *Degree*, to wit, .975899, &c. (by the third Rule of this Chapter.)

This to the Ingenious will be a sufficient Light for the finding of *Decimals* suitable to the *Shillings*, *Pence*, and *Farthings*, which are under a *Pound Sterling*: Also the *Decimals* of the known Parts of *Weight*, *Measure*, *Time*, &c. as they are express'd in the following Table, in which you may observe, that most of the *Decimals* consist of 7 or 8 Figures, yet in ordinary Practice, you'll have Occasion to use only the first Five, and sometimes fewer.

T H E
T A B L E
O F
R E D U C T I O N.

T A B L E T I. Of English Money, the In- teger being a Pound.		Pence with Farthings.	Decimals of a Pound.
Shillings.	Decimals of a Pound.		
19	.95		.0489583
18	.9		.0479166
17	.85		.046875
16	.8		.0458333
15	.75	11	.0447916
14	.7		.04375
13	.65		.0427082
12	.6	10	.0416666
11	.55		.040625
10	.5		.0395833
9	.45		.0385416
8	.4	9	.0375
7	.35		.0364583
6	.3		.0354166
5	.25	8	.034375
4	.2		.0333333
3	.15		.0322916
2	.1	7	.03125
1	.05		.0302083
		6	.0291666
			.028125
			.0270833
			.0260416
			.025

5	0104166	11	0982142
4	0083333	10	0892857
3	00625	9	0803371
2	0041666	8	0714285
1	0020833	7	0625
T A B L E T III. <i>Of Avoirdupois great Weight, the Integer being an hundred-weight, to wit, 112 pounds.</i>		6	0535714
<i>Quarters of 1 hundred.</i> <i>Decimals of 1 hund.</i>		5	0446428
3	.75	4	0357142
2	.5	3	0267857
1	.25	2	0178571
<i>Pounds.</i> <i>Decimals of 1 hund.</i>		1	0089285
27	.2410714	<i>Ounces.</i> <i>Decimals of 1 hund.</i>	
26	.2321428	15	0083705
25	.2232142	14	0078125
24	.2142857	13	0072544
23	.2053471	12	0066964
22	.1964285	11	0061383
21	.1875	10	0055803
20	.1785714	9	0050223
19	.1696428	8	0044642
18	.1607142	7	0039062
17	.1517857	6	0033482
16	.1428571	5	0027901
15	.1339285	4	0022321
14	.125	3	0016741
13	.1160714	2	0011160
12	.1071428	1	0005580
<i>Quarters of 1 Ounce</i> <i>Decimals of 1 hund.</i>		3	0004185
		2	0002790
		1	0001395

TABLET IV.
Of *Avoirdupois* little
Weight, the Integer
being a Pound.

Ounces.	Decimals of a Pound.
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15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

Drams.	Decimals of a Pound.
--------	-------------------------

15	.05859375
14	.0546872
13	.05078115
12	.046875
11	.04296875
10	.0390625
9	.03515625
8	.03125
7	.02734375

6	.0234375
5	.01953125
4	.015625
3	.01171875
2	.0078125
1	.00390625

Quarters of a Dram.	Decimals of a Pound.
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3	.0029296
2	.0019531
1	.0009765

TABLET V.
Of Liquid Measures, the
Integer being a Gallon.

Pints.	Decimals of a Gallon.
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7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Quarters of a Pint.	Decimals of a Gallon.
------------------------	--------------------------

3	.09375
2	.0625
1	.03125

TABLET

TABLET VI.

Of dry Measures, the Integer being a Quarter.

Bush els.	Decimals of a Quarter.
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Pecks.	Decimals of a Quarter.
3	.09375
2	.0625
1	.03125

Quarters of a Peck.	Decimals of a Quarter.
3	.0234375
2	.015625
1	.0078125

Pints.	Decimals of a Quarter.
3	.005859
2	.003906
1	.001953

TABLET VII.

Of Long Measures, one Yard being the Integer.

Quarters of one Yard.	Decimals of one Yard.
3	.75
2	.5
1	.25

Nails.	Decimals of one Yard.
3	.1875
2	.125
1	.0625

Quarters of 1 Nail.	Decimals of one Yard.
3	.046875
2	.03125
1	.015625

TABLET VIII.

Of the Reduction of Inches, &c. to Decimals, the Integer being a Foot in Length.

Inches	Decimals of a Foot.
11	.9166666
10	.8333333
9	.75

		Parts of a D. zen.	D. cimals of a Gross.
8	.666666		
7	.5833333		
6	.5		
5	.4166666	11	076388
4	.3333333	10	069944
3	.25	9	0625
2	.1666666	8	055555
1	.0833333	7	048611
		6	041666
		5	034722
		4	027777
		3	020833
		2	013888
		1	006944

Quarters of an Inch.	D. cimals of a Foot.
3	.0625
2	.0416666
1	.0208333
Half a quar- of an Inch.	.0104166

TABLET X.
Of Time, a day being the
Integer.

TABLET IX
Of Dozens, the Integer
being a Gross.

Dozens.	D. cimals of a Gross.
11	.9166666
10	.8333333
9	.75
8	.6666666
7	.5833333
6	.5
5	.4166666
4	.3333333
3	.25
2	.1666666
1	.0833333

Hours.

Hours.	D. cimals of a day.
23	.9583333
22	.9166666
21	.875
20	.8333333
19	.7916666
18	.75
17	.7083333
16	.6666666
15	.625
14	.5833333
13	.5416666
12	.5
11	.4583333
10	.4166666

9	.375	35	.0243055
8	.3333333	34	.0236111
7	.2916666	33	.0229166
6	.25	32	.0222222
5	.2083333	31	.0215277
4	.1666666	30	.0208333
3	.125	29	.0201388
2	.0833333	28	.0194444
1	.0416666	27	.01875
		26	.0180555
<i>Minutes.</i>	<i>Decimals of a day.</i>	25	.0173611
59	.0409722	24	.0166666
58	.0402777	23	.0159722
57	.0395833	22	.0152777
56	.0388888	21	.0145833
55	.0381944	20	.0138888
54	.0375	19	.0131944
53	.0368055	18	.0125
52	.0361111	17	.0118055
51	.0354166	16	.0111111
50	.0347222	15	.0104166
49	.0340277	14	.0097222
48	.0333333	13	.0090277
47	.0326388	12	.0083333
46	.0319444	11	.0076388
45	.0312500	10	.0069444
44	.0305555	9	.00625
43	.0298611	8	.0034722
42	.0291666	7	.0048611
41	.0284722	6	.0041666
40	.0277777	5	.0034722
39	.0270833	4	.0027777
38	.0263888	3	.0020433
37	.0256944	2	.0012888
36	.0249999	1	.0006944

V. This Table afore-going consists of Ten several Tablets, of which the First (entituled *English Money*) contains in the first Column of it the particular Fractions (*viz.* the *Shillings, Pence, and Farthings*) of a Pound Sterling; and in the other Column the Decimals, to which they may be respectively reduced: So in the same Tablet .65 is the Decimal answerable to 13 s. .0208333 to 5 d. and .003125 to 3 f. Likewise, .0489583 is the Decimal of 11 d. together with 3 Farthings: Also .03125, is the Decimal of 7 Pence Half-penny.

Tablet 1. Of *English Money.*

VI. The next Tablet (intituled *Troy Weight*) contains in the first Column the particular Fractions, (*viz.* the *Penny-weights and Grains*) of an Ounce-Troy, and in the other their respective Decimals: So .6 is the correspondent Decimal of 12 Penny-weight, and .0020833 of 1 Grain. Likewise .025 is the Decimal of 12 Grains.

2. Of *Troy-weight.*

VII. The Third Tablet (intituled *Averdupois great Weight*) contains in the first Column thereof the Fractions, (*viz.* the *Quarters, Pounds, Ounces, and the Quarters of an Ounce of an Hundred, according to Averdupois Weight*), and in the other their proper Decimals: So .5 is the Decimal of two Quarters or half a Hundred, .1517857 of 17 Pounds, .0033482 of 6 Ounces, and .0004185, the Decimal of three Quarters of an Ounce.

3. Of *Averdupois Great Weights.*

VIII. The Fourth (intituled *Averdupois little Weight*) shews the Fractions (*viz.* the *Ounces, Drams, and Quarters of a Dram*) of a Pound Averdupois, together with their respective Decimals: So the Decimal of 3 Ounces is .1875, the Decimal of 9 Drams is .03515625, and the Decimal of one Quarter of a Dram is .0009765.

3. Of *Averdupois Little Weights.*

IX. The Fifth (intituled *Liquid Measures*) has the Fractions (*viz.* the *Pints, and Quarters of a Pint*) of a Gallon, and likewise their several Decimals: So the Decimal of Five Pints is .625, and the Decimal of two Quarters or Half a Pint is .0625.

5 Of *Liquid Measures.*

X. The Sixth (intituled *Dry Measures*) gives the Fractions (*viz.* the *Busbels*, *Pecks*, *Quarters of Pecks*, and *6. Of Dry Pints*.) of a *Quarter*, together with their peculiar Decimals: So 375 is the Decimal of 3 *Busbels*, .03125 of one *Peck*, .0234375 of $\frac{3}{4}$ of a *Peck*, and .003906 of two *Pints*.

XI. The Seventh (intituled *Yards*) offers you the Fractions (*viz.* the *Quarters*, *Nails*, and *Quarters of 7. Of Long Nails*) and their respective Decimals: So .25 is the Decimal of one *Quarter of a Yard*, .125 of two *Nails*, and .046875 of three *Quarters of a Nail*.

XII. The Eighth (intituled *Reduction of Inches, &c. to Decimals of a Foot*) presents you with the Fractions (*viz.* the *Inches*, *Quarters*, and *Half-quarters of an Inch*) of a *Foot*, together, with their correspondent Decimals: So .4166666 is the Decimal of 5 *Inches*, .0625 of $\frac{1}{2}$ of an *Inch*, and .0104166 of $\frac{1}{8}$, or half a *Quarter of an Inch*.

XIII. The Ninth Tablet (intituled *Dozens*) yields you the Fractions (*viz.* the *Dozens* and *Particulars*) of a *Gross*, as also their respective Decimals: So .25 is the Decimal of 3 *Dozen*, and .048611 of 7 *Particulars*.

XIV. The Tenth and last Tablet (intituled *Time*) gives you the Fractions (*viz.* the *Hours* and *Minutes*) of a *Day*: So .625 is the Decimal of 15 *Hours*, .0375 of 54 *Minutes*, and .0006944 of one *Minute*,

XV. When a single Fraction of any of the premised Tablets is proposed to be reduced to a Decimal, find it in the first Column of the Tablet to which it belongs; this done, just against that Fraction so found, you'll have the Decimal required. So 13 *s.* being propounded, taking the first premised Tablet, I find 13 *s.* in the first Column of the Tablet of *Money*, and just against the same 13 *s.* I observe, 65, before which having prefixed a Point, and by that means signified it for a Decimal, (according to the third Rule of the twenty second Chapter of this Book;) I conclude the same .65 so ordered, to be the correspondent Decimal

mal of 13s. the Fraction propounded. In like manner, .0229166 is the Decimal of 11 Grains in the Tablet of *Troy-weight*; and .0357142 the Decimal of 4 l. in the Table of *Avoirdupois great Weight*, &c.

XVI. When two or more Fractions are propounded, and it is requir'd to find a Decimal equivalent to the Sum of them, find the Decimal of each of the Fractions given according to the last Rule; then adding together the Decimals so found, that entire Sum is the Decimal sought: So 13s. 5d. being reduced to a Decimal, .670833; for the Decimal of 13s. is 65, and the Decimal of 5d. .020833; which being added together (by the second Rule of the Twenty fourth Chapter of this Book) amount to .670833, viz. the Decimal which represents 13s. 5d. the Fraction propoted: In like manner the Decimal of 9 Penny-weight and 13 Grains, is .04770833, and the Decimal of $\frac{1}{2}$ C. 19 lb. 7 oz. is .67354, &c.

13 s.	.65
5 d.	.020833
	.670833
9 p.w.	.45
	.027083
	.4770833
$\frac{1}{2}$ C.	.5
19 lb.	.169642
7 unc.	.002906
	.673542

And here as you see meer Fractions reduced, so likewise may the Fractions of mixt Numbers be reduced to Decimals; for Example. These Numbers 97 lb. 7 Ounces, 13 $\frac{1}{2}$ Drams. Item 67 Gallons 5 $\frac{3}{4}$ Pints. Item 28 Quarters, 0 Bushels, and 2 $\frac{1}{2}$ Pecks, after Reduction are 97.4891, 67.187, and 28.0781.

97.4375	67 625	28.0625
.0607	.0937	.0156
.0009		
	67 7187	28.0781
97 4891		

Again, $22\frac{1}{2}$ Yards, $3\frac{1}{4}$ Nails; Item 36 Grofs, 3 Dozen and 5 Particulars, being reduced, are 22.7031, 36.2847.

22.5	
.1875	36.25
.0156	.0347
22.7031	36.2847

XVII. When a Decimal is proposed, to know what Fraction it represents, search the same Decimal in the second Column of the Tablet, to which it belongs; where if you find it expressly, the Number just against it in the first Column is the Fraction you look for: So .65 (representing the Fraction of a Pound *Sterling*.) being given, I find it in the second Column of the Tablet of *Money*, and over-against it in the first Column I find 13 s. which is the Fraction represented by .65, the Decimal propounded. In like manner, 3.025, (representing 3 Ounces and 025 of an Ounce *Troy*) being given, the Number represented by it, is 3 Ounces, 0 p. w. 12 Grains.

XVIII. When in the second Column of the Tablet to which you are directed, you cannot precisely find the *Decimal* prepos'd, search that which being less, comes nearest to it, and take the Number that answers it in the first Column, for the greatest Fraction of the Number required: Then deducting the *Decimal* so found, out of the *Decimal* given, find likewise the Remainder as another *Decimal*, and take its correspondent Number for the next Fraction of the Number required; and so proceed in that Order, till you have discovered the entire Number represented by the *Decimal* propounded.

Example, 6739 being given, I demand the Fraction of a Pound *Sterling* represented by it: The *Decimal* in the Tablet of *Money*, which being less, comes nearest to .6739 is .65, whose correspondent Number in that Tablet is 13, which are the Shillings of the Number required; then subtracting (by the first Rule of the 25th Chapter of this Book) .65 out of .6739 the Remainder is .0239, and the nearest *Decimal* in the same Tablet to .0239, is .0208, whose correspondent Number is 5, which are the Pence of the Number required. Last of all, deducting .0208 out of .0239, the Remainder is .0031, which gives

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gives you in the first Column 3, being the Farthings of the Number required: So that I conclude the entire Fraction represented by the Decimal 6739, is 13 s. 5 d. 3 f.

$$\begin{array}{r}
 \text{Subtract } 13 \text{ s.} \quad \underline{\quad .6739 \text{ l. Sterling.} \quad} \\
 \quad \quad \quad \underline{\quad .65 \quad} \\
 \quad \quad \quad \underline{\quad .0239 \quad} \\
 \text{Subtract } 5 \text{ d.} \quad \underline{\quad .0208 \quad} \\
 \quad \quad \quad \underline{\quad .0031 \quad} \\
 3 \text{ f.} \quad \underline{\quad .0031 \quad}
 \end{array}$$

In like manner, 7.359 C. being reduced by the Tablet of Avoirdupois great Weight, is $7\frac{3}{4}$ C. 12 l. 4 Ounces: and 94.58 lb. reduced by the Tablet of Avoirdupois little Weight, is 94 lb. 9 Ounces, and 6 Drums.

$$\begin{array}{r}
 \text{Subtract } 1 \text{ quarter.} \quad \underline{\quad 7.359 \text{ C.} \quad} \\
 \quad \quad \quad \underline{\quad .25 \quad} \\
 \quad \quad \quad \underline{\quad .109 \quad} \\
 \text{Subtract } 12 \text{ lb.} \quad \underline{\quad .107 \quad} \\
 \quad \quad \quad \underline{\quad .002 \quad} \\
 4 \text{ oz.} \quad \underline{\quad .56 \quad} \\
 \text{Subtract } 9 \text{ oz.} \quad \underline{\quad .02 \quad} \\
 6 \text{ Drums} \quad \underline{\quad .02 \quad}
 \end{array}$$

CHAP. XXIV.

Addition of Decimal Fractions.

I. TO such as well understand the Notation of Decimal Fractions, all the Varieties of their Numeration, to wit, Addition, Subtraction, &c. will be as easy as the Operations by whole Numbers; therefore he that would be a good Proficient in *Decimal Arithmetick*, must thoroughly understand the 22d and 23d Chapters foregoing.

II. When divers Decimal Fractions are given to be added together, they must first of all be orderly placed one under another

another according to the Doctrine of their Notation. So if these *Decimal Fractions*, to wit, .125, .39, and .7 were given to be added, they must be written thus;

$$.125.$$

$$.39$$

$$.7$$

or if you will have the same Number of Places to be in all the *Decimals* given, without altering their Values, they may be writ thus,

$$.125$$

$$.390$$

$$.700$$

Not thus,

$$.125$$

$$.39$$

$$.7$$

For the Figures or Cyphers, which are of like Degrees or Places, must be subscribed directly one under another, viz. *Tenth Parts* or *Primes* must be set down exactly under *Tenths*; also *Hundredth Parts* or *Seconds* are to be placed under *Hundredth Parts*: As you see in the first *Example*, where 3 or three tenth Parts in the second *Decimal*, stands directly under 1 or one tenth Part in the first *Decimal*; likewise 7 or seven Tenths in the third *Decimal*, stands directly under the tenths in the former, and so of the rest.

In like manner, when mixt Numbers, which consist of Integers and Decimal Parts, are given to be added, due Respect must be had to their Subscription one under another: So if these mixt Numbers, to wit, 32.056, 7.07, and 1.9 were given to be added, they are to be writ down thus,

$$32.056$$

$$7.07$$

$$1.9$$

III. Having placed the Decimals, and drawn a Line underneath in manner aforesaid, add them together, beginning with the outermost Rank towards the Right-hand (as has been taught in Addition of whole Numbers of one Denomination in the Third Chapter:) So if the Decimals in the first *Example* of the Second Rule of this Chapter were given to be added,

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added, I first subscribe 5, which is all that stands in the first Rank towards the Right-hand; then proceeding to the second Rank, I say 9 and 2 make 11; wherefore I set down 1, which is the excess of 11 above 10; and for the 10 I carry 1 in mind to the next Rank, saying 1 in mind added to 7 makes 8, which added to 3 and 1 make 12, wherefore I write 2, which is the excess of 12 above 10, under the Line, reserving 1 in mind for the 10; then I prefix a Point before 2, which stands in the first Place of Decimals; and on the Left-hand of the Point, to wit, in the Place of Units, or first Place of Integers, I write 1 (being the 1 in Mind;) which done, I find that the Sum of the Decimals given is, 1.215, that is, one Integer (whether it be a Perch, Yard, Foot, &c.) and $\frac{215}{1000}$ Parts of an Integer, as you see in the Example. In like manner, these mixt Numbers 32.056, 7.07, and 1.9 being given to be added, their Sum will be found to be 41.026, that is 41 Integers, and $\frac{26}{1000}$ Parts of an Integer, as you see in the Margin; more Examples for the Learner's Exercise are these :

$$\begin{array}{r}
 .125 \\
 .39 \\
 7 \\
 \hline
 1.215 \\
 \\
 32.056 \\
 7.07 \\
 1.9 \\
 \hline
 41.026
 \end{array}$$

.65	24 : 7	503 . 75
.025	0 . 35	0 . 32
.03	5 . 26	0 . 12
705	30 . 31	504 . 19

CHAP. XXV.

Subtraction of Decimal Fractions.

HAVING first set down the greater of the two Numbers given (whether it be a whole Number, Mixt Number, or Decimal) and the lesser under the greater, according to the Directions in the Second Rule of the 24th Chapter; proceed as you are taught in Subtraction of whole Numbers, (by the Rules of the 4th Chapter :) So if this Decimal Fraction .784 were given to be subtracted from this Decimal .837, the Remainder will be .053, that is, $\frac{53}{1000}$ Parts of an Integer; in like manner if this mixt Number

$$\begin{array}{r}
 .837 \\
 .784 \\
 \hline
 .053
 \end{array}$$

$$\begin{array}{r} 295.094 \\ 78919 \\ \hline \end{array}$$

$$216.175$$

ber 78.919 were given to be subtracted from 295.094, the Remainder will be $216\frac{175}{1000}$. In each of which *Examples* you may observe that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole Numbers of one Denomination: *Note* also, when the Decimals in both the Numbers given do not consist of the same Number of Places, that Decimal which is defective in Places towards the Right-hand, should have the void Places filled up with Cyphers, or at least Cyphers must be supposed to be annexed: So if this Decimal .04338 be given to

$$.65000$$

$$.04338$$

$$.60662$$

be subtracted from this .65, the Remainder will be found to be 60662, and the Work will stand as in the Margin; where you see the three void Places are supplied with Cyphers, and then the Operation is as in whole Numbers, by borrowing 10 as often as the lowest Figure cannot be subtracted from the upper. More *Examples* of Subtraction of Decimals are these following:

$$24.04338$$

$$.65$$

$$23.39338$$

$$37.000$$

$$0.104$$

$$36.896$$

$$.394$$

$$.35$$

$$.044$$

C H A P. XXVI.

Multiplication of Decimal Fractions.

WHEN two Numbers are given to be multiplied, and are both mixt Numbers, or both Decimal Fractions, or one of them a whole Number, and the other a Decimal or mixt Number, (which are all the Cases that can happen) there is no Necessity of writing them precisely one under the other, as in *Addition* and *Subtraction*; for the Product or Number sought in *Multiplication* depends not upon any regular placing of the two Numbers given: So if this mixt Number 56.3

$$1.30526$$

$$56.3$$

were given to be added to this mixt Number 1.30526, they ought to be set down one under the other, as you see (according to the Second Rule of the 24th Chapter;) but if they are to be multiplied one by the other, they may be writ thus:

$$1.30526$$

$$\begin{array}{r} 1.30526 \\ 56.3 \\ \hline \end{array}$$

II. In any of these Cases which may happen in Multiplication of Decimals, multiply the Numbers given as if they were whole Numbers; then cut off always from the Product by a Point, Comma, or Line, so many places towards the Right-hand, as there are places of decimal parts in both the Numbers given to be multiplied; that done, the Figure or Figures, if any happen to be, on the Left-hand of the said Point or Line of Separation, declare the Integer or Integers in the Product, and those on the Right-hand of the Point are decimal parts of an Integer: So if this mixt Number 1.30526 be given to be multiplied by this mixt Number 56.3 (that is, 56 Integers and $\frac{3}{10}$ of an Integer) the Product will be found 73.486138, that is, 73 Integers, and $\frac{486138}{1000000}$ parts of an Integer; for having chose that to be the Multiplier, which will cause least Work, and subscribed it under the Multiplicand (to wit, 56.3 under the 1.30526) I proceed according to the Rules of Multiplication of whole Numbers, viz. Having drawn a Line under the Numbers given, I multiply all the Multiplicand, to wit, 1.30526, as if it were a whole Number, by 3 the first multiplying Figure, and subscribe the Product thereof which is 391578 under the Line, and proceeding in like manner with the other multiplying Figures 6 and 5, at last I find the Total of the particular Products to be 73.486138; and because there are six places or decimal Parts in both the Numbers given (to wit, 5 places of decimal parts in the Multiplicand, and 1 place in the Multiplier) I cut off 6 places to the Right-hand from the Total before produced, so will it stand thus 73.486138: Wherefore I conclude, That the true Product is $73 \frac{486138}{1000000}$ or 73.486138, that is, 73 Integers and almost $\frac{1}{2}$ of an Integer.

In like manner, if this mixt Number 246.25 (that is 246 $\frac{25}{100}$ were given to be multiplied by 35 Integers, the true Product will be found 8618.75. that is 8618 Integers and $\frac{75}{100}$ parts of an Integer, as you see by the Operation in the Margin, where you may observe that two Places are cut off from the total Number produced by the Multiplication, towards the Right-

$$\begin{array}{r} 1.30526 \\ 56.3 \\ \hline 391578 \\ 783156 \\ 652630 \\ \hline 73486138 \end{array}$$

$$\begin{array}{r} 246.25 \\ 35 \\ \hline 123125 \\ 73875 \\ \hline 861875 \end{array}$$

T

hand,

hand, because there are two places of Decimals in the Multiplicand (the Multiplier consisting of Integers only ;) but if there had been decimal Parts also in the Multiplier, so many more places should have been cut off, as was shewed in the first *Example*.

Again, If these two Decimals .87 and .9 (to wit, $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true Product will be found to be, 783, that is $\frac{783}{1000}$ parts of an Integer, as you see in the *Example*; where you may observe that the Product is a Fraction only ; for after 3 places (being the Number of places of Decimals in both the Numbers given to be multiplied) are cut off to the Right-hand, there remains no Integer on the Left-hand.

$$\begin{array}{r} .87 \\ .9 \\ \hline .783 \end{array}$$

III. When the Multiplication is finish'd, if there arise not so many places in all as ought to be cut off by the second Rule of this Chapter, (which may often happen when the Product is a Fraction ;) in such Case, as many places as are wanting, so many Cyphers must be prefixed to the Product on the Left-hand thereof, and then a Point must be prefixt to sign the Product so encreased for a Decimal : So these Decimals .0375 and .05 being given to be multiplied one by the other, I multiply 375 by 5, and there arises 1875 : Now according to the second Rule of this Chapter, I should cut off 6 places to the Right-hand, and here are but 4 in all ; wherefore I prefix two Cyphers, to wit, as many as there are places wanting, and then prefixing a Point, the true Product will be .001875, or $\frac{1875}{1000000}$. In like manner if this mixt Number 5.525 be multiplied by this Decimal .0026, the true Product will be found to be .0143650 (or $\frac{143650}{10000000}$) as you may see by the Operation in the Margin, where one Cypher is prefixed to the Numbers arising from the total Multiplication to discover the true product.

$$\begin{array}{r} .0375 \\ .05 \\ \hline .001875 \\ \hline 5.525 \\ .0026 \\ \hline .33150 \\ .11050 \\ \hline .0143650 \end{array}$$

VI. Decimal parts of an Integer may be reduced to the known or accustomed parts of such Integer by Multiplication only ; for if the decimal Fraction given be multiplied by that Number which declares how many known parts are equal to the Integer, the product

To reduce Decimals to the known parts of the Integer.

duct gives the Number of known Parts required : So this Decimal Fraction of a Pound Sterling to wit, .8687 *l.* being proposed, I multiply it first by 20 (the Number of Shillings contained in a Pound) and the Product gives 17 Shillings and .3740 Parts of a Shilling ; which Decimal .3740 being multiplied by 12 (the Number of Pence in a Shilling,) produces 4 Pence, and .4880 Parts of a Penny ; Lastly, multiplying .4880 by 4 (the Number of Farthings which make a Penny) the Product gives one Farthing, and .9520 Parts of a Farthing, which are very near in Value to another Farthing ; so it appears that .8687 Parts of a Pound Sterling are 17 *s.* 4 *d.* 2 *f.* very near. After the same manner, a *Decimal Fraction* of any Integer whatsoever may be reduced into the known or accustomed Parts of such Integer.

	.8687 <i>l.</i>
	20
Shill.	17 3740
	12
	7480
	3740
Pence	4 4880
	4
Farth.	1.9520

A briefer Way to value any Decimal Part of a Pound of *English Money*, without Loss of a Farthing, may be this, *viz.* the Figure (if any happen) in the first Place of the Decimal being doubled, gives Shillings : Also if there be 5, or a Figure greater than 5 in the second Place, one Shilling more is to be added to the former ; Lastly, when 5 is taken from the Figure in the second Place, if every Unit in the Remainder be accounted as Ten, and the Figure in the third place as Units, these Tens and Units taken as one Number, and lessened by 1, give the Number of Farthings, which with the Shillings before found declare the Value of the Decimal propounded ; likewise if the Figure in the second place (when any happens) be less than 5 ; every Unit in such Figure is to be accounted Ten as before : So in the Decimal before-mentioned, to wit, .8687 *l.* the Figure 8 in the first place being doubled, gives 16 Shillings ; also because 5 is contained in 6 which stands in the second Place, one Shilling more is to be added to the aforesaid 16 Shillings, which will now be made 17 *s.* that done, the Remainder of the said 6 after 5 is subtracted, to wit, 1 being esteemed as 10 and added to 8, (which stands in the third place, and to be esteemed as Units,) gives 18, from which abating 1, the Remainder is 17 Farthings or 4 Pence and a Farthing ; so that the Value of the third Decimal .8687 *l.* is found as before to be 17 Shillings 4 Pence 1 Farthing.

A brief Way to find the Value of any Decimal Fraction of a Pound of English Money.

After the same manner this Decimal of a Pound of *English Money*, to wit, .319 l. will be reduced to 6 Shillings and 18 Farthings, or 6 Shilling 4 Pence 2 Farthings, which wants less than a Farthing of the exact Value of the Decimal .319 l.

V. Having explain'd all the Cases in *Multiplication of Decimals*, I shall here give the Learner a Taste of their excellent Use by some familiar Questions, whereby it will be evident, that what is often performed by many tedious *Multiplications and Divisions* in the vulgar Way, is effected for the most part by one or two *Multiplications in Decimals*.

The first *Example* may be this: Suppose there is a certain Piece of Wainscot, in form of a *Rectangled Parallelogram*, commonly called a *long Square*, whose Breadth is 3 Yards, $\frac{3}{4}$ of a Yard, 1 Nail, and $\frac{1}{4}$ of an Nail, and the Length 6 Yards, and $\frac{1}{2}$ of a Yard, the Question is to know how many square Yards are contained in that Piece of Wainscot; here, because it is desired that the superficial Content may be given in Yards, the parts of a Yard as well in the Breadth as in the Length of the Wainscot, which are before expressed by the accustomed parts of Quarters, Nails, &c. must be reduced to Decimal parts of a Yard, which are as easie to be found by a Yard subdivided decimally, as the common parts of Quarters and Nails are found by a Yard vulgarly subdivided; but for want of a Yard subdivided decimally, this Reduction may be performed by the seventh Tablet of the preceeding Table of Reduction, viz looking into the said Tablet, right against

$\frac{3}{4}$ of a Yard, I find this Decimal —	.75
Also the Decimal Correspondent to 1 Nail is —	.0625
And the Decimal of $\frac{1}{4}$ of a Nail is —	.015625
The Sum of these three Decimals is —	.828125
Therefore the Breadth of the Wainscot in Yards and Decimal parts is —	3.828125
Again, the Decimal of half a Yard is .5, wherefore the length of the Wainscot is —	6.5
The length and breadth being multiplied one by the other, produce the superficial Content therefore the Number of square Yards required is —	24.8828125

I conclude then that 24 square Yards, and somewhat more are contained in that piece of Wainscot; and it is evident by the first place of the Decimal, that what is above 24 Yards is more than $\frac{1}{10}$, but less than $\frac{1}{5}$ of a square Yard; or more strictly it is more than $\frac{3}{10}$, but less than $\frac{1}{2}$ of a square Yard: But by taking all the places in the Decimal, you have the exact Answer to this Question, because the common parts of Quarters, Nails, and Quarters of Nails, may be always exactly reduced to Decimals, but that seldom happens in other Things; nevertheless, tho' by Decimal Operations you cannot always hit the Mark, yet you may come as near it as is possible to be imagined, and that with much more Ease than by Vulgar Computations in Questions of this Nature; as will appear by comparing the preceding Operation with the common way of working here in your View, viz. the 3 Yards, 3 Quarters of a Yard, 1 Nail, and $\frac{1}{4}$ of a Nail, (which express the Breadth before-mentioned,) must all be reduced to Quarters of Nails by the sixth Rule of the Seventh Chapter; so there will be found 245 Quarters of Nails, as you see by the Operation.

y.	q.	n.	q.	n.
3	3	1	1	
	4			
	15			
	4			
	61			
	4			

245 quarters of Nails

Again, the 6 Yards and half which express the Length aforesaid, must likewise be reduced to Quarters of Nails by the aforesaid Rule; so there will be found 416 Quarters of Nails of a Yard, as you see by the Operation.

y.	q.
6	2
4	
26	
4	
104	
4	

416 quarters of Nails.

Then multiplying the Breadth and Length one by the other, we find, 245 by 416, the Product will give 101920 for the superficial

superficial Content of the Piece of Wainscot in square Quarters of Nails of a Yard; now these square Quarters of Nails of a Yard must be reduced to square Yards, and the readiest way to perform that, is to find first of all how many Quarters of Nails of a Yard are contained in one Yard in length, *viz.* since there are 16 Nails in a Yard, there are consequently 4 times 16 Quarters of Nails, to wit, 64 Quarters of Nails in a Yard in length; therefore 64 multiplied by 64 produces 4096 square Quarters of Nails in a Yard square: Lastly, I say by the Rule of Three, if 4096 square Quarters of Nails of a Yard, give 1 Yard square, How many Yards square will 101620 square Quarters of Nails give? So will the Answer be found $24\frac{3616}{4096}$ Yards, which is the same with 24.8828125, before found by the decimal Operation; (for $\frac{3616}{4096}$ is equal to the Decimal .8828125, as will appear by reducing them to a common Denominator by the fourteenth Rule of the seventeenth Chapter.) Now I leave it to the Reader to judge, which of these two Ways is the more expeditious, and so let him take which he likes best.

Example, 2. There is a squared Piece of Timber terminated at both ends by equal long Squares; *viz.* the Breadth of the Piece of Timber is 1 Foot, 5 Inches, 3 Quarters of an Inch, and 1 half quarter of an Inch; the Depth or Thickness is 1 Foot 3 Inches, 1 quarter of an Inch, and $\frac{1}{4}$ or half a quarter of an Inch; and the length of the Piece is 11 foot, 10 Inches, and 3 Quarters; the *Question* is how many solid or cubical Feet are contain'd in that Piece of Timber? The Answer may be found by decimal *Multiplication* in manner following, *viz.* Forasmuch as it is desired that the solid Content may be given in Feet, the Parts of a Foot as well in the Breadth, as Depth, and Length, which are before express'd by the accustomed parts of Inches, Quarters, and half Quarters, must be reduced to the decimal parts of a Foot, which are as easy to be found by a Foot subdivided decimally, as the other common Parts by a Foot vulgarly subdivided; but for want of a Foot subdivided decimally, this Reduction may be performed by the eighth *Tablet* of the preceding Table of *Reduction*, *viz.*

The Decimal Correspondent to 5 Inches is	_____	.4166
The Decimal of $\frac{3}{4}$ of an Inch is	_____	.0625
The Decimal of half a Quarter of an Inch is	_____	.0104
The Sum of these 3 Decimals is	_____	.4895
Therefore the Breadth of the piece of Timber is	_____	4895

In

In like manner the common Parts of Inches, &c. in the Depth or Thickness of the Piece of Timber, will be reduced by the said Tablet, to these Decimals, viz.

The decimal Correspondent to 3 Inches is—.25

The decimal of $\frac{1}{4}$ of an Inch is—.02

The decimal of half a quarter of an Inch is—.01

The Sum of these 3 decimals is—.28

The Depth then, or Thickness is—.28

Again, the accustomed Parts of Inches, &c. in the length of the Piece of Timber will be reduced to these Decimals, viz.

The decimal of 10 Inches—.833

The decimal of $\frac{1}{4}$ of an Inch is—.062

The Sum of these 2 decimals is—.895

Therefore the Length of the Piece is—.11.895

Now if the Breadth, Depth, and Length be multiplied continually, the last Product is the solid Content required, viz. 1.488 multiplied by .28 produces .41664, which multiplied by .11.895 gives 4.946, &c. I conclude therefore that 22 solid Feet, half a Foot, and somewhat more than half a quarter of a Foot are contained in that Piece of Timber.

Example 3. How many *Equinoctial Degrees* are correspondent to 136 Days, 21 Hours, and 40 Minutes? The Answer is found by multiplying the Time given by 360; for as 1 Day is to 360 Degrees, so 136 Days, 21 Hours, and 40 Minutes, to the *Equinoctial Degrees* required. But first the 21 Hours and 40 Minutes must be reduced to decimal Parts of a Day by the *Tenth Tablet*; thus,

The decimal of 21 hours is—.875

The decimal of 40 minutes is—.02777

The Sum of these 2 decimals is—.90277

Therefore the Time proposed is—.136.90277

Which being multiplied by 360, produces—49284.99, &c.

I conclude therefore, that 49284.99, or very near 49285 *Equinoctial Degrees* are Correspondent to 136 Days, 21 Hours, and 40 Minutes, which was required by the Question.

CHAP. XXVII.

Division of Decimal Fractions.

IN any of the Cases which may happen in Division, if the Dividend be greater than the Divisor, the Quotient will be either a whole Number, or else a mixt Number: But when the Dividend is less than the Divisor, the Quotient must necessarily be a Fraction; for a lesser Number is contained in a greater once at the least, but a greater is not contained once in a lesser.

II. Sometimes the Dividend, whether it be a whole Number, mixt Number, or decimal Fraction, is to be prepar'd by annexing a competent Number of Cyphers thereto, to make room for the Divisor: So if 32.5 were given to be divided by 17,325. the Dividend 32.5 must be encreased with Cyphers at pleasure after this manner 32.50000, &c. Likewise if 1 were given to be divided by 360, the Division cannot be made till the Dividend 1 be encreased with Cyphers, which being annexed, the Dividend will stand thus 1.000000, &c. Here note, that the Cyphers annexed in manner aforesaid, supply Places of decimal Parts, and will be useful in discovering the Quality of the Quotient according to the fourth Rule of this Chapter.

III. After the Dividend is prepared by annexing Cyphers, when occasion requires, (as in the last Rule,) all the Places thereof must be esteemed as one whole Number, (to wit, consisting of Units or Integers :) and so is the Divisor to be esteemed, whether it be a decimal Fraction or mixt Number; for in all Cases the Division must be performed in every Respect according to the Rules of Division of whole Numbers in the sixth Chapter. So if this mixt Number 326.25 were given to be divided by this mixt Number 12.3, you must divide in the same manner, as when you divide 32625 Integers by 123 Integers. Also if this Decimal .8356 were given to be divided by this Decimal .05, you are to divide in the same manner, as when you divide 8356 Integers by 5 Integers; and after the Quotient is found, the Degree, or Place of the first Figure which arises in the Quotient, is to be enquired after, viz. you must know how far such Figure is distant from the Place of Units, to the end, that the Point or Line which is used to separate between the place of Units (or first Place of Integers) and the first place of Decimals, may be duly fix'd: This is the only Knot in decimal Division, and may be resolved by the following Rule, viz.

IV. In

IV. In any of the Cases which may happen in Division of Decimals, the first Figure which arises in the Quotient, will be always of the same Place or Degree with that Figure or Cypher of the Dividend, which at the first Question stands over, or at least belongs to the Place of Units in the Divisor. To illustrate this Rule, I shall

A general Rule to discover the quality of the Quotient in all Cases of Division by decimal Fractions.

give Examples in all the principal Cases, and first let a mixt Number be given to be divided by a mixt Number, viz. Let it be required to divide 172.5 by 3.746. here (according to the Second Rule of this Chapter) the Dividend must be increased with Cyphers at pleasure, so will it stand thus 172.500000, &c. then Division being made according to the Rules of Division of whole Numbers in Chapter 6, the Quotient arising will be 46.049, &c.

$$3.746)172.500000(46.049, \text{ \&c.}$$

Now it remains to separate the Integers in this Quotient from the decimal Parts; to perform which, I subscribe the Divisor 3.746, orderly under the first Dividual 172.50 (being

$$3.746)172.500000(46.049, \text{ \&c.}$$

$$\begin{array}{r} \dots\dots \\ 3.746 \end{array}$$

that part of the Dividend, whereof the first Question must be asked) or at least I imagine the Divisor to be so subscribed, and so I find that the Figure 3 which stands in the place of Units in the Divisor will be placed under 7, which is the Place of Tens, (or second Place of Integers) in the Dividend; therefore by the fourth Rule before given, I conclude that the first Figure arising in the Quotient must likewise stand in the place of Tens (or second Place of Integers) and consequently the next Place on the Right-hand must be the Place of Units; so it is evident that the separating Point or Line must be placed between the Figure 6 and 0 in the Quotient; that done, the true Quotient is found to be 46.049, &c. to wit, 46 Integers and $\frac{49}{1000}$ parts of an Integer, and somewhat more; for $46\frac{49}{1000}$ is less than the true Quotient; but $46\frac{50}{1000}$ is greater than it; and therefore, tho', after the afore said Division of 172.500000 by 3.746 is ended, there will be a Remainder, to wit, 446, which seems to be greater, yet here it is less in

U

Value

Value than $\frac{1}{1000}$ part of an Unit or Integer ; and if to that Remainder you annex another Cypher and continue the Division, you will proceed nearer the Truth, and not miss $\frac{1}{1000}$ part of an Unit of the true Quotient, and in that Order you may proceed infinitely near, when you cannot obtain the Quotient exactly by Division of Decimals.

Example 2. Suppose this mixt Number 2.34 be given to be divided by this mixt Number 52.125 (where you may observe that the Dividend is less than the Divisor ;) first (as before) annex Cyphers at pleasure to the Dividend, to make room for the Divisor, then the Division being prosecuted as in whole Numbers, at length these Figures will arise in the Quotient, to wit,

$$\begin{array}{r} 52.125 \overline{) 2.3400000} \quad (.0448, \text{ \&c.} \\ \underline{52.125} \\ 0000000 \\ 0000000 \\ 0000000 \\ 0000000 \\ 0000000 \\ 0000000 \\ 0000000 \end{array}$$

448 ; and to the end the Degree or Quality of the first Figure 4 may be discovered, I subscribe the Divisor 52.125 under the Dividend 2.34000 (for so far the first Question did extend in the Division;) and thereby I find that the Figure 2, which stands in the Place of Units in the Divisor, will be seated under 4, which is in the second place of Decimals ; I conclude therefore that the first Figure arising in the Quotient must also stand in the second Place of Decimals, and consequently the first place of Decimals (which is next on the Left-hand to the second) must be supplied with a Cypher ; so that if a Cypher be prefixed on the Left-hand of 4, and then a Point placed before that Cypher, the Quotient will at length be discovered to be .0448, &c. or $\frac{448}{10000}$ and somewhat more, that is to say, $\frac{448}{10000}$ is less than the true Quotient, but $\frac{448}{10000}$ is greater than it ; and if you would proceed nearer the Truth, you may continue the Division, as is directed in the first Example of this Rule.

Example 3. Where a whole Number is divided by a decimal Fraction, viz. Suppose 82 Integers were given to be divided by this Decimal .056 ; After Cyphers are annexed to the Dividend at Pleasure, and the Division prosecuted as in

$$.056 \overline{) 82.00000} \quad (146428, \text{ \&c.}$$

whole Numbers (to wit, 8200000 being divided by 56) these Figures 146428 will arise in the Quotient ; now to the end the degree or seat of 1, the first Figure in the Quotient, may be known, I subscribe

I subscribe the divisor .056 under the first dividual 82 (for so far did the first Question in the division extend ;) and because the divisor is less than Unity, I supply the place of Units by a Cypher or 0 perfixed on the Left-hand of the Point of Separation

$$\begin{array}{r} .056 \overline{) 0082.00000} \quad (1464.28, \text{ \&c.} \\ \underline{0000} \\ 8200000 \\ \underline{00000} \\ 2800000 \\ \underline{00000} \\ 2800000 \\ \underline{00000} \\ 0000000 \end{array}$$

tion in the divisor ; also I prefix Cyphers before (to wit, on the Left-hand of) the Integers in the dividend, to represent a Succession of Places of Integers (for the Order of Places in Integers is from the Right-hand towards the Left ;) then I find that the Cypher or 0 which represents the Place of Units in the divisor, stands under that Cypher, which represents the fourth place of Integers in the dividend (as you see by the Example ; I conclude therefore that the first Figure arising in the Quotient must also be seated in the fourth place of Integers, and consequently the 4 first Places in the Quotient will be Integers, and the rest a Decimal : So that the true Quotient is 1464 Integers, and $\frac{28}{10000}$ parts of an Integer, and somewhat more, viz. 1464.28 is less than the true Quotient, but 1464.29 is greater than it.

Example 4. Suppose this decimal .0125 be given to be divided by this decimal .5 ; after division is finished according to the Rules of division of whole .5) .0125 (25 Numbers (to wit, after 125 is divided by 5) these Figures 25 will arise in the Quotient ; now to discover the Degree or Seat of 2 the first Figure in the Quotient, I subscribe the divisor 5 under the first dividual .012, and having (as in the last Example) prefixed a .5) 0125 (.025 Cypher on the Left-hand of the point of Separation in the divisor, to denote, or represent the 0.5 Place of Units, I find that such Cypher or Place of Units stands under the Figure which is seated in the second Place of decimals in the dividend, therefore I conclude by the Rule, that the first Figure which arises in the Quotient must also be in the second Place of decimals, and therefore prefixing a Cypher to supply the first Place of decimals, and putting a Point before that Cypher, the Quotient is at length discover'd to be .025 or $\frac{25}{1000}$.

Example 5. Suppose this decimal .8564 be given to be divided by this .008. first I annex Cyphers to the dividend at Pleasure, then prosecuting the division, as in whole Numbers,

to wit, dividing .856400 by 8, the Quotient arising is 107050 : Now to discover the Degree or Place of 1, the first Figure in the Quotient, I subscribe the divisor .008 under the first dividual .8, then I prefix a Cypher to set forth, or supply the place of Units in the divisor ; also I prefix Cyphers to represent Places of Integers in the dividend ;

that done, I find that the Cypher or 0 which supplies the place of Units in the divisor, stands under the Cypher which is seated in the third Place of Integers in the dividend : I conclude therefore by the Rule, that the first Figure arising in the Quotient must be also in the third place of Integers, and consequently the three first places in the Quotient will be Integers, and the rest a decimal ; so that the true Quotient is 107.050 or $107\frac{1}{2}$.

Example 6. Let it be required to divide this decimal Fraction .73952 by this .32 ; first, dividing 73952 by 32, as if they were whole Numbers, the Figures arising in the Quotient will be 2311. Now, to discover the Quality or Value of the said Figures, I subscribe the divisor .32 under the first dividual 73, then prefixing a Cypher as well on the Left-hand of the dividend, as of the divisor so subscribed (or imagined to be subscribed) as aforesaid, to represent the Place of Units in each of

them, I find the Cypher or 0, which supplies the place of Units in the divisor, to stand under the 0, which represents the place of Units in the dividend ; wherefore I conclude by the preceding fourth Rule, that the first Figure arising in the Quotient will stand in the place of Units, and consequently the following places of the Quotient will be a decimal Fraction, so that the true Quotient is 2.311 or $2\frac{11}{1000}$.

The Reason of the fore-going fourth Rule will appear from the following Considerations.

I. If the Product of the Multiplication of two Numbers be divided by one of them, the Quotient is the same with the other Number ; as, If 269.0625, the Product of 14.35, multiplied by 18.75, be divided by 14.35, the Quotient will give 18.75.

II. If the Divisor be multiplied by the first Figure in the Quotient, the Product is the first Number to be subtracted from the dividend (being the same with the last particular Product

Product in the Multiplication of the two Numbers that produced the Dividend;) and every particular Place of that Product is of the same Degree with that Figure or Cypher of the Dividend, which stands over such particular Place when the Subtraction is made; for a Figure of one Degree (or Place) cannot be subtracted from a Figure of a different Degree: As in the last mentioned *Example*, the Work whereof is here in view, the Divisor 14.35 being taken as in a whole Number and multiplied by 1, the first Figure in the Quotient, produces 1435, which must be conceived to consist of the same Degrees as are in 269.0 in the Dividend, from which the said Product is to be subtracted, and therefore the said Product 1435 is really but 143.5, as you may see by the last particular Product, in the Multiplication of the mixt Number 1435 by 18.75.

$$\begin{array}{r}
 14.35 \\
 18.75 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 7 \mid 175 \\
 100 \mid 45 \\
 1148 \mid 0 \\
 1435 \mid \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 14.35 \mid 269 \mid 0625 \text{ (18.75)} \\
 143 \mid 5 \\
 \hline
 125 \mid 56 \\
 114 \mid 80 \\
 \hline
 10 \mid 762 \\
 10 \mid 045 \\
 \hline
 \mid 7175 \\
 \mid 7175 \\
 \hline
 0
 \end{array}$$

III. And therefore to discover the Degree of the first Figure in the Quotient, is nothing else but to find out the Degree of that Figure, which multiplying the Figure or Cypher in any particular Place in the Divisor, will produce the same Degree as that Figure or Cypher in the Dividend is of, which stands over, or at least belongs to such particular Place of the Divisor,

Divisor, at the first Question ; because the Degree produced must be subtracted from the like Degree above it.

IV. Now among many Rules that might be given to discover the Degree of the first Figure in the Quotient, and consequently the Degrees of all the rest, the preceeding Fourth Rule of this Chapter is sufficient, namely, The first Figure which arises in the Quotient, is always of the same Place or Degree with that Figure or Cypher in the Dividend, which at the first Question stands over, or at least belongs to the Place of Units in the Divisor : The Reason is, because if a Figure standing in the Units Place of the Divisor, be multiplied by (or does multiply) a Figure of the same degree with that degree in the Dividend, which at the first Question belongs to the said Units Place of the Divisor, the first Place in the Product will be of that degree also, whether it be of Integers or decimal Parts : And consequently the rest of the Places in the said Product shall be of the same degree with their correspondent Degrees (or Places) in the Dividend ; as they ought to be, to the end that due Subtraction may be made, (according to Observation 2.)

So in the *Example* before given, the first Figure 1 in the Quotient, will be of the degree or place of Tens, because if the Figure 4 standing in the Units place of the Divisor 14.35, be multiplied by Ten, to wit, the Degree which the Figure 6 in the Dividend is of, that belongs to the said 4 at the first Question, it will produce Four Tens, to be subtracted from the said Six Tens : In like manner, if a Figure in the place of Units be multiplied by Units, the first place in the Product will be Units ; if by tenth parts of an Unit, or Integer, the first place in the Product will be Tenths, &c.

Having explained all necessary Rules in Division, concerning Decimal Fractions, I shall give a Taste of their excellent Use by the Two following Questions, and then conclude this Chapter.

Quest. 1. A Merchant bought of Gold Plate 356 Ounces, 13 Penny-weight, and 15 Grains, for 1160 Pounds sterling ; the Question is, what he paid for an Ounce ? *Answer,* 3*l.*—5*s*— $\frac{1}{2}$, very near. The Operation by *Decimals* may be after this manner, *viz.*

By

By the Second Tablet of Reduction the Decimal of 13 Penny weight is ——— } .65
The Decimal of 15 Grains is ——— } .03125

The Sum of these two Decimals is ——— .68125

Therefore the Quantity of Plate in Ounces } 356.68125
and Decimal Parts of an Ounce is ——— }

Then by the Rule of Three, I say, if 356.68125 Ounces cost 1160 Pounds, what will 1 Ounce? Here 'tis evident that if I divide 1160 by 356.68125, the Quotient will give the Value of an Ounce, to wit, 3.252 Pounds, or 3 Pounds, 5 Shillings and $\frac{1}{2}$ very near.

356.68125) 1160.000000 (3 252, &c.

Quest. 2. Suppose the Length of the Tropical Year, (or the Space of Time in which the Sun running through the whole Ecliptick Circle, consisting of 360 Degrees, is returned to the same Equinoctial or Solstitial Point from whence he departed) to consist of 365 Days, 5 Hours, and 49 Minutes, the Question is to know the Sun's mean or equal Motion for 1 Day, to wit, what part of 360 Degrees the Sun moves thro' in a whole Day? The Operation by Decimals, thus,

By the Tenth Tablet of Reduction the decimal correspondent to 5 Hours is ——— } .2083333

The Decimal of 49 Minutes is ——— .0340277

The Sum of those Decimals is ——— .2423610

Therefore the Time given in Days and Decimal Parts of a Day is ——— } 365.2423610

Then by the Rule of Three, if 365.242361 Days give 360 Degrees (or a total Circumference) what will 1 Day give? Here if I divide 360 by 365.242361, the Quotient will give the diurnal Motion required; which will be found very near .98564. &c. or $\frac{28564}{100000}$ Parts of a Degree, which Decimal being reduced to the common Sexagenary Parts (by the Fourth

Rule of the twenty sixth Chapter) will give $59^{\frac{1}{2}}$ — $8^{\frac{11}{2}}$, &c. and such is the Sun's diurnal Motion very near, according to the aforesaid Supposition of the Length of the Tropical Year.

I shall here add the Vulgar Sexagenary Resolution of this Question, that by comparing both ways together, the Excellency

lency of *Decimal Arithmetick* in Calculations of this Nature may be the more perspicuous.

The aforesaid Questions being stated according to the Rule of Three will stand thus,

Days. Hours. Min. Degrees. Day.

If 365 : 5 : 49 ——— 360 ——— 1

The first Term in the Rule must be reduced into *Minutes* (by the Sixth Rule of the Seventh Chapter;) so there will be found 525649 *Minutes*.

D.	H.	M.
365	5	49
24		
———		
1465		
730		
———		
8765	Hours.	
60		
———		
525949 <i>Minutes</i> .		

Likewise the third Term 1 *Day* must be reduced to *Minutes*, which will be found to be 1440, as you see by the following Operation.

1 Day or 24 Hours.
60
———
1440

Then multiply the third Term by the second, to wit, 1440 by 360, the Product is 518400, which being divided by the first Term 525949 (according to the Note in the Ninth Rule of the Sixteenth Chapter) the Quotient will give $\frac{5}{2} \frac{1}{2} \frac{8}{4} \frac{0}{4} \frac{0}{4}$ parts of a Degree, which Fraction being reduced to the accustomed Sexagenary parts (by the Ninth Rule of the Seventeenth

Chapter) will give as before $59^{\circ} 8'$, &c. for the Sun's mean diurnal Motion; now which of these two ways is the more expeditious, I leave to him who is skilled in both to determine.

C H A P. XXVIII.

The Rule of Three Direct in Fractions.

TO repeat such Things as have already been declared in reference to the Definition of this Rule, as also to the due placing of the Three given Numbers, would be superfluous; and if Respect be had to the Rules of *Multiplication* and *Division* in *Fractions* delivered in the 20, 21, 26 and 27 Chapters; the Working of the *Rule of Three Direct in Fractions* as well *Vulgar* as *Decimal*, is the same with that in whole Numbers, *viz.* multiply the second Number by the third, (or the third by the second,) and divide the Product by the first Number, so the Quotient is the fourth Number sought for; to wit, the Answer of the Question.

Otherwise thus in Vulgar Fractions.

Multiply the Denominator of the first Number by the Numerator of the second, also multiply that Product by the Numerator of the third Number, and reserve this last Product for a new Numerator; again multiply the Numerator of the first Number by the Denominator of the second, also multiply this Product by the Denominator of the third Number, so will this last Product be a new Denominator: *Lastly*, the new Fraction (whose Numerator and Denominator are found as aforesaid) is the fourth Number sought, which if it be a proper Fraction, may (as occasion requires) be reduced to the known Parts of the Integer, (by the Ninth Rule of the Seventeenth Chapter;) if an improper Fraction, it must be reduced to its equivalent whole Number or mixt Number by the Thirteenth Rule of the Seventeenth Chapter.

Example, If $\frac{3}{4}$ of a Yard of Velvet be sold for $\frac{2}{3}$ of a Pound Sterling, what will $\frac{5}{6}$ of a Yard cost? *Answer* $\frac{40}{7}$ or 14 s. 9 $\frac{2}{7}$ d.

For according to the Rule I multiply the Denominator 4 by the Numerator 2, and the Product is 8; this 8

I again multiply by the Numerator 5, and the Product gives 40 for a new Numerator:

$$\begin{array}{cccc} y. & l. & y. & l. \\ \frac{3}{4} & - \frac{2}{3} & - \frac{5}{6} & = \frac{40}{7} \end{array}$$

Moreover, I multiply the Numerator 3 by the Denominator 3, and the Product which is 9, I again multiply by the Denominator 6, so the last Product is 54 for a new Denominator;

wherefore I conclude that $\frac{40}{7}$ is the fourth Number sought; which if it be reduced (according to the Ninth Rule of the

Seventeenth Chapter) gives $14\text{ s. } 9\frac{4}{7}\text{ d.}$ (or $9\frac{7}{8}$ for the Answer of the Question.

II. When any of the Three given Numbers is a whole Number, or mixt Number, such Number must first of all be reduced to an improper Fraction (by the Tenth or Eleventh Rule of the Seventeenth Chapter) to the End that all the Three given Numbers may be Three Fractions: And farther if after such Reduction, the first and third Numbers be not Fractions of Integers of the same particular Denomination, such of the said Numbers as is of the lesser Denomination, must be reduced to a Fraction of the greater, (by the Sixteenth Rule of the Seventeenth Chapter;) which Preparation being performed, the Rest of the Work is to be prosecuted according to the First Rule of this Chapter. An Example of this Second Rule here follows. If a Quantity of *Amber-grieze* weighing $1\frac{2}{7}\text{ lb. Troy}$, be sold for 60 l. Sterling , what are $19\frac{7}{8}\text{ Grains}$ worth at that Rate? Answer, $3\frac{6}{7}\frac{2}{3}\frac{4}{5}\text{ or } 2\text{ s. } 4\frac{1}{2}\text{ d.}$

This Question being stated according to the Seventh Rule of the Eighth Chapter will stand thus: —————

These Three Numbers will be reduced (by the Tenth and Eleventh Rules of the Seventeenth Chapter) to these improper Fractions. —————

But since the Third Number $19\frac{7}{8}\text{ Grains-Troy}$ is not a Fraction of an Integer of the same Name with the first (which is a Fraction of a Pound Troy,) it must be reduced to the Fraction of a Pound Troy, thus, $19\frac{7}{8}\text{ gr.}$ is $\frac{19\frac{7}{8}}{72}$ of $\frac{1}{72}$ of $\frac{1}{72}$ of a Pound Troy, which compound Fraction will be reduced (by the Sixteenth Rule of the Seventeenth Chapter) to this single Fraction, to wit, $\frac{19\frac{7}{8}}{4800}\text{ lb. Troy}$, and so the Three Numbers will at length stand thus in the Rule.

$$1\frac{2}{7}\text{ lb.} \quad \frac{60}{1}\text{ l.} \quad \frac{19\frac{7}{8}}{4800}\text{ lb.}$$

Then working as in the first Example of this Chapter, the Answer will be found $5\frac{6}{7}\frac{2}{3}\frac{4}{5}\text{ l.}$ which being reduced (according to the Ninth Rule of the Seventeenth Chapter) is found equal to $2\text{ s. } 4\frac{1}{2}\text{ d.}$

Another Example. When $\frac{2}{3}$ of $\frac{3}{4}$ of a Ship is valued at $147\text{ l.} \text{---} 11\text{ s.} \text{---} 3\text{ d.}$ how much is the whole Ship worth? Answer, $491\text{ l.} \text{---} 17\text{ s.} \text{---} 6\text{ d.}$

Note

Note. When in any Question whatsoever, a compound Fraction, to wit, a Fraction of a Fraction, is one of the given Numbers, such compound Fraction must first of all be reduced to a single Fraction, (by the Sixteenth Rule of the Seventeenth Chapter;) so here the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ being reduced to a single Fraction gives $\frac{1}{2}$ or $\frac{3}{6}$; then say, if $\frac{3}{6}$ be worth 147 l.—11 s.—3 d.

what is 1 or the whole Ship Ship. l. s. d. Ship.
worth? After due Reduction $\frac{4}{3}$ —147—11—3—1
is made by converting the 147 l.

11 s.—3 d. into Pence, and that Number of Pence, as also the Third Number 1 to improper Fractions, the Three Numbers will stand in the Rule thus,

$$\begin{array}{r} \text{Ship.} \quad \text{Pence.} \quad \text{Ship.} \\ \frac{3}{10} \quad \frac{35415}{1} \quad \frac{1}{1} \end{array}$$

Lastly, Proceeding as in the First Rule of this Chapter, the Fourth Number will be found to be $\frac{354150}{1}$, which being reduced first by the Thirteenth Rule of the Seventeenth Chapter, and then by the Seventh Rule of the Seventh Chapter, the Answer, at length is 49 l.—17 s.—6 d.

An Example of the Rule of Three Direct in Decimals may be this that follows. If 19 Ounces, 3 Penny-weight, and 5 Grains of Gold be worth 62 l.—10 s.—6 d. what is the value of $1\frac{1}{2}$ Ounce? Answer, 4 l.—17 s.—10 d. $\frac{3}{4}$ very near.

By the second Tablet in the Table of Reduction in the Twenty third Chapter, the Decimal Fraction correspondent to Three Penny-weight, is .15

Also the Decimal of 5 Grains is .010416

The Sum of those two Decimals is .160416

Therefore the first Number in the Rule of Three } oz.
is ————— } 19.160416

Again, by the first Tablet of the before-mentioned Table, the Decimal of 10 Shillings is .5

Also the Decimal of 6 d. is .025

The Sum of these two Decimals is .525

The second Number then in the Rule of Three } l.
is ————— } 62.525

And farther, by the said Tablet 2, the Decimal of $\frac{1}{4}$ of an Ounce or Ten Penny-weight is .5, therefore the third Number in the Rule of Three is } oz.
} 1.5

X 2

So

So that after the said *Reduction* is finished, the Three given Numbers will stand in the Rule thus:

$$\begin{array}{ccc} \text{oun.} & & \text{lb.} & & \text{oun.} \\ 19.160416 & \text{---} & 62.525 & \text{---} & 1.5 \end{array}$$

Lastly, Multiplying the second Number by the third, and dividing the Product by the first Number (according to the Rules of *Multiplication* and *Division of Decimals* deliver'd in the Twenty sixth and Twenty seventh Chapters) the fourth Number will be this, to wit, 4.8948, &c. that is 4 Pounds Sterling, and $\frac{8948}{10000}$ parts of a Pound, which *Decimal* being reduced (according to the Fourth Rule of the Twenty sixth Chapter) gives 18 s. — 10 d. — 3 far.

The Proof of the Rule of Three Direct in Fractions is the same as in whole Numbers, respect being had to the Rules of *Multiplication of Fractions*.

CHAP. XXIX.

The Inverse Rule of Three in Fractions.

1. **A**FTER a Question belonging to this Rule is duly stated, (according to the Seventh Rule of the Eighth Chapter,) and prepared if need require, according to the Second Rule of the Twenty eighth Chapter; the Operation will be the same as in the Rule of Three Inverse in whole Numbers, respect being had to the Rules of *Multiplication* and *Division in Fractions*, viz. multiply the first Number by the second, and divide the Product by the third; the Quotient is the fourth Number sought, to wit, the Answer of the Question.

Or thus, in Vulgar Fractions.

Multiply the Denominator of the Third Fraction by the Numerator of the Second; also multiply that Product by the Numerator of the first Fraction, and reserve the last Product for a new Numerator; again, multiply the Numerator of the Third Fraction by the Denominator of the second; also multiply this Product by the Denominator of the first Fraction, so is the last Product a new Denominator: *Lastly*, This new Fraction is the Fourth Number sought, or Answer of the Question.

Example,

Example. If Cloth, which is $1\frac{1}{4}$ yard in breadth and $3\frac{1}{2}$ yards in length will make a Cloak, how much in length of Stuff which is $\frac{5}{8}$ yard in breadth will make a Cloak of the same bigness with the former? *Answer*, $9\frac{4}{5}$ yards

The Three Numbers being duly placed will stand thus—
 $\left. \begin{array}{l} \text{bread.} \quad \text{length.} \quad \text{bread.} \\ 1\frac{1}{4} \text{ y} \quad 3\frac{1}{2} \text{ y} \quad \frac{5}{8} \text{ y} \end{array} \right\}$

Then after the first and second Numbers are reduced to improper Fractions, the three Numbers will stand thus—
 $\left. \begin{array}{l} \frac{5}{4} \quad \frac{7}{2} \quad \frac{5}{8} \end{array} \right\}$

Lastly, 8, 7 and 7 being multiplied continually give 392 for a Numerator; also 5, 2 and 4 being multiplied continually give 40 for a Denominator, whereby this improper Fraction $\frac{392}{40}$ arises, which (by the Thirteenth Rule of the Seventeenth Chapter) will be found to be $9\frac{3}{5}$, or (the Fraction being reduced to its least Terms) $9\frac{4}{5}$, which is the *Answer* of the Question.

Example 2. Suppose when Wheat is at 2 l.—10 s.—6 d. the Quarter, the Penny White Loaf ought to weigh 8 Ounces and $1\frac{5}{16}$ Penny-weight of Troy-weight; what ought it to weigh when Wheat is at 36 Shillings the Quarter? *Answer*, 9 Ounces and $1\frac{3}{16}$ Penny weight.

The Three given Numbers being duly placed in the Rule and reduced will stand thus—
 $\left. \begin{array}{l} \text{pence} \quad \text{p. w.} \quad \text{pence} \\ 48\frac{5}{8} \quad 46\frac{7}{8} \quad 44\frac{1}{2} \end{array} \right\}$

And if the Operation be prosecuted according to the Rule before given, the *Answer* will be found 181 $\frac{3}{16}$ $\frac{9}{16}$ $\frac{6}{16}$ Penny weight or 9 Ounces, $1\frac{3}{16}$ Penny weight.

CHAP. XXX.

Double Rule of Three in Fractions.

1. **T**HE Double Rule of Three is so called, because it is composed of two single Rules, and may either be resolved at one Work by the Rule compound of Five Numbers, or else by two distinct Single Rules of Three; which latter way, to such as understand the Rule of Three in Fractions, is (as I conceive) less troublesome in the stating, and (in the Method whereby I intend to prosecute it) the same in Operation with the former. This I shall manifest first in whole Numbers, then in Fractions.

Example

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Example 1. If I pay 28 Shillings for the Carriage of 3 C. weight for 50 Miles, how much ought I to pay for the Carriage of 17 C. for 84 Miles? *Answer,* 13 l.—6 s.—6 d¹⁸/₅.

Of the Five given Numbers I make choice of Three such as will make a *Single Rule of Three*, and say,

$$\begin{array}{ccc} \text{C.} & \text{Skill.} & \text{C.} \\ \text{If } 3 & \text{--- } 28 & \text{--- } 17 \end{array}$$

which Rule I find (by the Third Rule of the Ninth Chapter) to be direct, and therefore I multiply the third Number 17 by the second 28, and the Product which is 476 I place as a Numerator over the Divisor as Denominator. Then ⁴⁷⁶ with this Fraction (whether it happen to be a proper or improper Fraction) and the remaining two Numbers in the Question, which have not yet been used, I form a second Rule of Three, and say,

$$\begin{array}{ccc} \text{miles.} & \text{Skill.} & \text{miles.} \end{array}$$

$$\text{If } \frac{50}{1} \text{ --- } \frac{476}{3} \text{ --- } \frac{84}{1}$$

which being a Rule of Three Direct, I work as a Rule of Three in Fractions, according to the First Rule of the Twenty eighth Chapter, and so find the fourth Number to be ³⁹⁹⁸⁴ 13 l.—6 s.—6 d¹⁸/₅.

Or the Single Rule being varied, the Operation will be thus,

$$\begin{array}{ccccccc} & & \text{miles} & \text{C.} & \text{miles} & \text{C.} & \\ 1. \text{ By a Rule Inverse.} & 50 & \text{---} & 3 & \text{---} & 84 & \text{---} \left(\frac{150}{84} \right) \end{array}$$

$$\begin{array}{ccccccc} & & \text{C.} & \text{lb.} & \text{C.} & \text{lb.} & \\ 2. \text{ By a Rule Direct} & \frac{50}{84} & \cdot & \frac{28}{1} & \cdot & \frac{17}{1} & \cdot \left(\frac{39984}{150} \right) \end{array}$$

Otherwise thus,

$$\begin{array}{ccccccc} & & \text{C.} & \text{m.} & \text{C.} & \text{m.} & \\ 1. \text{ By a Rule Inverse,} & 3 & \text{---} & 50 & \text{---} & 17 & \text{---} \left(\frac{150}{17} \right) \end{array}$$

$$\begin{array}{ccccccc} & & \text{m.} & \text{lb.} & \text{m.} & \text{lb.} & \\ 2. \text{ By a Rule Direct,} & \frac{3}{17} & \cdot & \frac{28}{1} & \cdot & \frac{84}{1} & \cdot \left(\frac{39984}{150} \right) \end{array}$$

Thus you see the two Single Rules varied Three manner of Ways in resolving the Question propos'd, and every Way produces the same Answer; the like Diversity may be found in all Questions resolvable by the Double Rule of Three, or Rule compound of Five Numbers.

Example

Chap. XXXI. *The Rule of False in Fractions.* 159

Example 2. If $40\frac{3}{5}$ l. in $\frac{2}{3}$ of a Year, gain $2\frac{1}{2}$ l. what will 100 l. gain after that Rate in $\frac{7}{12}$ of a Year? *Answer,* $5\frac{2500}{9744}$ l. or 5 l. — 7 s. — 9 $\frac{3}{4}$ d.

By 2 Single Rules of Three, thus,

$$1. \text{ By a Rule Direct, } \begin{array}{c} \text{l.} \\ 40\frac{3}{5} \end{array} : \begin{array}{c} \text{l.} \\ 2\frac{1}{2} \end{array} :: \begin{array}{c} \text{l.} \\ 100 \end{array} : \begin{array}{c} \text{l.} \\ 5\frac{2500}{9744} \end{array}$$

$$2. \text{ By a Rule direct, } \begin{array}{c} \text{year} \\ \frac{2}{3} \end{array} : \begin{array}{c} \text{l.} \\ 2\frac{1}{2} \end{array} :: \begin{array}{c} \text{year} \\ \frac{7}{12} \end{array} : \begin{array}{c} \text{l.} \\ 5\frac{2500}{9744} \end{array}$$

Or by these two single Rules.

$$1. \text{ By a Rule Direct, } \begin{array}{c} \text{year} \\ \frac{2}{3} \end{array} : \begin{array}{c} \text{l.} \\ 2\frac{1}{2} \end{array} :: \begin{array}{c} \text{year} \\ \frac{7}{12} \end{array} : \begin{array}{c} \text{l.} \\ 5\frac{2500}{9744} \end{array}$$

$$2. \text{ By a Rule Direct, } \begin{array}{c} \text{l.} \\ 40\frac{3}{5} \end{array} : \begin{array}{c} \text{l.} \\ 2\frac{1}{2} \end{array} :: \begin{array}{c} \text{l.} \\ 100 \end{array} : \begin{array}{c} \text{l.} \\ 5\frac{2500}{9744} \end{array}$$

Otherwise thus :

$$1. \text{ By a Rule Inverse, } \begin{array}{c} \text{l.} \\ 40\frac{3}{5} \end{array} : \begin{array}{c} \text{year} \\ \frac{2}{3} \end{array} :: \begin{array}{c} \text{l.} \\ 100 \end{array} : \begin{array}{c} \text{year} \\ \frac{7}{12} \end{array}$$

$$2. \text{ By a Rule Direct, } \begin{array}{c} \text{l.} \\ 40\frac{3}{5} \end{array} : \begin{array}{c} \text{year} \\ \frac{2}{3} \end{array} :: \begin{array}{c} \text{year} \\ \frac{7}{12} \end{array} : \begin{array}{c} \text{l.} \\ 5\frac{2500}{9744} \end{array}$$

Thus by 2 single Rules of Three, varied three several ways, you see the Answer of the Question to be $5\frac{2500}{9744}$ l. to wit, 5 l. — 7 s. — 9 $\frac{3}{4}$ d.

C H A P. XXXI.

The Rule of False in Fractions.

I. **W**HEN a Question proposed cannot readily be applied to the *Rule of Three*, or any of the vulgar Rules in *Arithmetick*; the best Refuge for such as are not acquainted with *Algebra*, is the Rule of *two false Positions*, which, for that it has already been handled in *whole Numbers*, I shall the more briefly touch upon in *Fractions*,

II. When

II. When a Number is sought by a Question, you are to feign or suppose some Number taken by guess to be the Number sought, and to make Tryal whether that feigned Number will answer the Conditions in the Question or not, by comparing the Number resulting at the end of the Work, with the given Number proceeding from the true Number sought; and if you find both those Results to be the same, then is the Number which you first took by guess, the true Number or Answer of the Question: But if the Number resulting from the supposititious Number be either greater or less than the given Result, with which it ought to be compared, (to see whether you have hit the Mark or not.) such Excess or Defect must be noted for the Error of the first Position, to wit, an *Excess* must be signified by this Note $+$, and a Defect by this—.

III. In like manner, a second Number must be feigned, and after Tryal is made therewith, to see whether it will perform the Conditions prescribed in the Question, by comparing the Results as aforesaid, the Error of this second Position, if too much is to be added by $+$, if too little by—, as before.

IV. After the Errors of both Positions are discover'd, the two Numbers before supposed or feigned to be the Number sought, must be multiplied by the altern Errors, that is, the first Position by the second Error, and the second Position by the first Error; then if the Notes of the Errors be unlike, to wit, one of them $+$, and the other—, the Sum of the said Products is to be taken for a Dividend, and the Sum of the Errors for a Divisor: But if the Notes of Errors be both alike, to wit, both of them $+$, or both—, the Difference of the said Products is to be taken for a Dividend, and the Difference of the Errors for a Divisor; lastly, the Quotient arising from the Division (made by the said Dividend and Divisor) gives the true Number sought, or Answer of the Question, if it be solvable by the *Rule of False*. These Rules are the same in Substance with those deliver'd in the 15th Chapter, and may be farther illustrated by the following Questions.

Quest. 1. A Gentleman hired a Servant for a Year, for 6 Pounds Sterling and a Livery Cloak valued at a certain Rate, but it happened that $\frac{7}{12}$ of the Year being expired, they fell at Variance, and the Gentleman put away his Servant, giving him the Cloak together with 50 Shillings in Money, which was the Servant's full due for the Time of his Service, the Question is to find what the Cloak was valued at? *Answ.* 2 l.—8 s.—0 d.

1. I suppose the Cloak to be valued at 3 Pounds, and then seek how much thereof was due to the

Servant; saying, if one Year give 3 *l.* $y.$ $l.$ $y.$
how much $\frac{1}{2}$ of a Year? Answer $\frac{3}{2}$ *l.* $1 \text{ --- } 3 \text{ --- } \frac{1}{2} \text{ --- } (\frac{3}{2} \text{ l.})$

2. I likewise find what part of the 6 Pounds was due to the Servant at the end of $\frac{1}{2}$ of the Year,

saying, If 1 Year give 6 Pounds, how $y.$ $l.$ $y.$
much $\frac{1}{2}$ of the Year? Answer, $\frac{3}{2}$ *l.* $1 \text{ --- } 6 \text{ --- } \frac{1}{2} \text{ --- } (\frac{3}{2} \text{ l.})$

3. Since the Cloak, together with the Money which the Servant receiv'd, ought to be equal to the Price of the Cloak, together with part of the 6 Pounds, Wages due to him at the end of $\frac{1}{2}$ of the Year; therefore 3 *l.* (the supposed Value of the Cloak) together with 2 $\frac{1}{2}$ *l.* (the Money which the Servant received) should be equal to $\frac{3}{2}$ of a Pound, (the Value of part of the Cloak due to the Servant at the end of $\frac{1}{2}$ of the Year,) together with $\frac{3}{2}$ *l.* (the Wages due for the same time;) that is to say, $5 \frac{1}{2}$ *l.* (the Sum of 3 *l.* and 2 $\frac{1}{2}$ *l.*) should be equal to $1 \frac{3}{4}$ *l.* (the Sum of $\frac{3}{2}$ *l.* and $\frac{3}{2}$ *l.* but it is greater by $\frac{1}{4}$; wherefore the first Position for the Value of the Cloak being 3 Pounds, the Error is found to be $\frac{1}{4}$ too much.

4. I make a second Supposition, guessing the Value of the Cloak to be 2 Pounds, and proceeding in every respect as with the first Supposition, I find the Error to be $\frac{1}{2}$ too little, so that the two Positions with their Errors will be as you see:

Posit.	Er.
3	$+$ $\frac{1}{4}$
2	$-$ $\frac{1}{2}$

Now in regard the Errors are Fractions, I may take in their stead whole Numbers in the same Proportion, to wit, multiplying the Numerator of the first Fraction (or first Error) by the Denominator of the second, I take the Product which is 6 instead of the first Error $\frac{1}{4}$; likewise multiplying the Numerator of the second Fraction by the Denominator of the first, I take the Product which is 4 instead of the second Error $\frac{1}{2}$, or instead of the said 6 and 4, I may take 3 and 2, (which are in the same Proportion with 6 and 4, or with $\frac{1}{2}$ and $\frac{1}{4}$; (then multiplying the Positions and new Errors cross-wise, and adding the Products together, (because the Signs are unlike,)

Pos.	Err.
3	$+$ $\frac{1}{4}$ 63
2	$-$ $\frac{1}{2}$ 42
6	
6	
5	(12 ($\frac{2}{5}$ pound.
	the

the Sum is 12 for a *Dividend*, and the Sum of the *Errors* 3 and 2 is 5 for a *Divisor*, so the Quotient will be found to be $2\frac{2}{5}$ *l.* so much therefore was the Value of the Cloak, as will easily appear if Tryal be made with $2\frac{2}{5}$ *l.* in the same manner as with the first feigned Number.

Quest. 2. *Vitruvius* (in *lib. 9. cap. 3.*) reports, That King *Hiero* having given Orders for the making of a Crown of pure Gold, was informed that the Workman had detained Part of the Gold; the King being much displeased at the Deceit, recommended the Examination of the Business to [the famous *Archimedes* of *Syracuse*, who without defacing the Crown discover'd the Cheat in this manner; *viz.* Experience telling him that a Quantity of Gold would possess less Room or Space than the same Quantity of Silver, and consequently that a mixt Mass of Gold and Silver of the same Quantity would take up some mean space between the two former; he made a Mass of pure Gold of the same Weight with the Crown, likewise another Mass of Silver of the same Weight; then having put the Crown, as also the other two Masses severally into a Vessel filled up to the Brim with Water, he diligently reserv'd the Water flowing over into another Vessel, and from those three several Quantities of Water so expell'd, he found out the Quantity of Gold and of Silver in the Crown: But since *Vitruvius* has not deliver'd the practical Operation, I shall shew the same after the manner of *Cardanus*, *Gemma Frisius*, and other *Arithmeticians*.

Let us therefore suppose the Weight of the Crown, as also of the two several Masses to have been 5 *l.* Suppose also that by putting the Mass of Gold into the Vessel, 3 *l.* of Water was expell'd; by putting in the Crown $3\frac{1}{4}$ *l.* and by putting in the Mass of Silver, $4\frac{1}{2}$ *l.* The Question therefore is to know how much Gold and how much Silver the Crown was composed of. This may be resolv'd after this manner. Suppose

4 — 3 — 3 — ($1\frac{1}{2}$ there remain'd 2 *l.* of Silver; now say
5 — $4\frac{1}{2}$ — 2 — ($1\frac{1}{2}$ by the Rule of Three, if 5 *l.* of Gold expel 3 *l.* of Water, how much $3\frac{1}{4}$ *l.* of

Gold? *Ans.* $1\frac{4}{7}$ *l.* Also if 5 *l.* of Silver expel $4\frac{1}{2}$ of Water, how much 2 *l.* of Silver? *Answer*, $1\frac{2}{7}$ *l.* of Water; add therefore the Water of the Silver and of the Gold together, to wit, $1\frac{4}{7}$ and $1\frac{2}{7}$, so there will arise $3\frac{3}{7}$ *l.* of Water: This ought to have been $3\frac{1}{4}$ *l.* (for so much overflow'd by putting in the Crown;) but it is too much by $\frac{5}{28}$; therefore $\frac{5}{28}$ is to be noted with +, for

for the Errour of the first Position 3 l. Again, feign another Quantity of Gold to have been in the Crown, to wit, 2 l. therefore there remain'd 3 l. of Silver; then say, if 5 l. of Gold expel 3 l. of Water, how much 2 l. of Gold? *Ans.* $1\frac{1}{5}$ l. of Water: Also if 5 l. of Silver expel $4\frac{1}{5}$ l. of Water, how much 3 l. of Silver? *Ans.* $2\frac{1}{5}$, then add $1\frac{1}{5}$ l. unto $2\frac{1}{5}$, the Sum will be $3\frac{2}{5}$ of Water. This ought to have been $3\frac{1}{4}$ l. but it is too much by $\frac{1}{20}$. Therefore $\frac{1}{20}$ is to be noted with + for the Errour of the second Position 2 l. Here because the Errours are Fractions having a common Denominator, I take their Numerators, 7 and 13 instead of the Errours, then multiplying cross-wise, to wit, 3 by 13 the Product is 39; also 2 by 7, the Product is 14, which subtracted from the former Product 39, (because the Errours are alike,) leaves 25 for a Dividend; also the Difference between the Errours 7 and 13 is 6 for a Divisor: Lastly, dividing 25 by 6 the Quotient is $4\frac{1}{6}$; so much Gold, therefore, was in the Crown, and consequently (because the Weight of the Crown was 5 l.) there was $\frac{5}{6}$ of Silver, which may be proved thus: Say if 5 l. of Gold expel 3 l. of Water, how much $4\frac{1}{6}$ l. of Gold? *Ans.* $2\frac{1}{6}$ l. of Water: Again, if 5 l. of Silver expel $4\frac{1}{6}$ of Water, how much $\frac{5}{6}$ of Silver? *Answer,* $\frac{3}{4}$ l. of Water, which being added to $2\frac{1}{6}$ l. the Sum is $3\frac{1}{4}$ l. of Water, to wit, as much as flowed over when the Crown was put into the Vassel.

Here note, that in making a Tryal of this Nature, there is no Necessity that the Mass of Gold or of Silver be of the same Weight with the Crown, or whatsoever thing is to be examined, but of what notable part of Weight you please.

Note also, That for the more easy discovering of the Dividend and Divisor by the Notes of X and — according to the fourth Rule of this Chapter, the following Verse may be a Help; to wit,

Addito dissimiles, subtrahitoque pares.

Or thus,

*Notes being unlike, Addition make;
If like, lesser from greater take.*

<i>Pof.</i>		<i>Err.</i>
3	+	7
2	—	13
39		
14		
6		(25 ($4\frac{1}{6}$ l. of Gold.

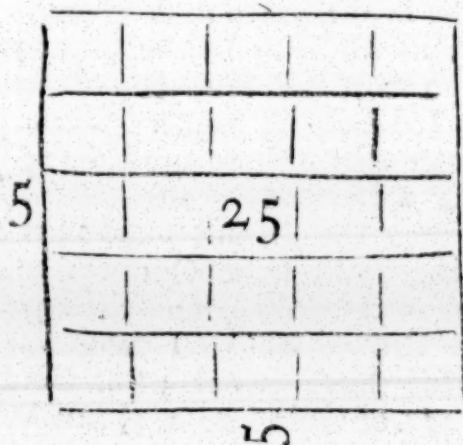
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The Reader may see more *Questions* to exercise the *Rule of False* in the Tenth Chapter of the *Appendix*, and the Demonstration of it in the Ninth Chapter of the same.

C H A P. XXXII.

The Extraction of the Square (or Quadrate) Root.

I. THE Extraction of the Square-root is that, by which having a Number given, we find out another Number, which being multiplied by itself, produces the Number given.



II. In the Extraction of the Square-root, the Number proposed is always conceiv'd to be a square Number, that is, a certain Number of little Squares comprehended within one entire great Square, and the Root or Number required is the Side of that great Square; as will readily appear by this Diagram, where you see the 25 little Squares contained within one great Square; now if the said Content 25 be given, and the Side or Root of the Square containing the said 25 little Squares is required; the Invention of such Side or Root is called the *Extraction* of the Square-root; which Root must be such, that if it be squared, that is, multiplied by it self, the Product will be equal to the square Content first given: So 5 is the Square-root of 25, for 5 times 5 is 25. Likewise this square Number 49 being propounded, its Root is 7.

III. Square Numbers are either single or compound.

IV. A

IV. A single square Number is that, which being produced by the Multiplication of one single Figure by itself, is always less than 100: So 25 is a single square Number produced by 5; likewise 4 is a square Number produced by 2.

V. All the single square Numbers together with their respective Roots are expressed in the following Table.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost Rank of the Table are placed the single square Numbers of every particular Figure, and in the other their respective Roots; and therefore if it were demanded, What is the square root of 36, the Answer will be 6. So the square-root of 16 is 4; the square-root of 9 is 3, &c. And contrarily the square of the Root 6 is 36: Also the square of 3 is 9.

VI. When a square Number is given, that exceeds not 100, and yet is none of the square Numbers mentioned in the Table, for its root you are to take the root of the square Number that being less yet comes nearest to it: So 45 being given, the root that belongs unto it is 6, and 10 being given, its correspondent root is 3.

VII. A compound square Number is that which being produced by a Number (that consists of more Places than one) multiplied by itself, is never less than 100: So 1024 is a compound square Number produced by the Multiplication of 32 multiplied by itself.

VIII. To prepare any square Number given for Extraction, put a point over the first place thereof on the Right hand, (being the place of Units;) then proceeding towards the Left-hand, pass over the second place, and put another point over the third place; also passing over the fourth place put another point over the fifth, and so forward in such manner that between every two Points which are next to the other, one place will be intermitted: So if the square root of 1024 be required, the first point is to be placed over 4, and the second over 0 as you see, and so many points as are in that manner placed, of so many Figures the root demanded will consist.

IX. Having

IX. Having thus prepared your Number, you may see it distributed by the Points into several Squares : So in the last Example, 10 is the first Square, and 24 the second ; likewise if this Number 144 were propounded for Extraction, after Points are duly placed according to the last Rule, you will see 1 to be the first Square and 44 the second.

X. Having drawn a crooked Line on the Right-hand of the Number given for Extraction, (after the same manner as is usually done in *Division* to denote the Place of the Quotient,) find the root of the first square, and place it in the Quotient : So I find, by the sixth Rule aforesaid, 3 to be the correspondent root of 10 ; wherefore I write 3 in the Quotient, and then the Work will stand as you see.

XI. Subscribe the square of the Figure placed in the Quotient under the first square of the Number given, as you see in the Margin.

XII. Having drawn a Line under the square (of the Figure placed in the Quotient, subscribed as aforesaid,) subtract the same out of the first square of the Number proposed, and place the *Remainder orderly under the Line* ; so the square of 3 which is 9, being subtracted from 10, the remainder is 1, and the Work will appear as you see in the Margin.

XIII. To the said Remainder bring down the next square of the given Number, that is, write down the Figures or Cyphers standing in the two following Places of the Number propounded on the Right-hand of the said remainder, so the square 24 be placed next to the remainder 1, there will be found this Number 124, which may be called the

Resolvend.

XIV. Double the root being the Number placed in the Quotient, and place the said Double on the Left-hand of the *Resolvend*, like a *Divisor* : So the double of 3 is 6, which being placed before a crooked Line on the Left hand of the *Resolvend*, 124, the Work will stand as you see.

XV. Let the whole *Resolvend*, except the first Place thereof on the Right hand (being the Place of Units,) be always esteemed as a Dividend, then demanding how often the Divisor before found, is contained in the said Dividend,

dend, and observing in that behalf the Rules before taught in Division, write the Answer in the Quotient, and also on the Right-hand of the Divisor, to wit, between the Divisor and the crooked Line: So if you ask how often the Divisor 6 is found in the Dividend 12, the Answer is 2; therefore I write 2 in the Quotient and also after the Divisor 6; as you see in the Margin.

XVI. Multiply all the Numbers which stand on the Left-hand of the Resolvend (to wit, before the crooked Line) by the Figure last placed in the Quotient, and write the Product orderly under the Resolvend, (to wit, Units under Units, Tens under Tens, &c.) then having drawn a Line under the said Product, subtract it from the Resolvend, and subscribe the Remainder under the Line: So 62 being multiplied by 2, the Product is 124, which I subtract out of the Resolvend 124, the Remainder is 0; and thus the whole Work being finished, the square root of 1024 (the Number proposed) is found to be 32.

$$\begin{array}{r}
 . . \\
 1024 \quad (32 \\
 9 \\
 \hline
 62 124 \\
 124 \\
 \hline
 0
 \end{array}$$

Note, 1. When the Product before mentioned exceeds the Resolvend placed above it, the Work is erroneous, and then you are to reform it by placing a lesser Figure in the Quotient.

Note, 2. For every one of the particular Squares (distinguished by the Points) except the first on the Left-hand, a Resolvend is to be set a-part, by bringing down to the Remainder the congruent particular Square, as is directed in the thirteenth Rule; and as often as a Resolvend is set a-part, so often a new Divisor is to be found by doubling or multiplying by 2 all the Root in the Quotient (consisting of what Number of Places soever.)

Note, 3. The Work of the 10, 11, and 12 Rules for finding out the first Figure in the Root, is but once used in the Extraction of the Square Root of a Number consisting of what Number of Places soever; but the Work of the 13, 14, 15, and 16 Rules is to be repeated for the finding of every Place in the Root, except the first.

The Practice of these Three Notes will be seen in the following Examples.

Example 1. Let it be required to extract the Square Root of 43623.

Having

Having distributed the Number propos'd into several Squares

...
43623 (2

4

0

this Chapter, I demand the square root of 4 the first square, which I find by the Fifth Rule of this Chapter to be 2; therefore placing 2 in the Quotient, and the square thereof, which is 4, under the first square 4, I draw a Line, and subtracting 4 from 4, the Remainder is 0, which I subscribe under the Line. This is always the first Work, which is no more repeated in the whole Extraction, (as was intimated in the Third *Note* afore-going.)

Then bringing down the next square, which is 36, and placing it next after the Remainder 0, the Resolvend is 36; and doubling the Root 2 in the Quotient, the Product is 4 for a Divisor, (by the Thirteenth and Fourteenth Rules,) and the Dividend will be 3. (by the Fifteenth Rule;) I demand

...
43623 (20

4

40) 036

therefore how often the Divisor 4 is contained in the Dividend 3, and not finding it once contained in it, I place 0 in the Quotient, and also next after the Divisor 4; and because the Product of 40 multiplied by 0 (the last Character in the Quotient) is 0, the Resolvend 36, from which the said Product ought to be deducted,

remains the same without Alteration; therefore I bring down 23 the next square, and place it after the Remainder 36, so will 03623 be a new Resolvend; then doubling the whole Root in the Quotient, which is 20, the Divisor will be 40,

...
43623 (20

4

40) 03623

(according to the second *Note* before mentioned,) and the Dividend will be 362 (to wit, all the Resolvend except the first place on the Right-hand by Rule Fifteenth) therefore I demand how often the Divisor 40 is contain'd in the Dividend 362, or how often 4 in 36, and though it be 9 Times in it, yet (according to first *Note* afore-going) I can take but 8, (for if

I should take 9, and proceed according to the Fifteenth and Sixteenth Rules, a Number would arise greater than the Resolvend, from which such Number arising ought to be subtracted:) Therefore I write 8 in the Quotient, and also after the Divisor 40; this done, I multiply 408 (the Number on the Left-hand of the Resolvend) by 8 the Figure last placed in the Quotient, and the Product, to wit, 3264, I subscribe

und

underneath, and subtract from the Resolvend 3264, so there will remain 359; thus the Work being finished I find 208 to be the Number of Units contained in the Root sought; and because after the Extraction is ended there happens to be a Remainder, to wit, 359, I conclude that the Root is greater than the said 208, but less than 209; yet how much it is greater than 208, no Rules of Art hitherto known will exactly discover, tho' we may proceed infinitely near, as in the next Rule will be manifest.

$$\begin{array}{r}
 43623(208 \\
 \underline{4} \\
 0 \\
 408)03623 \\
 \underline{3264} \\
 359
 \end{array}$$

XVII. To find the Fractional part of the root very near, a competent Number of Pairs of Cyphers, to wit, 00, 0000, 000000, or 00000000, &c. are to be annexed to the Number first proposed; then esteeming the said given Number with the Cyphers annexed, to be but one entire Number, the Extraction is to be made according to the preceeding Rules, and look how many Points were placed over the Number first given, so many Places of Integers will be in the Root, the rest of the Root towards the Right-hand will be the Numerator of a decimal Fraction, which Numerator consists of so many Places as there were Points over the Cyphers annexed: So if 43623 were given as before, to find the Root thereof (according to this Rule,) annex Cyphers in this manner, and then if you extract it according to the Rules foregoing, you'll find the

43623.000000 (208.861, 0.000000)

Root arising in the Quotient to be 208.861, that is $208 \frac{861}{1000}$; and because after the Extraction is finished there happens to be a Remainder, I conclude that $208 \frac{861}{1000}$ is less than the true or exact Root, but $208 \frac{2661}{1000}$ is greater than it; so that by annexing three Pairs of Cyphers to the Number propounded, you will not miss $\frac{1}{1000}$ part of an Unit of the true Root; also by annexing 4 Pairs of Cyphers, you will not miss $\frac{1}{10000}$ part of an Unit; and in that Order you may proceed infinitely near, when you cannot obtain the true Root. The whole Operation of the said *Example* here follows.

43623.000000 (208.861, &c.
4 The Root.

408) 03623
3264

4168) 35900
33344

41766) 255600
250596

417721) 500400
417421
82 79

Again, if 10 were propos'd to be extracted, you must prepare it thus,

10.0000000000000000.

And then the Root thereof being extracted } will be 3.1622776, &c.

Which (according to the Third Rule of the Twenty second Chapter) may be writ } thus 3.1622776, &c.

See here part of the Work in the Extraction of the Root 10, which may give you a Light and understanding of the rest.

10.0000000000000000 (3 16227, &c.

9
61) 100
61

616) 3900
3756

6522) 14400
12644

63242) 171600
126484

632447) 4911600
4427129

484471

XVIII. The

XVIII. The Extraction of the square root is proved by multiplying the root by it self, for that done, the Product (in such Case where there is no remainder after the Extraction is finished) will be equal to the root whose square was enquired; so in the first *Example* of this *Chapter*, the root 32 being multiplied by it self, produces 1024 the Number propounded: But when, after the Extraction is finished, there happens to be a remainder, and that the root is found as near as you please, in a mixt Number of Integers and Decimal Parts (by annexing Cyphers, as in the Seventeenth Rule;) then such mixt Number being multiplied by it self must produce a mixt Number less than the Number first given for Extraction; yet so near it, that if the Figure standing in the last place of the Numerator of the Decimal Fraction in the root, be made greater by 1, and then the mixt Number so encreased be multiplied by it self, the Product must be greater than the Number first proposed: So in the *Example* of the Seventeenth Rule, if 208.861 be multiplied by it self, it produces 43622.917, &c. which is less than the given Number 43623; but if 208.802 be multiplied by it self, the Product will be 43623.335, &c. which is greater than 43623.

XIX. The square root of a Fraction is found in this manner, viz. Extract the root of the Numerator (by the preceeding Rules of this Chapter) which root shall be a new Numerator. Also the root of the Denominator is to be taken for a new Denominator: So the new Fraction will be the square root of the Fraction first proposed. Thus the square root of $\frac{9}{16}$ is $\frac{3}{4}$, viz. the root of 9 is 3 for a new Numerator, also the root of 16 is 4 for a new Denominator. In like manner the square root $\frac{1}{4}$ is $\frac{1}{2}$. But here note diligently, That if the Fraction whose square root is required be not in its least Terms, it must first of all be reduced by the Fourth Rule of the Seventeenth Chapter, before any Extraction be made; for it frequently happens that the Fraction first given has not a perfect root, but when such Fraction is reduced to its least Terms, the root thereof may be extracted: So in this Fraction $\frac{8}{18}$ each Term is incommensurable to its square root, viz. neither 8 nor 18 has a square root expressible by any true or rational Number; but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the root of this may be extracted; for the root of 4 is 2 for a new Numerator; also the root of 9 is 3 for a new

new Denominator; so that $\frac{2}{3}$ is found to be the square root of $\frac{4}{9}$ (equivalent to $\frac{2}{3}$.)

XX. When either the Numerator or Denominator of a Fraction has not a perfect square root, such root is usually expressed by prefixing this Character, $\sqrt{}$ or $\sqrt{q.}$ before the Fraction given: So the square root of $\frac{1}{2}$ is signified thus, $\sqrt{\frac{1}{2}}$ or thus $\sqrt{q. \frac{1}{2}}$, because the root of $\frac{1}{2}$ cannot be expressed by any true or rational Number whatsoever, yet it may be found very near, as in the next Rule.

XXI. The square root of a Fraction which is incommensurable to its root, may be found near, in this manner, *viz.* reduce the Fraction proposed to a Decimal by the Third Rule of the Twenty third Chapter: The more places are in the Decimal, the nearer will the root be found; but the Decimal must consist of an even Number of places, *viz.* either of Two, Four, Six, Eight, or Ten, &c. Places; then extract the square root of that Decimal, as if it were a whole Number, according to the Rules foregoing, which root found shall be a Decimal expressing near the square root of the given Fraction.

So if the square root of $\frac{1}{2}$ be required near, reduce the said $\frac{1}{2}$ to a Decimal (by the Third Rule of the Twenty third Chapter) which is found .81250000, &c. then extracting the square root thereof as if it were a whole Number, it will be found .9013 very near.

XXII. The square root of a mixt Number commensurable to its root is found in the same manner as in the 19th Rule of this Chapter, the mixt Number being reduced to an improper Fraction by the Tenth Rule of the Seventeenth Chapter.

So the square root, of $34\frac{3}{4}$ will be found $5\frac{7}{8}$, *viz.* $34\frac{3}{4}$ being reduced to the improper Fraction $\frac{119}{4}$, the square root of the Numerator .2109 will be 47 for a new Numerator; also the square root of the Denominator 64 is 8, for a new Denominator; so we find $\frac{47}{8}$ which (by the Thirteenth Rule of the Seventeenth Chapter) is $5\frac{7}{8}$ the square root sought. And here the same Caution is to be observed as in the Nineteenth Rule of this Chapter; *viz.* The Fractional part of the mixt Number, or the improper Fraction equivalent to the mixt Number, must be in the least Terms before any Extraction be made.

XXIII. When

XXIII. When the mixt Number given is incommensurable to its square Root, prefix this Character before it, viz. $\sqrt{}$ or $\sqrt{q}$. So the square Root, of $7\frac{2}{3}$ will be thus expressed, $\sqrt{7\frac{2}{3}}$ or $\sqrt{q 7\frac{2}{3}}$: But if you desire to find the square Root near of a mixt Number incommensurable to its Root, reduce the fractional Part of the mixt Number, to a Decimal of an even Number of Places, as in the Twenty first Rule of this Chapter, and annex the Decimal so found to the whole part of the mixt Number; then esteeming the said whole Number and Decimal as one entire Number, extract the square Root thereof according to the preceeding Rules of this Chapter, and from the Root found, cut off always to the Right-hand, so many Places as there are Points over the Decimal annexed, which Number so cut off shall be a Decimal, shewing the Fractional part of the Root, and that on the Left hand will be the whole part of the Root; so the square Root of $7\frac{2}{3}$ is found 2.7688 very near.

To find the Square Root near, of a mixt Number incommensurable to its Root.

C H A P. XXXIII.

The Extraction of the Cube-Root.

THE Extraction of the Cube-Root is that, by which having a Number given, we find another Number, which being first multiplied by it self, and then by the Product, produces the Number given.

II. In the Extraction of the Cube-root, the Number proposed is always conceiv'd to be a Cube number, that is, a certain Number of small Cubes, comprehended within one entire great Cube, and the Root or Number required is the side

A Cubical Number.

of that great Cube: What a Cube is may be well express'd by a Die, which indeed is a little Cube it self; therefore if you place Four Dice in a square Form, that is laying two and two in a Rank, you'll have a Square containing Four Dice, upon which if you yet erect such another Square of Dice, you shall have another great entire Cube comprehending two times four, that is 8 Dice or small Cubes; and here 8 is the Cube Number given, and 2 is the Root or Number required; In like manner if you rank 25 Dice in a square Form, viz. laying

laying 5 in a rank, you have a Square containing 25 Dice; now upon this Square of Dice if you erect four other such Squares one upon another, you'll have a great entire Cube comprehending 5 times 25, that is 125 little Cubes, and in this case 125 is the Cube-number propounded, and 5 the Root or Number required.

II. A Cube-number is either single or compound.

IV. A single Cube-number is that, which being produced by the Multiplication of one single Figure, first by it self and then by the Product, is always less than 1000. So 125 is a single Cube number produced by 5 multiplied first by it self, and then by 25 the Product; for 5 times 5 is 25, and 5 times 25 is 125.

V. All the single Cube-numbers, and Square-numbers, together with their respective Roots, are expressed in the following Table.

Cubes.	1	8	27	64	125	216	343	512	729
Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here in the uppermost Rank of the Table are placed the single Cube-numbers of the particular Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; in the next, the Squares of those Figures, and in the lowest Rank the Figures themselves being the respective Roots of the Cubes and Squares in the uppermost Rank; and therefore the Cube-root of 125 being demanded, the Answer is 5, and the Cube-root of 216 being required, the Table will give 6, and so of the rest.

VI. When a Cube-number is given, that exceeds not 1000 and yet is none of the Cube-numbers mention'd in the Table, for its Root you are to take the Root of the Cube-number, that being less, yet comes nearest to it: So 157 being given, the Root that belongs to it is 5.

VII. A compound Cube-number is that, which being produced by a Number (that consists of more places than one) first multiplied by it self, and then by the Product, is never less than 1000. So 157464 is a compound Cube-number, produced by 54 multiplied first by its self, and then by 2916 the Product, for 54 times 54 is 2916, and then 54 times 2916 is 157464, the compound Cube-number proposed.

VIII. To

VIII. To prepare a Cube-number for Extraction, put a point over the first place thereof towards the right-hand (to wit the place of Units;) then passing over the second and third places, put another point over the fourth, and passing over the fifth and sixth, put another point over the Seventh, and in that Order (to wit, two places being intermitted between every two adjacent points,) place as many points as the Number will permit: So 157464 being given, you are to place the points as in the Margin, and so many points as are in that manner fixed, of so many Figures the root demanded will consist,

$$\begin{array}{r} \cdot \cdot \cdot \\ 157464 \end{array}$$

IX. Having thus prepared your Number, you may see it distributed by the points into several Cubes.

So in the same Example 157 is the first Cube, and 464 the second. In like manner, if this Number 7464 were proposed for Extraction, after points are duly placed as before, you'll see 7 to be the first Cube, and 464 the second.

$$\begin{array}{r} \cdot \cdot \cdot \\ 157464 \end{array}$$

$$\begin{array}{r} \cdot \cdot \cdot \\ 7474 \end{array}$$

X. Having drawn a crooked Line on the right-hand of the Number prepounded to signifie a Quotient, seek for the Cube-root of the first Cube and set it in the Quotient: So I finding (by the sixth Rule of this Chapter) 5 to be the Correspondent root of 157, I write 5 in the Quotient, and then the Work will stand as you see in the Margin.

$$\begin{array}{r} \cdot \cdot \cdot \\ 157464 \end{array} \begin{array}{l} (5 \end{array}$$

XI. Subscribe the Cube of the root placed in the Quotient under the first Cube of the Number given: So 125 being the Cube of 5, the root, (by the fifth Rule of this Chapter,) I write it under 157 the first Cube of the Number given, as you see in the Example.

$$\begin{array}{r} \cdot \cdot \cdot \\ 157464 \end{array} \begin{array}{l} (5 \\ 125 \end{array}$$

XII. Draw a Line under the Cube, subscribed as aforesaid, (to wit, the Cube of the root set in the Quotient) and subtract this Cube from the first Cube of the Number proposed, placing the remainder orderly under the Line: So 125 the Cube of 5 being subtracted from 157, the remainder is 32, and the Work will stand as you see.

$$\begin{array}{r} \cdot \cdot \cdot \\ 157464 \end{array} \begin{array}{l} (5 \\ 125 \\ \hline 32 \end{array}$$

XIII. To

XIII. To the said remainder bring down the next Cube of the Number propounded (to wit, the Figures or Cyphers that stand in the three next places) placing the said Cube next after, to wit, on the right-hand of the remainder; so the next Cube 464 being set after the remainder 32, there will be found this Number 32464, which may be called the *Resolvend*.

$$\begin{array}{r} 157464 \quad (5 \\ 125 \\ \hline 32464 \text{ resolv.} \\ \hline \end{array}$$

XIV. Having drawn a Line under the *Resolvend*, square the root in the Quotient; that is, multiply it by it self, and subscribe the Triple of the said Square or Product under the *Resolvend* in such manner, that the first place, (to wit, the place of Units of the said triple square, may stand directly under the third place, or place of Hundreds) in the *Resolvend*: So the square of the root 5 is 25, the Triple whereof is 75; which I subscribe under the *Resolvend* in such manner, That the Figure 5 which is in the first place (to wit, the place of Units) in the triple Product 75, may stand under 4, which is seated in the third place of the *Resolvend*, as you see in the Margin.

$$\begin{array}{r} 157464 \quad (5 \\ 125 \\ \hline 32464 \text{ resol.} \\ \hline 75 \end{array}$$

XV. Triple the root or Number in the Quotient. and subscribe this Triple Number in such manner, that the first place thereof, (to wit, the place of Units) may stand directly under the second place (to wit, the place of Tens) in the *Resolvend*: So the triple of the root 5 is 15, which I subscribe after such manner, that the Figure 5 which is in the first place (to wit the place of Units) in the said triple Number, may stand directly under 6, which is seated in the second place of the *Resolvend*, and the Work will appear as in the Margin.

$$\begin{array}{r} 157464 \quad (5 \\ 125 \\ \hline 32464 \text{ resolv.} \\ \hline 75 \\ 15 \end{array}$$

XVI. The triple Square of the root, and the Triple of the root being placed one under the other, as is directed in the 14th and 15th rules aforesaid, draw a Line underneath, and add them together in such order as they are seated, and let the Sum be esteemed as a Divisor: So the triple square 75, and the triple Number 15, being added together as they are ranked in the Work, the sum will be 765 for a Divisor.

$$\begin{array}{r} 157464 \quad (5 \\ 125 \\ \hline 32464 \text{ resolv.} \\ \hline 75 \\ 15 \\ \hline 765 \end{array}$$

XVII. Let

XVII. Let the whole *Resolvend*, except the first place thereof towards the Right-hand, (to wit, the place of *Units*) be esteemed as a *Dividend*; then demanding how often the first Figure (towards the Left-hand) of the *Divisor* is contained in the Correspondent part of the *Dividend*, and observing in that behalf the Rules before taught in *Division*. write the Answer in the *Quotient*: So if I ask how often 7 (the first Figure of the *Divisor* towards the Left-hand) is contained in 32 (the Correspondent part of the *Dividend* placed above) the Answer will be 4; wherefore I write 4 in the *Quotient*, as you see in the *Example*.

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 \hline
 \hline
 \end{array}$$

XVIII. Having drawn another Line under the Work, multiply the Triple Square before subscribed (as is directed in the Fourteenth Rule,) by the Figure last placed in the *Quotient*, and subscribe this Product under the said Triple Square; (to wit, Units under Units Tens under Tens, &c.) So 75 being multiplied by 4, the Product is 300; which I subscribe under 75 (the Triple Square,) and the Work will stand as you see in the Margin.

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 300 \\
 \hline
 \hline
 \end{array}$$

XIX. Multiply the Figure last placed in the *Quotient* first, by it self, and then the Product by the Triple Number before subscribed, (as is directed in the Fifteenth Rule of this Chapter;) this done, subscribe the last Product under the said Triple Number (to wit, Units under Units, Tens under Tens, &c.) so 4 being squared or multiplied by it self, the Product is 16, which being multiplied by the Triple Number 15, the Product is 240; this therefore I subscribe under the aforesaid Triple Number 15, and the Work will appear as you see.

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 \hline
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 300 \\
 240 \\
 \hline
 \hline
 \end{array}$$

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 75 \\
 15 \\
 \hline
 765 \text{ Divisor} \\
 \hline
 300 \\
 204 \\
 64
 \end{array}$$

$$\begin{array}{r}
 157464 \text{ (54)} \\
 125 \\
 \hline
 32464 \text{ Resolv.} \\
 75 \\
 15 \\
 \hline
 765 \text{ Divisor.} \\
 \hline
 300 \\
 240 \\
 64 \\
 \hline
 32464 \\
 0
 \end{array}$$

XX. Subscribe the Cube of the Figure last placed in the Quotient under the Resolvend, in such manner, that the first place of this Cube, (to wit, the place of Units,) may stand under the place of Units in the Resolvend: So 64 being the Cube of 4, I write it under the Resolvend 32464, in such manner as, that the Figure 4, which is in the place of Units in the Cube 64, may stand under the Figure 4, which is seated in the place of Units of the Resolvend: Observe the Work in the Margin.

XXI. Drawing yet another Line under the Work, add the Three last Numbers together in the same Order as they are seated, and subtract the Sum of them from the Resolvend, placing the Remainder orderly underneath: So the Sum of the three last Numbers, as they are ranked in the Work, is 32464, which, if you subtract out of the Resolvend 32464, the Remainder is 0. Thus the whole Work being finished, the Cube-root of 157414 (the Number propos'd) is found to be 54.

Note, 1. When the Sum of the Three last Numbers before-mentioned is greater than the Resolvend, the Work is erroneous, and then you are to reform it by placing a lesser Figure in the Quotient.

Note, 2. For every one of the particular Cubes distinguished by the Points) except the first Cube on the Left-hand, a Resolvend is to be set a-part, by bringing down to the Remainder the next Cube (as is directed in the thirteenth Rule.) And as often as a Resolvend is set a-part, so often is a new Divisor to be found, by adding the Triple of all the Root in the Quotient (consisting of what Number of Places soever,) to the Triple of the Square of such Root, after they are orderly placed according to the 14 and 15th Rules

Note,

Note, 3. The Work of the 10th, 11th and 12th Rules for finding the first Figure in the root is but once used in the Extraction of the Cube-root of any Number whatsoever; but the Work of all the following Rules is but once used for finding of every place in the Root, except the first.

The Practice of these three Notes will be seen in the following Examples.

Example 2. Let it be required to extract the Cube-root of 8302348.

Having distributed the Number given into several Cubes by Points, as is directed in the eighth Rule of this Chapter. I demand the Cube root of 8, the first Cube on the Left hand) which I find by the fifth Rule of this Chapter to be 2; wherefore placing 2 in the Quotient, and 8 the Cube thereof under 8 the first Cube, I draw a Line, and subtracting 8 out of 8, the Remainder is 0, which I subscribe under the Line. This is always the first Work, and is no more repeated in the whole Extraction, (as was intimated in the third Note aforegoing;) then bringing down the next Cube, (to wit, the Figures standing in the three following Places of the Number proposed,) which is 302, I place it after the Remainder 0, so is 302 the *Resolvend*: This done, having drawn a Line under the *Resolvend*, I seek for the Triple of the Square of the Root, viz. the Root in the Quotient 2, which multiplied by it self, produces the Square 4, the Triple whereof is 12, this I subscribe under the *Resolvend*, in such manner that the Figure 2 in the Units place of this Triple Square 12, may stand directly under the Figure 3, which is seated in the third Place of the *Resolvend*, (to wit, the Place of Hundreds,) according to the Fourteenth Rule aforegoing. Again, I triple the Root 2, which produces 6, and subscribe this Triple Number 6 under the second Place (or Place of Tens) in the *Resolvend*, to wit, under 0, (according to the Fifteenth Rule of this Chapter;) then drawing a Line under the Work, and adding together the said two Numbers last subscribed, as they are ranked, the Sum of them is 126 for a Divisor (according to the Sixteenth Rule before laid down)

$$\begin{array}{r}
 8302348 \quad (2 \\
 8 \\
 \hline
 0302 \text{ Resolvend.} \\
 12 \\
 06 \\
 \hline
 126 \text{ Divisor.}
 \end{array}$$

That done, esteeming 30, to wit, all the Places except the first or place of Units in the *Resolvend*, as a *Dividend*, I demand how often the Divisor 126 is contained in 30, and not finding it once contained therein, I write 0 in the Quotient; and now, because the Sum of the three Numbers, which ought to have been produced (according to the 18th, 19th, and 20th Rules of this Chapter) by the Multiplication of 0, (which was last placed in the Quotient) amounts to 0; the *Resolvend* 302, out of which the said Sum should have been subtracted, remains the same without Alteration: Therefore, having drawn a Line under the Work, I write down a new the old *Resolvend* 302, and bringing down the next Cube 348, I annex it to the said 302, so there will be a new *Resolvend*, to wit, 302348.

Then squaring the Root 20, (that is multiplying it by itself) the Product is 400, which I triple or multiply by 3, and subscribe the Product 1200 under the new *Resolvend* in such manner, that the Place of Units in this triple Quadrate 1200 may stand under the Place of Hundreds, or third Place of the *Resolvend* 302348, to wit, under 3 (according to the 14th Rule.) Again, I subscribe the Triple of the Root 20, which is 60, in such manner, that the Place of Units in this Triple Root 60 may stand under the Place of Tens, or second Place of the *Resolvend*, to wit, under 4, then adding together the two Numbers last subscribed, to wit, 1200 and 60, in such Order as they are ranked in the Work, the Sum is 1260 for a Divisor.

Again

$$\begin{array}{r}
 \begin{array}{r}
 302348 \quad (202 \\
 \hline
 0302 \text{ Resolvend.} \\
 \hline
 12 \\
 66 \\
 \hline
 126 \text{ Divisor.} \\
 \hline
 302348 \text{ Resolvend.} \\
 \hline
 1200 \\
 60 \\
 \hline
 1260 \text{ Divisor.} \\
 \hline
 2400 \\
 240 \\
 08 \\
 \hline
 242408 \text{ Ablatitium.} \\
 \hline
 59904
 \end{array}
 \end{array}$$

Again, esteeming the whole Resolvend, except the first Place (or Place of Units,) as a Dividend, to wit, 30234, I demand how often 1 (the first Figure of the Divisor towards the Left-hand) is contained in 3 the correspondent Part of the Dividend; and though it be Three Times contained in it, yet, (according to the first Note at the End of the Twenty first Rule of this Chapter,) I dare take but 2, for if I should take 3, and proceed according to the Eighteenth, Nineteenth, Twentieth, and Twenty first Rules of this Chapter, a Number would arise greater than the Resolvend (from which such Numbers arising ought to be subtracted) therefore I write 2 in the Quotient.

Then multiplying the Triple Square 1200 before subscribed, by 2 (the Figure last placed in the Quotient,) the Product is 2400, which I subscribe under the said 1200 (to wit, Units under Units, and Tens under Tens. &c.) Also multiplying the Triple Root 60 before subscribed, by 4 (the Quadrate of 2 the Figure last placed in the Quotient,) the Product is 240, which I subscribe under the said Triple Root 60; last of all I subscribe 8 the Cube of the said new Root 2, under the Place of Units or first Place of the Resolvend, to wit, under 8; and having added together those Three Numbers last subscribed, to wit, 2400, 240, and 8, as they stand in Ranks in the Work, the Sum of them is 242408, which being subducted from the Resolvend 302348, there will remain 59940. Therefore the Work being finished, I find 202 to be the Number of Units contained in the Cube-root of 8302348 the Number propounded: And because after the Extraction is ended there happens to be a Remainder, to wit, 59940, I conclude that the Cube-root sought is greater than the said 202, but less than 203; yet how much it is greater than 202, no Rules of Art hitherto known will exactly discover, tho' we may proceed infinitely near, as by the next Rule will be manifest.

XXII. To find the fractional part of the Root very near, ternaries of Cyphers, to wit, 000, 000000, or 000000000 &c. are to be annexed to the Number first proposed; then esteeming the said proposed Number with Cyphers annex'd, to be but one entire Number, the Extraction is to be made according to the preceeding Rules of this Chapter, and look how many Points were placed over the Number first given, so many of the foremost Places in the Quotient are the Integers or Units contained in the Cube-root sought for, and the

the rest of the Places in the Quotient are to be esteemed as the Numerator of a Decimal Fraction, which Numerator consists of so many Places as there were Points over the Cyphers first annexed: So if 8302348 were given as before, to find the *Cube-root* thereof (according to this Rule) annex Cyphers in this manner.

8302348,000000(

And then if you prosecute the Extraction according to the Rules aforegoing, you'll find the *Cube-root* sought to be 202.48, &c. that is $202\frac{48}{1000}$ and somewhat more; therefore you may conclude: that $202\frac{48}{1000}$ is less than the *true Root*, but $202\frac{49}{1000}$ is greater than it: Thus by annexing two Ternaries of Cyphers, to wit, 6 Cyphers to the Number propounded, you will not miss $\frac{1}{1000}$ part of an Unit of the true Root; also by annexing 3 Ternaries of Cyphers, to wit, Nine Cyphers, you will not miss $\frac{1}{1000000}$ part of an Unit of the true Root; and in that Order you may proceed infinitely near, when you cannot obtain the true Root. The whole Operation of the said *Example* here follows, where you may observe, that for the more certain and easy placing, as well of the Numbers which constitute the several Divisors, as of those that make up the ablatitious Numbers to be subtracted from the several and respective Resolvends, down-right Lines are drawn between the particular Cubes of the Number proposed, first distinguish'd by Points as before.



8	302	384	000	000 (102 .48, &c.
8	—	—	—	
0	302	—	—	Resolvend.
1	2	—	—	
	06	—	—	
1	26	—	—	Divisor.
	302	348	—	Resolvend.
	120	0	—	
	—	60	—	
	120	60	—	Divisor.
	240	0	—	
	2	40	—	
	—	08	—	
	242	408	—	Ablatitium.
	59	940	000	Resolvend.
	12	241	2	
	—	6	06	
	12	247	26	Divisor.
	48	964	8	
	—	96	96	
	—	—	64	
	49	061	824	Ablatitium.
	10	878	176	000 Resolvend.
	1	228	972	8
	—	—	60	72
	1	229	033	52 Divisor.
	9	831	728	4
	—	3	885	08
	—	—	—	512
	9	835	668	992 Ablatitium.
	1	042	507	008

In like manner the *Cube-root* of 2 will be found to be near equal to 1.25992, &c. that is $1\frac{25992}{1000000}$ and more.

XXIII. The Extraction of the *Cube-root* is proved by multiplying the Root cubically, to wit, the
The Proof. root being first multiplied by it self, and then the Product multiplied by the root, the Num-

ber arising or last Product (in case there be no Remainder after the Extraction is finished) will be equal to the Number proposed: So in the first Example of this Chapter, the *Cube-root* 54 being multiplied first by it self produces 2916, which being multiplied again by 54 produces 157464, to wit, the Number whose *Cube-root* was required. But if after the Extraction is finished, there happens to be a Remainder, and the root is found as near as you please in *Integers* and *Decimal Parts*, (by annexing Cyphers as in the Twenty second Rule of this Chapter;) then such mixt Number expressing the root being multiplied cubically, must produce a mixt Number less than the Number first propounded, yet so near to it, that if the Figure standing in the last Place of the *Decimal Fraction* in the root be made greater by 1, and the mixt Number so increased be multiplied cubically, the Product will be greater than the Number first proposed: So in the Example of the Twenty second Rule of this Chapter, if 202.48, be multiplied cubically, it produces 8301355.48, &c. which is less than the propounded Number 8302348; but if 202.49 be multiplied cubically, there will arise 8302535.49, &c. which is greater than the said given Number.

XXIV. The *Cube-root* of a Fraction is found in this manner, viz. extract the *Cube-root* of the Numerator (accord-

To extract
the *Cube-root*
of a Fraction.

ing to the fore-going Rules,) which root reserve for a new Numerator; also the *Cube-root* of the Denominator shall be a new Denominator; lastly, This new Fraction will be the *Cube-root* of the Fraction first proposed: So the *Cube-root* of $\frac{8}{27}$ is $\frac{2}{3}$; for the *Cube-root* of 8 is 2 for a new Numerator; also the *Cube-root* of 27 is 3 for a new Denominator. In like manner the *Cube-root* of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently, that the Fraction whose *Cube-root* is required, must be in its least Terms before any Extraction be made; for it frequently happens that the Fraction first given has not a perfect root, though, when such Fraction is reduced to its least Terms, the root thereof may be extracted: So in this Fraction $\frac{1}{54}$ neither the Numerator nor

Denom-

Denominator has a perfect Cube-root, yet the said $\frac{16}{34}$ being reduced to its least Terms $\frac{8}{17}$ (by the Fourth Rule of the Seventeenth Chapter) the Cube-root of this may be extracted; for the Cube-root of 8 is 2 for a new Numerator, also the Cube-root of 27 is 3 for a new Denominator; so that the Cube-root of $\frac{8}{27}$ (which is equal to $\frac{16}{34}$) is found to be $\frac{2}{3}$.

XXV. The Cube-root of a Fraction which has not a perfect Cube-root may be found near in this manner; viz. reduce the Fraction given to a Decimal Fraction, by the Third Rule of the Twenty third Chapter; the more Places are in the Decimal, the nearer will the Root be found: But the Decimal must consist of Ternaries of Places, to wit, either of Three, Six, Nine, or Twelve, &c. Places; then extract the Cube-root of the Numerator of that Decimal, as if it were a whole Number, (according to the Rule before given,) which Root found shall be a Decimal expressing near the Cube-root of the Fraction proposed.

So if the Cube-root of $\frac{2}{3}$ were required, I reduce the said $\frac{2}{3}$ to a Decimal, whose Numerator may consist of Ternaries of Places, to wit, into this, 666666666666, &c. then extracting the Cube-root thereof, I find .8735, which is very near the Cube-root of $\frac{2}{3}$.

XXVI. The Cube-root of a mixt-Number commensurable to its Root may be found in the same manner as in the Twenty fourth Rule of this Chapter; the mixt Number being first reduced to an improper Fraction, (by the Tenth Rule of the Seventeenth Chapter.)

So the Cube root of $12 \frac{1}{2}$ is discovered to be $2 \frac{1}{2}$, viz. reducing $12 \frac{1}{2}$ to this improper Fraction $\frac{25}{2}$, the Cube-root of $\frac{25}{2}$ will be found $\frac{5}{2}$ or $2 \frac{1}{2}$. And here the same Caution is to be observed as in the Twenty fourth Rule of this Chapter, viz. the fractional part of the mixt Number, or the improper Fraction equivalent to the mixt Number, must be expressed by a Numerator and Denominator in the least Terms, before any Extraction is made.

XXVII. When the mixt Number, whose Cube root is required, has not a perfect Cube-root, this Character, $\sqrt[3]{\text{c}}$ is usually prefixed before such mixt Number; so the Cube-root of $2 \frac{1}{3}$ is thus expressed, $\sqrt[3]{\text{c}} 2 \frac{1}{3}$. Likewise $\sqrt[3]{\text{c}} \frac{5}{8}$

denotes the Cube-root of $\frac{1}{8}$, which is a Fraction whose Cube-root is inexpressible by any true or rational Number: But if you desire to know the Cube-root near, of a mixt Number which has not a perfect Cube-root, reduce the fractional Part of the mixt Number to a Decimal, (as in the Twenty fifth Rule of this Chapter,) and annex the Decimal so found to the Integers of the mixt Number; then esteeming the said Integers with the Decimal so annexed as one entire Number, extract the Cube-root thereof and from the Root found, cut off always to the Right hand so many Places as there were Points over the said Decimal annexed, which Places so cut off will be the fractional Part of the Root, and those remaining on the Left hand shall be the Integers of the Root: So the Cube-root of $2\frac{3}{8}$ is 1.334, and somewhat more.

XXVIII. I might here proceed to shew the Extraction of the Roots of the *Biquadrate*, (or Fourth Power,) the Fifth Power, &c. but their Operation being exceeding tedious, and hardly intelligible without the Knowledge of *Algebra*, I shall only in this Place touch upon the Extraction of the *Biquadrate-root*, because it may be extracted by the Rules delivered in the thirty second Chapter, and refer the more curious Arithmetician for further Satisfaction in this Matter, to my Treatise of the *Elements of Algebra*.

XXIX. A quadrate or square Number multiplied by it self produces a Biquadrate Number: So 4 multiplied by it self produces the Biquadrate 16. Therefore if a Number be proposed and the Biquadrate-root of it is required, first extract the quadrate or Square-root of the said given Number, and then extract the Square-root of that Root for the Biquadrate-root sought. Thus if 20736 be a Number propounded, the Biquadrate-root thereof will be found 12: For the Square-root of 20736 is 144, and the Square-root of 144 is 12. When the given Number has not a perfect Biquadrate-root, you are to annex Quaternaries of Cyphers, to wit, either 4, 8, 12, or 16, &c. Cyphers, and then proceed as before; so you'll find the Root near, whose fractional Part will be a Decimal. Thus the Biquadrate-root of 7 is near 1.62.

C H A P. XXXIV.

The Relation of Numbers in Quantity.

I. **T**HUS far single Arithmetick: Comparative Arithmetick ensues, which is work'd by Numbers, as they are consider'd to have Relation one to another.

II. This Relation consists in Quantity, or Quality. *Boetius Arith; l. 1. cap. 21.*

III. Relation in Quantity is the Reference or Respect, that the Numbers themselves have one to another: As when the Comparison is made between 6 and 2, or 2 and 6: 5 and 3, or 3 and 5.

IV. Here the Terms or Numbers proposed are always two, of which the first is called the *Antecedent*, and the other the *Consequent*: So in the first Example, 6 is the Antecedent, and 2 the Consequent: And in the second, 2 is the Antecedent, and 6 the Consequent.

V. Relation in Quantity consists either in the Difference, or else in the Rate or Reason that is found between the Terms propounded.

VI. The Difference of two Numbers is the Remainder left after Subtraction of the less out of the greater: So 6 and 2 being the Terms given, 4 is the Difference between them; for if you subtract 2 from 6, the Remainder is 4. *Difference.*

VII. The Rate or Reason between Two Numbers is the Quotient of the Antecedent divided by the Consequent: So if it be demanded what Rate or Reason 6 has to 2, I answer, triple Reason: For if you divide 6 the Antecedent, by 2 the Consequent, the Quotient is 3; 2 being contained just 3 times in 6. In like manner there is a subtriple Reason between 2 and 6; for if you divide 2 by 6, the Quotient is $\frac{2}{6}$, or (which is all one) $\frac{1}{3}$; because 6 being not once found in 2, there remains 2 for the Numerator, 6 the Divisor being the Denominator of the Fraction produced in the Quotient, according to the Ninth Rule of the Sixteenth Chapter aforegoing.

VIII. This Rate or Reason of Numbers is either equal or unequal.

IX. Equal Reason is the Relation that Equal Reason. equal Numbers have to one another: As 5 to 5, 6 to 6, 7 to 7, &c.

X. Here the one being divided by the other, the Quotient is always an Unit: For if it be demanded how often 5 is in 5, the Answer is 1.

XI. Unequal Reason is the Relation that unequal Numbers have one to another; and this either of Unequal Reason. the greater to the less, or of the less to the greater.

XII. Unequal Reason of the greater to the less, is when the greater Term is Antecedent: as of 6 to 2, 5 to 3, and the like.

XIII. Here the Quotient of the Antecedent divided by the Consequent is always greater than an Unit: So 6 divided by 2, the Quotient is 3, and 5 divided by 3, the Quotient is $1\frac{2}{3}$.

XIV. Unequal Reason of the less to the greater, is when the less Term is Antecedent: as of 2 to 6, 3 to 5, &c.

XV. Here the Quotient of the Antecedent divided by the Consequent is always less than an Unit: So 2 divided by 6, the Quotient is $\frac{2}{6}$ or $\frac{1}{3}$; and 3 divided by 5, the Quotient is $\frac{3}{5}$.

XVI. Each of these Kinds of unequal Reason is again subdivided into Five other Sorts of Varieties, of which the Three first are simple, and the other two are mixt.

XVII. The simple Kinds of unequal Reason are 1, Manifest. 2. Superparticular. 3. Superpartient.

XVIII. Manifest Reason of the greater to the less is, when the Consequent is contained in the Antecedent divers times without any part remaining: As 4 to 2, 8 to 4, 16 to 8, which is also call'd double Reason, because the less is contained twice in the greater; so 6 to 2 is triple Reason, 8 to 2 is fourfold Reason, &c.

XIX. Here the Quotient of the Antecedent divided by the Consequent is always a whole Number: So 8 divided by 2, the Quotient is 4.

XX. The

XX. The opposite of this Kind, viz. of the less to the greater, is termed *Submanifold*: Examples hereof are 2 to 4, 4 to 8, 8 to 16, &c. Likewise 2 to 6, 2 to 8, 2 to 10, &c. *Submanifold.*

XXI. Superparticular is, when the Antecedent contains the Consequent once, and besides an Aliquot part of the Consequent; that is, an half, a third, a fourth, or a fifth Part, &c. of the Consequent; as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here 3 divided by 2, the Quotient is $1\frac{1}{2}$ and 4 being divided by 3, the Quotient is $1\frac{1}{3}$. In like manner 5 divided by 4, the Quotient is $1\frac{1}{4}$, and 6 divided by 5 the Quotient is $1\frac{1}{5}$; therefore I say 2 and half 2 (that is 3) constitute 4: So likewise 3 and one third Part of 3 (viz. 1.) make 4, and so of the rest.

XXII. Here the Quotient of the Antecedent divided by the Consequent is a mixt Number, whose whole Part, as also the Numerator of the Fraction annexed, is always an Unit: As is observable in the Examples last mentioned.

XXIII. The opposite Reason of this kind is Subsuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, &c. *Subsuperparticular.*

XXIV. Superpartient is, when the Antecedent contains the Consequent once, and besides several Parts of the Consequent: As 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, &c. here 5 divided by 3, the Quotient is $1\frac{2}{3}$, and therefore 5 contains 2 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3 (viz. 2) constitute 5.

XXV. Here the Quotient of the Antecedent divided by the Consequent is a mixt Number, whose whole Part being an Unit, has always for the Numerator of the Fraction annexed to it a Number composed of more Units than one: So the Comparison being made between 5 and 3, and 5 the Antecedent being divided by 3 the Consequent, the Quotient is $1\frac{2}{3}$.

XXVI. The opposite of this Reason is Subsuperpartient: Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like. *Subsuperpartient.*

XXVII. The mixt Kind of unequal Reasons are Manifold Superparticular, and Manifold Superpartient.

XXVIII. Manifold Superparticular Reason is, when the Antecedent contains the Consequent divers Times, and besides an Aliquot part of the Consequent; as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

Manifold Superparticular.

XXIX. Here the Quotient of the Antecedent divided by the Consequent, is a mixt Number, whose whole part consisting of more Units than one, has always an Unit for the Numerator of the Fraction annexed to it; so 5 divided by 2, the Quotient is $2\frac{1}{2}$, and 21 divided by 5, the Quotient is $4\frac{1}{5}$.

XXX. The opposite of this Reason is Submanifold Superparticular; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, &c.

Submanifold Superparticular.

XXXI. Manifold Superpartient is, when the Antecedent contains the Consequent several times, and besides, divers Parts of the Consequent; as 8 to 3, 17 to 5, 19 to 4, 27 to 5, &c.

Manifold Superpartient.

XXXII. Here the Quotient of the Antecedent, divided by the Consequent, is a mixt Number, whose whole Part, as also the Numerator of the Fraction, annexed to it, is always a Number composed of more Units than one: So 8 divided by 3, the Quotient is $2\frac{2}{3}$, and 28 divided by 5, the Quotient is $5\frac{3}{5}$.

XXXIII. The opposite here is Submanifold Superpartient; as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

Submanifold Superpartient.

These are the several Kinds or Varieties of the Rates, or Reasons that are found amongst Numbers; so that no two Numbers whatsoever can be named, but the Rate or Reason between them is comprehended under one of these five Kinds.

C H A P. XXXV.

The Relation of Numbers in Quality, where of Arithmetical and Geometrical Proportion.

I. Relation in Quality (otherwise called Proportion) is either the Reference or Respect that the Reasons of Numbers have one to another, or else, which the Differences of Numbers have one to another. *Vide Euclid l. 3. d. 5. & Alsted Arith. c. 5.*

II. Therefore here the Terms proposed ought always to be more than two; for otherwise there cannot be a Comparison of Reasons or Differences in the plural Number.

III. This Proportion is either Arithmetical, or Geometrical.

IV. Arithmetical Proportion is, when several Numbers differ according to an equal Difference, as 2, 4, 6, 8, 10, &c. here 2 is the common Difference between 2 and 4, 4 and 6, 6 and 8, 8 and 10, &c. 1, 2, 3, 4, 5, 6, 7, 8, &c. differ by Arithmetical Proportion, 1 being the common Difference between them. *Arithmetical Proportion.*

V. Arithmetical Proportion is either continued or interrupted.

VI. Arithmetical Proportion continued, is, when divers Numbers are linked together by a continual Progression of equal Differences. Such are the Examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, &c. And 100000, 200000, 300000, 400000, &c. *1. Continued.*

VII. In a Rank of Numbers that differ by Arithmetical Proportion continued, the Sum of the first and last Terms being multiplied by half the Number of the Terms, the Product is the Total Sum of all the Terms: So it being demanded, how many Strokes the Clock strikes between Midaight

Midnight and Noon: the Terms of the Progression in this Question are Twelve; viz. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; for in that Order the Clock strikes. Therefore if I multiply 13, the Sum of 12 and 1 (the first and last Terms,) by 6 (being half the Number of the Terms,) the Product is 78, which is the total Sum of all the Terms given being added together.

VIII. Or thus, Multiply the Number of the Terms by the half Sum of the first and last Terms, and then likewise the Product will give you the Total of all the Terms: So 13, 11, 9, 7, 5, 3, being given, their Total is 48; for 8 the half Sum of 13 and 3, the first and last Terms, being multiplied by 6, the Number of the Terms, the Product is 48.

IX Three Numbers being proposed that differ by Arithmetical Proportion continued, the Mean being doubled, is equal to the Sum of the Extremes: So if 5, 6, 7, be given, 6 being doubled, is equal to the Sum of 5 and 7, the two Extremes.

X. Arithmetical Proportion may be continued either upwards or downwards.

XI. Upwards, when the Terms of the Progression encrease, as these, 2, 4, 6, 8, 10, 12, &c. or
Upwards. these, 1, 2, 3, 4, 5, 6, &c. And this last Rank is more particularly termed *Natural Progression*.

XII. Here, when the first Term is also the common Difference of the Terms, the last Term being divided by the Number of the Terms, the Quotient will give you the first Term of the Rank: Again in this Case, the first Term multiplied by the Number of the Terms produces the last Term: So this Rank 3, 6, 9, 12, 15, 18, 21, being proposed, in which 3 is both the first Term, and also the common Difference of the Terms; I say 21, the last Term being divided by 7 the Number of the Terms, the Quotient is 3 the first Term; contrariwise the first Term multiplied by 7 produces 21 the last Term.

XIII. Arith-

XIII. Arithmetical Proportion continued downwards, is, when the Terms of the Progression decrease:

Such as are 35, 32, 29, 26, 23, 20: And 40, *Downwards.*
35, 30, 25, 20, 15, 10, 5.

XIV. Here, when the last Term is also the common Difference of the Terms, the first Term being divided by the Number of the Terms, the Quotient will give you the last Term: Again, the last Term multiplied by the Number of the Terms, produces the first Term of the Rank.

This Rule is the Inverse of the 12th Rule afore-going.

For Example; This Rank 40, 35, 30, 25, 20, 15, 10, 5, being proposed, in which 5 is both the last Term, and likewise the Common Difference of the Terms; I say, 40, the first Term being divided by 8 the Number of the Terms, the Quotient is 5 the last Term: On the other side, 5 the last Term being multiplied by 8, the Product is 40 the first Term.

XV. Arithmetical Proportion interrupted, is, when the Progression is discontinued: As in these Numbers, 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10, differ according to Arithmetical Proportion; but so do not 4 and 8 differ; for 2 is the common Difference betwixt 2 and 4, 8 and 10; whereas the Difference betwixt 4 and 8 is 4. In like manner 8, 14, 17, 23, differ by Arithmetical Proportion interrupted.

XVI. Four Numbers being given, that differ by Arithmetical Proportion either continued or interrupted, the Sum of the two Means is equal to the Sum of the two Extremes: So 5, 6, 7, 8, being given, the Sum of 6 and 7, the two mean Numbers, is equal to the Sum of 5 and 8, the two Extremes; and 8, 14, 17, and 23, being propounded, the Sum of 14 and 17 being added together, is equal to the Sum of 8 and 23.

XVII. Geometrical Proportion is, when several Numbers differ according to like e Rate or Reason; that is, when the Reasons of Numbers being compared together are equal, So 1, 2, 4, 8, 16,

Geometrical Proportion.

32, &c. which differ one from another by double Reason, are said to differ by *Geometrical Proportion*; for as 1 is half 2, so 2 is half 4, 4 half 8, 8 half 16, 16 half 32, &c.

XVIII. *Geometrical Proportion* is either continued or interrupted.

XIX. *Geometrical Proportion continued*, is, when divers Numbers are linked together by a continual Progression of the like Reason: Of this Sort is the Example last given; for as 1 is to 2, so is 2 to 4, 4 to 8, 8 to 16, 16 to 32, &c. So likewise the Numbers 3, 9, 27, 81, 243, 729, &c. differ by *Geometrical Proportion continued*, viz. by triple Reason, every one of them being contained three Times in the next Number that follows it.

XX. In Numbers continually proportional from 1, the first Number from 1 is the Root or first Power, the second is the Square or second Power, the third the Cube or third Power, the fourth the Biquadrate or fourth Power, the fifth the fifth Power, the sixth the sixth Power, &c. So in this Rank of Numbers, 1, 3, 9, 27, 81, 243, 729, &c. 3 is the Root, 9 the Square, 27 the Cube, 81 the Biquadrate, 243 the fifth Power, 729 the sixth Power, &c.

XXI. The Root being multiplied by it self produces the Square, which being again multiplied by the Root, produces the Cube, and so every Proportional being multiplied by the Root, produces the proportional next above it, and then the Numbers comprehended between 1, and the last Number produced, are called mean Proportionals: So in this Rank of proportional Numbers, 1, 2, 4, 8, 16, 32, &c. 2 the Root being multiplied by itself produces 4 the Square, which being again multiplied by 2, produces 8 the Cube; then 8 being multiplied by 2, the Product is 16 the Biquadrate, and so of the rest in their Order; and here, 2, 4, 8, and 16, are the mean Proportionals in the Rank proposed.

Continual Means.
Briggius Arith. Log. c. 6.

XXII. If you multiply the Root by it self, and consequently the subsequent Numbers by themselves, the Numbers intercepted between 1 and the Number last produced may not unfitly be called continual Means: So if 2 be

be given for the Root, that multiplied by it self, the Product is 4, which being again multiplied by it self, produces 16, then 16 in like manner squared produces 256, which likewise multiplied by itself produceth 65536; I say then, 2, 4, 16, and 256 are continual Means between 1 and 65536.

XXIII. The continual Means comprehended between any Number given and 1, are discovered by a continued Extraction of the Square Roots; for *Example*, 65536 being given, the Root thereof extracted is 256, whose Root is 16; then the Root of 16 is 4, and the Root of 4 is 2; so that at last I find 256, 16, 4, and 2 to be continual Means intercepted betwixt 65536 and 1 as before.

XXIV. In Numbers that encrease by Geometrical Proportion continued, if you multiply the last Term by the Quotient of any one of the Terms divided by another Term, which being less, is next to it, and then deducting the first Term out of that Product, divide the Remainder by a Number that it is an Unit less than the Quotient, the last Quotient will give you the Total of all the Terms proposed in the Progression. So this Rank 2, 6, 18, 54, 162, 486, 1458, being propounded, wherein the Proportionals differ by subtriple Proportion, I first take 2 and 6 the two first Terms, and dividing 6 by 2, I find the Quotient 3: Then multiplying 1458 the last Term, by 3 the Quotient, the Product is 4374, out of which if I deduct 2 the first Term, the Remainder is 4372, which being divided by 2 (*viz.* a Number that is an Unit less than three the Quotient,) the last Quotient gives me 2186, which is the Total Sum of the given Proportionals.

XXV. Three Proportionals being given, the Square of the Mean is equal to the Product of the Extremes: So 4, 8, and 16 being propounded, 8 times 8 being 64, is equal to 4 Times 16, which is likewise 64.

XXVI. Geometrical Proportion interrupted is, when the Progression of like Reason is discontinued, in such sort, that the four Numbers being given, 2. *Interrupted*, the like Reason is not found between the second and third, that is between the first and second, and the third and fourth; of this sort are these Numbers 2, 4, 16,

32; here, as 2 is to 4, so is 16 to 32; for they differ by double Reason; but as 2 is to 4, so is not 4 to 16, for 4 and 16 differ by fourfold Reason, 4 being contained 4 Times in 16: So likewise 4, 8, 16, differ according to Geometrical Proportion interrupted.

XXVII. The Numbers of *Multiplication* and *Division* are proportional; for in Multiplication, as 1 is to the Multiplier, so is the Multiplicand to the Product, or as 1 is to the Multiplicand, so is the Multiplier to the Product: Again, in Division, as the Divisor is to 1, so is the Dividend to the Quotient: Or as the Divisor is to the Dividend, so is 1 to the Quotient.

XXVIII. Four proportional Numbers whatsoever being given, the Product of the two Means is equal to the Product of the two Extremes: So 2, 4, 16, 32, being proposed, 4 times 16 (which is 64) is equal to 2 times 32, which is likewise 64.

Here ends the First Book, which contains all that is absolutely necessary, for the full understanding of *Common or Practical Arithmetick*. Such as desire to see how the same is performed by Artificial, or Borrow'd Numbers, called *Logarithms*, may peruse Mr. *Wingate's Second Book*, being a distinct *Treatise of Artificial Arithmetick*.

A N
APPENDIX,
CONTAINING

Choice Knowledge in *Arithme-*
tick, both *Practical* and *Theoretical* ;
the *Contents* of which are exprefs'd
in the following Page.

Composed by JOHN KERSEY, late Teacher
of the *Mathematicks*, now carefully Revised,
and accurately Corrected.



Vox audita perit, litera scripta manet.

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OF THE
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U

*An Explication of several NOTES
or CHARACTERS, which, for Brevity's
sake, are used in this APPENDIX.*

THIS $+$ is a Note of *Addition*, signifying that the Number which follows such Sign is to be added to the Number preceeding it; so $3 + 4$ implies that 4. is to be added to 3: Sometimes also when no Number is placed next after the said Note, it shews that the Number preceeding is not exactly express'd; so the Square-root of 2 is $1.414 +$, or 1.414 , &c. that is, $1\frac{414}{1000}$ and somewhat more.

This $-$ is a Sign of *Subtraction*, signifying that the Number which follows such Sign is to be subtracted from the Number preceeding it; so $6 - 2$ signifies the Difference between 6 and 2, or 2 to be subtracted from 6

This $\times$ is a Sign of *Multiplication*, denoting that the Number which preceeds such Sign is to be multiplied into, or by the Number following the Sign: So 3×4 implies that 3 is to be multiplied by 4; likewise by $3 \times 4 \times 8$ is understood the continual *Multiplication* of the Numbers 3, 4, and 8; viz. 3 is to be multiplied by 4, and the Product is to be multiplied by 8. Sometimes also the said Sign has Reference to as many of the preceeding or following Numbers as have a little Line placed over them; so $3 \times 2 \times 6$ or $2 \times 6 \times 3$ signifies that 3 is to be multiplied by the Sum of 2 and 6. Likewise $8 - 5 \times 3$. or $3 \times 8 - 5$ implies that 3 is to be multiplied by the Difference between 8 and 5. And farther, if A and B represent Two Numbers, then $A + B$ or AB implies the Product of the Multiplication of those Numbers: Likewise $B - C + A$ signifies the Product arising from the Multiplication of the Excess of the Number B above the Number C, by (or into) the Number A. Again, if AB and AC represent Two Lines, then $AB \times AC$ implies a rectangular Figure or long Square made of the Lines AB and AC.

Numbers placed as you see in the Margin, denote a *Divisor*, a *Dividend*, and a *Quotient*, to wit, 3 the *Divisor*, 18 the *Dividend*, and 6 the *Quotient*; the $3 \overline{) 18} \ 6$ like is to be understood of other Numbers so disposed.

Numbers placed after the manner of a Fraction denote a *Quotient*, which arises from dividing the *Numerator* by

$$2 \times 5 \times 6$$

the *Denominator*; so $\frac{2 \times 5 \times 6}{3 + 4}$ is equal to the *Quotient*,

$$3 + 4$$

which arises from dividing the *Product* of the *continual Multiplication* of 2, 5, and 6 by the *Product* of 3 multiplied by 4.

Four Numbers set as you see in the Margin are *Geometrical Proportionals*, viz. As 2 is to 4, so is 6 to 12: or if 2 give 4, then 6 will give 12. Sometimes also they are placed thus, 2...4...6...12.

This $=$ is a Note of Equality or Equation; so, by $3 \times 4 = 5 \times 2$, is signified that the Sum of 3 and 4 is equal to the Sum of 5 and 2: Also $7 - 3 = 9 - 5$ signifies that the Difference between 7 and 3 is equal to the Difference between 9 and 5; that is, 7 lessened by 3, leaves the same Remainder, as 9 lessened by 5. Also $4 \times 3 = 12$ implies that the *Product* of the Multiplication of 4 by 3 is equal to 12.

$>$ This is a Sign of *Majority*, signifying that the Number on the Left-hand of such Sign is greater than the Number on the Right-hand of it; so $5 > 3$ implies that 5 is greater than 3.

$<$ This is a Sign of *Minority*, denoting that the Number on the Left-hand of such Sign is less than the Number on the Right-hand thereof; so $3 < 5$ implies that 3 is less than 5.

This Character $\sqrt{\quad}$ or $\sqrt[3]{\quad}$ signifies the Square-root of the Number which follows it; so $\sqrt{144}$ implies the Square-root of 144, to wit 12.

Also this $\sqrt[3]{\quad}$ denotes the Cube-root of the Number which follows it: So $\sqrt[3]{1728}$ signifies the Cube-root of 1728, which Cube-root will be found to be 12.



AN APPENDIX.

CHAP. I.

Of Contractions in the Rule of Three.

SUCH as are well skill'd in the Parts of *Arithmetick*, which have been fully laid open in the Preceding Book, and are mindful of the *Notes* or *Symbols* before explained, will find no Difficulty in the 1, 2, 3, 4, 5, and 10th Chapters of this *Appendix*; in which several compendious Operations, no less delightful than useful, are methodically handled, and the rest will be as easy to those that are but meanly acquainted with *Geometrical Demonstration*.

II. To repeat the brief Ways of *Multiplication* set forth in the Tenth, Eleventh, and Twelfth Rules of the Fifth Chapter, or those of *Division*, in the Eleventh, Fifteenth, and Sixteenth Rules of the Sixth Chapter afore-going, would be a superfluous Work; and therefore I suppose the Reader to be thoroughly acquainted with them, as also with competent Knowledge in the Operations of *Fractions* both *Vulgar* and *Decimal*.

III. It will be no small Advantage to the *Practical Arithmetician*, to have by Heart not on-

ly the common Table of *Multiplication*, but this also in the *Margin*, to the End that when a Number is given to be multiplied or divided by 12 (which happens in the *Reduction* of *Shillings* to *Pence* and the converse) the *Product* or *Quotient* may be set down in one Line only, as in the following *Examples*.

$$\begin{array}{r} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \times 12 = \begin{array}{r} 24 \\ 36 \\ 48 \\ 60 \\ 72 \\ 84 \\ 96 \\ 108 \end{array}$$

$$\begin{array}{r} 3472 \\ 12 \\ \hline 12) 41664 (3472 \end{array}$$

$$\begin{array}{r} 4736 \\ 12 \\ \hline 12) 56832 (4736 \end{array}$$

VI. When a whole Number is given to be divided by a *Divisor*, which is equal to the *Product* of the *Multiplication* of the two single Figures, instead of dividing by that *Divisor*, you may first divide by one of those single Figures, and then divide the *Quotient* by the other; so will the last *Quotient* be the same as if the *Division* had been finished by the *Divisor* first given: Thus if 3466 *Farthings* be given to be reduced to *Shillings*, because $8 + 6 = 48$, I first divide 3466 by 8, so there will arise 433 for a new *Dividend*, and 2 *Farthings* remain; then I divide the said 433 by 6, so there will arise $72\frac{1}{2}$ or 72 *Shillings* 2 *Pence*, which with the 2 *Farthings* remaining of the first *Division* make in all 72 s. $2\frac{1}{2}$ d. which is the very *Quotient*, when 3466 *Farthings* are divided by 48. Note, that you are to reserve a *Farthing* for every Unit remaining of the first *Division* by 8, and 2 *Pence* for every Unit remaining of the second *Division* by 6. The Reason of the Operation is evident, for $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.

In like manner, if 7136 *Pence* are given to be reduced to *Pounds*, because $240 \text{ d.} = 1 \text{ l.}$ also $6 \times 40 = 240$, therefore, if 7136 *Pence* be first divided by 6, the *Quotient* will give 1189 six *Pences*, and 2 *Pence* remain; then if 1189 be divided by 40, (that is by 4, after 9 the last Place of the *Dividend* towards the *Right-hand* is cut off,) the *Quotient* will be 29 l. and there will remain 29 six *Pences*, or 14 s. 6 d. which

which together with the 2 Pence remaining of the first Division, 6) 7136 and the said 29 l. makes in all l. s. d. 29 l. 14 s. 8 d. which is the same 40) 118/9 (29 : 14 : 8 with the Quotient, when 7136 Pence are divided by 240, for $\frac{1}{40}$ of $\frac{1}{2} = \frac{1}{20}$.

Again, suppose 3463 Pence are given to be reduced to Shillings; forasmuch as $4 \times 3 = 12$, I first divide 3463 by 4, so there will arise 865 for a new Dividend, and 3 Pence remain: Then I divide the said 865 by 3, so there will arise $288\frac{1}{3}$ or 288 s. 4 d. which with the 3 Pence before remaining make 288 s. 7 d. which 4) 3463 is the same with the Quotient, when s. d. 3463 Pence are divided by 12, for $\frac{1}{3}$ of 3) 865 (288.7 $\frac{1}{3} = 13$.

V. In the Rule of Three as well Direct as Inverse, when the Divisor with either of the other Two given Numbers may be severally divided by some common Measure, without leaving any Remainder, the Quotients may be taken for new Terms, and proceeding in like manner as often as possible, the Operation according to the Tenth Rule of the Eighth Chapter, or the Second Rule of the Ninth Chapter, will be much contracted: So if it be demanded, what 52 Yards of Cloth will cost at the Rate of 21 l. for 14 Yards; the Answer will be found 78 Pounds in manner following.

y.	l.	y.
14 . . .	21 . . .	52
2 . . .	3 . . .	52
1 . . .	3 . . .	26 . . (78

In the first Rank you may observe that the Divisor 14 and the second Term 21, being severally divided by their common Measure 7, the three new Terms (in the second Rank) will be 2, 3, 52. Again, in the second Rank, the Divisor 2 and the third Term 52 being severally divided by their common Measure 2, the three new Terms (in the third Rank) will be 1, 3, 26. Lastly, working with these according to the Rule of Three Direct, the Answer to the Question, (or Fourth Term,) will appear to be 78.

Another

Another *Example*. If 21 Men can finish a Work in 16 Days, what Time must be allowed to 12 Men for the finishing of such a Work? *Answer* 28 Days.

<i>Men.</i>	<i>Days.</i>	<i>Men.</i>
21 . . .	16 . . .	12
7 . . .	16	4
7	4	1 (28 Days.

In the first Rank you may observe, That the *Divisor* 12 (for the *Rule is Inverse*;) and the first Term 21 being severally divided by their common Measure 3, the three new Terms in the second Rank, will be 7, 16, 4. Again, in the second Rank, the Divisor 4, and the second Term 16 being severally divided by their common Measure 4, the three new Terms in the third Rank will be 7, 4, 1. *Lastly*, working with these as the *Rule of Three Inverse* requires, the *Answer* to the *Question* (or fourth Term) will be found 28.

VI. In the *Rule of Three*, as well *Direct* as *Inverse*, when the Divisor and either of the other two Terms are Fractions having a common Denomination, the Denominators may be rejected, and their Numerators retained as new Terms: So if it be demanded what is the Value of $\frac{7}{8}$ of an Ell, when $\frac{1}{8}$ of an Ell are worth 66 Pence, the *Answer* will be found 154 Pence, and the Work will stand as you see.

$\frac{7}{8}$. .	66 . .	$\frac{7}{8}$
3 . .	66 . .	7
1 . .	22 . .	7 (154

Another *Example*. If $3\frac{3}{4}$ Yards of Scarlet-Cloth cost 8 l. 15 s. What is the Price of one Yard at that Rate? *Answer*, 2 l. 6 s. 8 d.

$3\frac{3}{4}$. .	$3\frac{3}{4}$. . .	1
15 . .	35 . . .	1
3 . .	7 . . .	1 ($2\frac{1}{3}$

VII. In the *Rule of Three* as well *Direct* as *Inverse*, when the Divisor only is a Fraction, either of the other two Terms may be reduced to a Fraction of the same Denominator, and then the Denominators may be rejected, as before in

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in the sixth Rule. Also when one of the three given Terms is a Fraction, and is not the Divisor, the Divisor may be reduced to a Fraction of the same Denomination with the Fraction first given, and then the common Denominators may be likewise cancelled.

An Example of the first Case may be this, if $\frac{7}{8}$ of a Yard cost 14 s. what is the Price of 1 Yard? Answer, 16 Shillings.

$$\begin{array}{rcl}
 \text{yards} & \text{shill.} & \text{yard} \\
 \frac{7}{8} \dots 14 \dots 1 & & \\
 \frac{7}{8} \dots 14 \dots \frac{8}{8} & & \\
 7 \dots 14 \dots 8 & (16 \text{ shill.}) &
 \end{array}$$

An Example of the second Case; if of Stuff that is $\frac{3}{4}$ of a Yard in Breadth, 7 Yards in Length, will make a Garment; how much of that Stuff which is one Yard in breadth will be sufficient for the same Purpose? Answer $5 \frac{1}{4}$ Yards.

Rules of Three Inverse $\left\{ \begin{array}{l} \frac{3}{4} \dots 7 \dots 1 \\ \frac{3}{4} \dots 7 \dots \frac{4}{4} \\ 3 \dots 7 \dots 4 \end{array} \right. \left(\frac{21}{4} \text{ or } 5 \frac{1}{4} \right).$

C H A P. III.

Rules of Practice by Aliquot Parts.

I. AN Aliquot Part takes its Name from the Latin Word *Aliquoties*; for (according to Euclid) an Aliquot Part is of a greater Number such a Part, which being taken (*Aliquoties* or) certain Times, precisely constitutes that greater Number; so 3 is an Aliquot Part of 12; for 3 taken 4 Times, exactly makes 12 without any Excess or Defect: In like manner 4 is an Aliquot Part of 20, because 4 taken 5 times, precisely makes 20; but 7 is not an Aliquot Part of 20, for 7 taken twice, wants of 20, and being taken thrice, exceeds 20; this kind of Part last mentioned, is by Euclid called *pars aliquanta*, of which there will be no Use in this Place.

II. When

II. When the *Rule of Three Direct* has 1 or an Integer for the first Term, it is commonly called a *Rule of Practice*, either from the great Use and Practice of it in common Affairs, or else for that, Questions of this Nature may be resolved by Operations more speedy and practical than those of the *Rule of Three*.

III. The choicest of these *Rules of Practice* may be reduced to 5 Cases, viz.

When the Price of 1 or an Integer consists, {

- 1, Of shillings under 20.
- 2, Of pounds and shillings.
- 3, Of pence under 12.
- 4, Of shillings and pence.
- 5, Of pounds, shillings, and pence, with parts of a penny.

All these Cases, with others of the like Nature, are handled in their Order.

IV. Any even Number of Shillings is either $\frac{1}{10}$ of a Pound (that is 2 Shillings) or else is composed of $\frac{1}{10}$ l. (to wit 2 s.) taken certain Times: So 8 s. is composed of $\frac{1}{10}$ l. (or 2 Shillings) taken four Times; in like manner 18 s. is composed of $\frac{1}{10}$ l. taken 9 Times.

V. When the Price of 1, or an Integer of what Name soever, is 2 Shillings, the Price of as many Integers as one will of that Name is discoverable at first Sight; to wit, by accounting the Double of the Figure which stands in the first Place (towards the Right-hand) of the said Numbers of Integers as Shillings, and the rest of the said Number as

yard shill. yards.

1 . . . 2 . . . 345

Answer 34 l. 10 s.

Pounds: So 345 Yards at 2 Shillings the Yard will cost 34 l. 10 s. for the Double of 5 is 10, which I write down a-part as Shillings; then esteeming the remaining Figures towards the Left-hand, to wit, 34, as an entire

Number of Pounds, the Answer will be 34 l. 10 s. This Contraction is nothing else, but dividing the Number of Integers, whose Price is required by 10.

More

More Examples hereof are these;

$$\begin{array}{r}
 \text{oz.} \quad \text{shill.} \quad \text{oz.} \\
 1 \dots 2 \dots 2057 \\
 \hline
 \text{l.} \quad \text{s.} \\
 \text{Answ. } 205 \dots 14
 \end{array}$$

$$\begin{array}{r}
 \text{yard} \quad \text{shill.} \quad \text{yards.} \\
 1 \dots 2 \dots 120 \\
 \hline
 \text{l.} \quad \text{s.} \\
 \text{Answ. } 12 \dots 0
 \end{array}$$

VI When the given Price of 1 or an Integer is any even Number of Shillings greater than two Shillings, multiply the Number of Integers, whose Price is required, by half the given Number of Shillings, with this Caution, That the Double of the Figure which arises, in the first Place of the Product be written a-part as Shillings, and the rest of the Product as Pounds: So if it be demanded what 218 Yards, at 8 Shillings the Yard, amount to, the Answer will be found 87*l.* 4*s.* For I multipo 218 by 4 (which is half 8 the given Number of Shillings) saying 4 times 8 is 32; here the double of 2 (to wit, of that Figure which is to possess the first Place in the Product) is 4, which I set a-part as Shillings, keeping 3 in Mind for the three Tens; again, 4 times 1 is 4, which with 3 in Mind makes 7; lastly, 4 times 2 makes 8, so I conclude that the Answer to the Question is 87*l.* 4*s.* The Reason of this Contraction is evident from the fourth and fifth Rules afore-going. More Examples of this Rule are these following.

$$\begin{array}{r}
 \text{yard.} \quad \text{s.} \quad \text{yards.} \\
 1 \dots 14 \dots 436 \\
 \hline
 7
 \end{array}$$

$$\begin{array}{r}
 \text{l.} \quad \text{s.} \\
 \text{Anw. } 305 \dots 4
 \end{array}$$

$$\begin{array}{r}
 \text{yard.} \quad \text{s.} \quad \text{yards.} \\
 1 \dots 18 \dots 320 \\
 \hline
 9
 \end{array}$$

$$\begin{array}{r}
 \text{Answ. l. } 307 \dots 0
 \end{array}$$

E e

VII.

VII. Any odd Number of Shillings is either compos'd of $\frac{1}{10}l.$ (or 2 s.) and of $\frac{1}{20}l.$ (or 1 s.) or else it is compos'd of $\frac{1}{10}l.$ (or 2 s.) taken certain times, and of $\frac{1}{20}l.$ (or 1 s.) So 3 s. is compos'd of 2 s. and 1 s. Also 7 s. is compos'd of 2 s. taken three times and of 1 s. Likewise 13 s. is compos'd of 2 s. taken six times, and of 1 s.

VIII. When the given Price of 1 or an Integer is an odd Number of Shillings, work for the greatest even Number of Shillings contained in that odd Number, according to the fifth or sixth Rule afore going: then for the odd Shilling remaining, take $\frac{1}{10}$ of the Number of Integers whose Price is required, (by the 16th Rule of the sixth Chapter of the preceeding Book.) These two Results added together, give the Answer to the Question: So, if it be demanded what 2344 Ounces, at 13 s. the Ounce, will cost, the Answer will be

oz.	shill.	oz.	
1.	13	2344	
		6	
		—	
l.		s.	
	1406	8	
	117	4	
	—		
l.		s.	

found 1523 l. 12 s. For if (according to the sixth Rule of this Chapter) I multiply 2344 by 6, (to wit, by half the Remainder, when 1 is abated from 13 the given Number of Shillings,) there will arise 1406 l. 8 s. Then taking $\frac{1}{10}$ of 2344, there will arise 117 l. 4 s, which being added to the former Product, gives 1523 l. 12 s. for the Answer to the Question.

Ans. 1523...12

Note, When 5 Shillings is the given Price of 1 or an Integer, the shortest way will be to take $\frac{1}{4}$ of the Number of Integers, whose Value is required; for such Quotient will give the Pounds and Shillings, which answer the Question: So 2347 Ounces at 5 s. the Ounce amount to 586 l. 15 s. For $\frac{1}{4}$ of 2347 is 586 $\frac{3}{4}$ or 586 l. 15 s. But when the given Price of 1 is any other odd Number of Shillings, this eighth Rule will be as compendious as any other whatsoever.

More Examples of this Rule are these following.

yard	shill.	yards.
1	19	739

	l.	s.
	665	2
	36	19
	—	

Ans. 7:2...1

yard

yard. shill. yards.	
1 . . . 17 . . . 345	
l. s.	
276 . . 0	
17 . . 5	
Answ. 293 . . 5	

IX. When the Price of 1, or an Integer, consists of Pounds and Shillings, first multiply the Number of Integers, whose Price is required, by the Number of Pounds in the said given Price, and subscribe the Product as Pounds; then proceed with the Shillings in the said given Price, according to the sixth or eighth Rule of this Chapter, and having subscribed that which arises under the aforesaid Product of Pounds, add them all together for the Answer of the Question, So, if it be demanded what 328 Hundred-weight, amounts to at 2*l.* 17*s.* per C. (or one Hundred-weight,) the Answer will appear to be 934*l.* 16*s.* as by the Operation is evident.

C. l. s. C.	
1 . . . 2 . . . 17 . . . 328	
l. s.	
656 . . . 0	
262 . . . 8	
16 . . . 8	
Answ. 934 . . . 16	

More Examples to illustrate this Rule are these following :

C. l. s. C.	
1 . . . 7 : 12 . . . 504	
l. s.	
3528 . . . 0	
302 . . . 8	
Answ. 3830 . . . 8	

C. l. s. C.

1 ... 5 : 7 ... 129

l. s.

645 ... 0

38 ... 14

6 ... 9

Answ. 680 ... 3

X. Any Number of Pence under 12 is either an Aliquot part of a Shilling, or else compos'd of Aliquot parts thereof; so 3 Pence is an Aliquot part, to wit, $\frac{1}{4}$ of a Shilling. Likewise 4 is $\frac{1}{3}$ of 12: But farther. 5 Pence is compos'd of 2 Aliquot parts, to wit, of 3 Pence, which is $\frac{1}{4}$ of a Shilling, and of 2 Pence, which is $\frac{1}{6}$ of a Shilling; all which will readily appear by the following Table.

Pence.	Aliquot parts of a Shilling.
1	$\frac{1}{12}$ or $\frac{1}{4}$ of $\frac{1}{3}$
1 $\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4} \times \frac{1}{6}$
6	$\frac{1}{2}$
7	$\frac{1}{3} \times \frac{1}{2}$
8	$\frac{1}{3} \times \frac{1}{4}$
9	$\frac{1}{2} \times \frac{1}{4}$
10	$\frac{1}{2} \times \frac{1}{3}$
11	$\frac{1}{3} \times \frac{1}{2} \times \frac{1}{4}$

XI. When the given Price of 1 or an Integer is an Aliquot part of a Shilling, divide the Number of Integers whose Value is required by the Denominator of such Aliquot Part; so will the Quotient be the Number of Shillings that answers the Question; which Number of Shillings

Shillings (when there is Occasion) may be reduced to Pounds by the brief way of dividing by 20: So if it be required to know what 2686 Ounces at 4 Pence the Ounce amount to; the *Answer* will be found 44 *l.* 15 *s.* 4 *d.* For since 4 *d.* is an Aliquot part, to wit, $\frac{1}{3}$ of a Shilling, I divide 2686 by 3, so will the Quotient be 895 $\frac{1}{3}$ *s.* or 895 *s.* 4 *d.* which Shillings being divided by 20, give 44 *l.* 15 *s.* 4 *d.* for the *Answer* to the *Question*, as you see by the following Operation.

$$\begin{array}{r}
 \text{oz.} \quad \text{d.} \quad \text{oz.} \\
 1 \dots 4 \dots 2686 \\
 \hline
 \text{s.} \quad \text{d.} \\
 20) 89 \mid 5 \dots 4 \\
 \text{Answ. } 44 \dots 15 \dots 4
 \end{array}$$

More Examples of this Rule are these following.

$$\begin{array}{r}
 \text{yard.} \quad \text{d.} \quad \text{yards.} \\
 1 \dots 6 \dots 759 \\
 \hline
 \text{s.} \quad \text{d.} \\
 20) 37 \mid 9 \dots 6 \\
 \text{Answ. } 18 \dots 19 \dots 6
 \end{array}$$

$$\begin{array}{r}
 \text{yard.} \quad \text{d.} \quad \text{yards.} \\
 1 \dots 1 \dots 204 \\
 \hline
 \end{array}$$

Answ. 17 Shillings.

XII. When the given Price of an *Integer* is compos'd of Aliquot parts of a *Shilling*, divide the Number of *Integers* whose Price is required, by the several Denominators of the Aliquot Parts contained in the given Number of Pence, then add the Quotients together, and the Sum will be the Number of *Shillings* which answer the *Question*. So if it be demanded what 2347 Yards of Linnen-cloth will cost at 9 pence the Yard, the *Answer* will be found 88 *l.* 0 *s.* 3 *d.* For since 9 *d.* is compos'd of 6 *d.* and 3 *d.* to wit, of the Aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a Shilling, I first divide 2347 by 2, (the Denominator of the Aliquot part $\frac{1}{2}$.) so there arises 1173 $\frac{1}{2}$, or 1173 *s.* 6 *d.* Again, dividing the said 2347 by 4, (the Denominator of the other Aliquot part) there will arise

$$\begin{array}{r}
 \text{yard.} \quad \text{d.} \quad \text{yards.} \\
 1 \dots 9 \dots 2347 \\
 \hline
 \text{s.} \quad \text{d.} \\
 1173 : 6 \\
 586 : 9 \\
 \hline
 20) 1760 : 3 \\
 \text{l.} \quad \text{s.} \quad \text{d.} \\
 \text{Answ } 88.0 : 3 \\
 586 \frac{3}{4}
 \end{array}$$

586 $\frac{3}{4}$, or 586 s. 9 d. which two Quotients being added together give 1760 s. 3 d. or 88 l. 0 s. 3 d. which is the *Answer* of the *Question*.

More *Examples* to illustrate this *Rule* are these;

yard. d. yards.
1 . . . 8 . . . 782

s. d.
260 . . . 8
260 . . . 8

20) 52 | 1 . . . 4
l. s. d.
*Ans*w. 26 . . 1 . . 4

oz. d. oz.
1 . . . 11 . . . 540

180

180

135

0 49 | 5
l. s. d.

*Ans*w. 49 . . 15 . . 0

XIII. When the given Price of an Integer consists of Shillings and Pence, first multiply the Number of Integers whose Value is required by the said given Number of Shillings, and subscribe the Product as Shillings; then divide the said Number of Integers by the several Denominators which are correspondent to the *Aliquot* parts contained in the given

yard. s. d. yards.

1 . . 7 : 10 . . 347

s. d.
7 + 347 = 2419 :
2) 347 . . (173 : 6
3) 347 . . (115 : 8

20) 27 | 18 : 2

l. s. d.

*Ans*w. 135 : 18 : 2

Number of Pence, and subscribe the Quotient or Quotients under the aforesaid Product of Shillings, all which being added together give the Number of Shillings which *answers* the *Question*: So if it be demanded what 347 Yards of Cloth will cost at the rate of 7 s. 10 d. the Yard, the *Answer* will be found 135 l. 18 s. 2 d. For first 347 being multiplied by 7 (the given Number of Shillings,)

lings,) produces 2429 Shillings, then dividing 347 by 2 and 3 severally, (because 10d. is composed of $\frac{1}{2}$ and $\frac{1}{3}$ of a Shilling,) the Quotients will be $173\frac{1}{2}$ and $115\frac{2}{3}$, that is 173 s. 6 d. and 115 s. 8 d. Lastly, the Sum of all is 2718 s. 2 d. or 135 l. 18 s. 2 d.

More Examples of this kind are these.

yard. s. d. yards.
1 . . . 17 : 9 . . . 540

17 + 540 = $\left\{ \begin{array}{l} 3780 \\ 56 \\ 270 \\ 135 \end{array} \right.$
2 } 540 (..
4 } 540 (..

20) 958 | 5
l. s. d.

Ans^w 479 : 5 : 0

y. s. d. y.
1 . . . 14 . . . 6 . . . 313

14 + 313 = $\left\{ \begin{array}{l} 1252 \\ 313 \\ 156 \end{array} \right.$
2) 313 (..

20) 453 | 8

Ans^w, 226 l. 18 s. 6 d.

XIV. When the Price of an Integer consists of Shillings and Pence, and that such Shillings and Pence jointly considered do make an *Aliquot* part of a Pound, it will often be a briefer Way than that in the last Rule, to divide the Number of Integers, whose Value is required, by the Denominator of such *Aliquot* part, so will the Quotient give the Answer to the Question in Pounds and known Parts of a Pound. Thus if it be demanded what 767 Yards cost at the rate of 6 s. 8 d. the Yard, the Answer will be found 255 l. 13 s. 4 d. For since 6 s. 8 d. is an *Aliquot* part, to wit, $\frac{2}{3}$ of a Pound, I divide 767 by 3. so there arises in the Quotient $255\frac{2}{3}$, or 255 l. 13 s. 4 d. which is the Answer of the Question. Note, that the *Aliquot* parts of a Pound convenient for this Rule are these express'd in the following TABLE.

y. s. d.
1 . . . 6 : 8 . . . 767
l. s. d.
3) 767 (255 . . 13 : 4

sh. d.

sh.	d.	Aliquot Parts of a Pound.
6	8	$\frac{1}{3}$
3	4	$\frac{1}{6}$
2	6	$\frac{1}{8}$
1	8	$\frac{1}{12}$
1	4	$\frac{1}{5}$
1	3	$\frac{1}{7}$

XV. When the given Price of 1, or an Integer, consists of Pounds, Shillings, and Pence, reduce the Pounds and Shillings into Shillings, then proceed according to the Thirteenth Rule of this Chapter: So 517 C. at 3 l. 17 s. 5 d. per C. will be found to amount to 2001 l. 4 s. 5 d. For having reduced 3 l. 17 s. into 77 s. I multiply 517 by 77, and set down the particular Products; then for the 5 Pence which is compos'd of the Aliquot parts $\frac{1}{4}$ and $\frac{1}{8}$ of a Shilling, I take $\frac{1}{4}$ and $\frac{1}{8}$ of 517, and subscribe the Quotients orderly under the aforesaid Products: Lastly, adding all together the Sum is 40024 s. 5 d. or 2001 l. 4 s. 5 d. for the Answer of the Question.

$$\begin{array}{r}
 \text{C.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{C.} \\
 1 \dots 3 : 17 : 5 \dots 517 \\
 \hline
 77 \overline{) 517} = \begin{array}{l} \{ 3619 \\ \{ 3619 \end{array} \\
 4) 517 \dots 129 : 3 \text{ d.} \\
 6) 517 \dots 86 : 2 \\
 \hline
 20) 40024 : 5 \\
 \text{Answ. } 2001 : 4 : 5
 \end{array}$$

More Examples of this Rule are these following.

$$\begin{array}{r}
 \text{C.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{C.} \\
 1 \dots 5 : 13 : 8 \dots 108 \\
 \hline
 113 \overline{) 108} = \begin{array}{l} \{ 324 \\ \{ 108 \end{array} \\
 3) 108 \dots 108 \\
 3) 108 \dots 36 \\
 \hline
 20) 12276 \\
 \text{l.} \quad \text{s.} \\
 \text{Answ. } 613 : 16 : 6
 \end{array}$$

Answ.

C. l. s. d. C.
1 ... 2 : 10 : 6 ... 84

$$\begin{array}{r} 50 \times 84 = 4200 \\ 2) \quad 84 \quad (42 \end{array}$$

20) 424 | 2 (212 : 2 : 0 *Answ.*

C. l. s. d. C.
1 ... 1 : 12 : 4 1/4 ... 306

$$\begin{array}{r} 32 \times 306 = \left\{ \begin{array}{l} s. \quad d. \\ 612 \\ 918 \\ 102 \\ 6 : 4 1/2 \end{array} \right. \end{array}$$

20) 990 | 0 : 4 1/2
l. s. d.
Answ. 495 : 0 : 4 1/2

Note, When the given Price of an *Integer* consists of certain Pence, together with 1/2 d. or 3/4 d. it will be convenient to take due aliquot Parts of the Number of *Integers* proposed for all the given Price of an *Integer*, except 1 d. and the said 1/2 d. or 3/4 d. Then for that Penny, and 1/2 d. take 1/2 of the said *Integer* propounded, and if there be yet a farthing, take 1/4 of the said *quotient* which arises by taking 1/2; both which *quotients* give the value in shillings correspondent to 1 1/2 d. This will be evident by the following *Examples*.

yard d. yards
1 ... 8 1/2 ... 326

$$\begin{array}{r} s. \quad d. \\ 3) 326 (.. | 108 .. 8 \\ 4) 326 (.. | 81 .. 6 \\ 8) 326 (.. | 40 .. 9 \\ 6) 40 (.. | 6 .. 8 \\ 6) 9 (.. | 0 .. 1 1/2 \end{array}$$

$$20) 23 | 7 .. 8 1/2$$

l. s. d.
Answ. 11 : 17 : 8 1/2
F f

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 1 \dots 3 : 6\frac{1}{2} \dots 720 \end{array}$$

$$\begin{array}{r} 3 \times 720 = 2160 \\ 4) 720 \dots 180 \\ 6) 720 \dots 120 \\ 8) 720 \dots 90 \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 20) 255 | 0 \quad (127 : 10 : 0 \text{ Answ.} \end{array}$$

XVI. When the Price of an Integer is given, and the Price of many Integers of the same name *together* with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an Integer is required, the value of those Integers may be first found by some of the preceeding Rules, and then for the price of $\frac{1}{2}$ of an Integer, take $\frac{1}{2}$ of the given price of an Integer; likewise for $\frac{1}{4}$ of an Integer, take $\frac{1}{4}$ of the said given price; also for $\frac{3}{4}$ of an Integer, take the composed of $\frac{1}{2}$ and $\frac{1}{4}$ of the said given price: So if it be *demanded* what 34 C. 3 qu. (to wit, 34 hundred weight, and $\frac{3}{4}$ of an hundred weight) of Sugar will cost at 4l. 16 s. 3 d. per C. the *Answer* will be found 167 l. 4 s. 8 $\frac{1}{4}$ d. as by the subsequent Operation is manifest.

$$\begin{array}{r} \text{C.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \quad \text{C.} \quad \text{q.} \\ 1 \dots 4 : 16 : 3 \dots 34 : 3 \end{array}$$

$$\begin{array}{r} \text{s.} \quad \text{d.} \\ 96 \times 34 = \left\{ \begin{array}{l} 204 \\ 306 \end{array} \right. \end{array}$$

$$\begin{array}{r} \text{The Quotients for} \left\{ \begin{array}{l} 4) 34 \dots 8 \dots 6 \\ \frac{1}{2} \text{ C. } \left\{ \begin{array}{l} 48 \dots 1\frac{1}{2} \\ 24 \dots 0\frac{3}{4} \end{array} \right. \end{array} \right. \end{array}$$

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 20) 334 | 4 \dots 8\frac{1}{4} \end{array}$$

$$\text{Answ. } 167 \dots 4 \dots 8\frac{1}{4}$$

An Example of *Avoirdupois* greater Weight, where the quantity whose *Price* is sought consists of entire Hundred Weights, quarters of an Hundred, and of some number of Pounds, which is not an Aliquot Part of 28 or $\frac{1}{4}$ C.

C.	l.	s.	d.	C.	q.	lb.
1	5	15	$7\frac{3}{4}$	218	3	24
$115 \times 218 =$				1090		
				218		
				218		
2) 218 (..				109	d.	far.
8) 218 (..				27	:	3 : 0
$\frac{1}{6}$ of 27 s. 3 d. . .				4	:	6 : 2
The Quotients arising for				57	:	9 : $3\frac{1}{2}$
				28	:	10 : $3\frac{3}{4}$
				14	:	5 : $1\frac{3}{4} +$
				7	:	2 : $2\frac{3}{4} +$
				3	:	1 : 0 +
				20)	2532	2 : 3 : 2 +
						l. s. d.
				Answ. 1266 : 2 : $3\frac{1}{2} +$		

The *Example* last mentioned being (of those questions which ordinarily happen in Trade) one of the *hardest* to be resolved by the *Rule of Practice*, I shall touch upon the afore-going operation, where you may observe the price of 218 C. 3 qu. to be found after the manner of former *Examples*; then for 14 lb. part of the 24 lb. in the *question*, I take $\frac{1}{2}$ of the *price* of $\frac{1}{4}$ C. Likewise for 7 lb. I take half the *Price* of 14 lb. and so there yet remains 3 lb. whose *price* is found by taking $\frac{3}{7}$ of the *price* of 7 lb. viz. the *price* of 7 lb. being very near 7 s. $2\frac{1}{2}$ d. or $86\frac{1}{2}$ d. I multiply $86\frac{1}{2}$ by 3, and divide the *quotient* by 7, so there arises 37 d. or 3 s. 1 d. very near. Lastly, all being added together, the sum is found to be very near 2532 s. $3\frac{1}{2}$ d. or 1266 l. 2 s. $3\frac{1}{2}$ d.

Note, That a quarter of a farthing (or $\frac{1}{16}$ of a penny) is the smallest Money express'd in the example; and where any thing arises less than a quarter of a farthing, it is omitted, but it is supposed to follow this note +, for which surpluses some respect ought to be had in adding all together: Now although in resolving questions after this practical manner there will be some error, yet the loss for the most part will be less than a Farthing, which is inconsiderable.

XVII. When the price of 1 or an Integer consists of diverse Denominations, as Pounds, Shillings, Pence; and the price of a certain number of Integers, which does not exceed a single figure, is required, work as in the following Example; viz. If it be required to find what 8 C. must cost at 3 l. 13 s. $7\frac{1}{2}$ d. per C. it is evident that 8 C. must cost 8 times 3 l.

F f 2

C

$$\begin{array}{rcccccc} C. & l. & s. & d. & C. \\ 1 & . & 3 & : & 13 & : & 7\frac{1}{2} & . & 8 \end{array}$$

$$Ans. 29 : 9 : 0$$

13 s. 7½d. Therefore I multiply $\frac{1}{2}$ by 3, saying, 8 half Pence make 4 Pence, which I reserve in mind; again, 8 times 7 Pence makes 4s. 8d. (to wit, 8 six-pences make 4 s. and there are 8 Pence besides) to which adding 4 pence in mind, there will arise 5 s. which I reserve in mind, and subscribe a cipher under the place of Pence; again, I say 8 times 13 Shillings make 5l. 4 s. (to wit, 8 Angels make 4l. and 8 times 3 s. make 1 l. 4 s.) to which adding 5 s. in mind, the sum will be 5l. 9 s. wherefore I subscribe 9 s. (the excess above the Pounds) under the Shillings, and keep 5 l. in mind; lastly, I say 8 times 3 pounds make 24 pounds, which with 5 pounds in mind make 29 pounds; so that the total Product or answer of the question is found to be 29 l. 9 s.

More Examples of this kind are these.

$$\begin{array}{rcccccc} C. & l. & s. & d. & C. \\ 1 & . & 17 & : & 15 & : & 5\frac{1}{4} & . & 7 \end{array}$$

$$Ans. 124 : 8 : 0\frac{1}{4}$$

$$\begin{array}{rcccccc} C. & l. & s. & d. & C. \\ 1 & . & 18 & : & 12 & : & 6\frac{1}{2} & . & 8 \end{array}$$

$$Ans. 149 : 00 : 6$$

XVIII. When the price of 1 lb. weight is known, and the price or value of 1 C (to wit 112 lb.) is required, the answer may sometimes be given more speedily, than by any of the former Rules, by this Rule which follows, viz. Find the number of farthings contained in the given price of 1 lb. weight, then take twice that number of shillings, and once that number of groats, and having added them together, the sum will give the value of 1 C, to wit, 112 lb. weight: So if it be demanded what 1 C. or 112 lb. weight of Cheese will cost at the rate of 3 $\frac{1}{4}$ pence the pound weight, the answer will be 1 l. 10 s. 4 d.

For according to the said Rule, the number of farthings contained

tained in $3\frac{1}{4}d.$ (the price of 1 pound weight) is $l. \quad s. \quad d.$
 $13,$ therefore the double of 13 shillings is $.. \quad 1 : 6 : 0$
 13 Groats make $.. \quad 0 : 4 : 4$
 Therefore the sum (which is the price of $\frac{1}{2}$ C. or $112 lb.$ weight) is $.. \quad 1 : 10 : 4$

The reason of this Rule is evident; for if $1 lb.$ weight cost 13 farthings, then $112 l.$ must necessarily cost 112 times 13 farthings, or (which is the same) 13 times 112 farthings; but 13 times 112 farthings, are equal to twice thirteen shillings together with once thirteen groats; because 112 farthings are composed of twice 48 farthings (or two Shillings) and of 16 farthings (or one groat;) wherefore the truth of the said Rule is evident.

Another Example. When Sugar is at $5\frac{1}{2}d.$ the pound weight, what is the value of $1 C.$ (or $112 lb.$ weight?) *Answer* $2 l. 11 s. 4 d.$ For in $5\frac{1}{2}d.$ are contrained 22 farthings, therefore the double of 22 shillings is $.. \quad 2 : 4 : 0$
 22 groats make $.. \quad 0 : 7 : 4$

Which added together give the price of $1 C.$ or $112 l.$ to wit. $\frac{1}{2} \quad 2 : 11 : 4$

XIX. When the gain of (or allowance for) 100 Integers consists of some number of pounds not exceeding 10 , the gain of as many like Integers and known parts of an Integer as one will, may be found very briefly by the following method, *viz.* If $100 l.$ gain $3 l.$ what is the gain of $246 l. 18 s. 10 d.$ *Answer* $7 l. 8 s. 1\frac{28}{100} d.$ First, I multiply $246 l. 18 s. 10 d.$ by 3 (the second term) after the manner delivered in the 17 Rule of this Chapter, and write down the *Product* which is $740 l. 16 s. 6 d.$ Then I divide the said *Product* by 100 (the first term in this Rule of Three,) in this manner, *viz.* I divide 740 pounds by 100 , which is performed by cutting off, towards the right hand, the

Compendious way of computing Interest and Factors allowances.

$$\begin{array}{r}
 \begin{array}{ccccccc}
 l. & l. & l. & s. & d. & & \\
 100 & .. & 3 & .. & 246 & : & 18 : 10 \\
 & & & & & & 3 \\
 \hline
 & & 7 & | & 40 & : & 16 : 06 \\
 & & & & 20 & & \\
 \hline
 & & 8 & | & 16 & & \\
 & & & & 12 & & \\
 \hline
 & & 1 & | & 98 & &
 \end{array}
 \end{array}$$

two last places of 740, so the *quotient* gives 7 Pounds, and there will be a *remainder* of 40 Pounds, which 40 Pounds I reduce into Shillings, so there will arise 800 s. to which adding the 16 s. which stand in the place of Shillings, the sum will be 816 Shillings; these are also to be divided by 100 (by *cutting off* two places as before,) so the *quotient* will give 8 Shillings, and there will remain 16 Shillings, which being reduced to Pence, and unto them 6 Pence being *added* (to wit, the 6 Pence which *stands* in the place of Pence) there will arise 198 Pence; these also are to be *divided* by 100 (by *cutting off* two places to the *right hand* as before) so the *quotient* gives 1 penny, and there will remain 98 pence; so the exact *quotient* or answer of the question is found to be 7*l.* 8*s.* 1⁹⁸/₁₀₀ *d.*

More Examples of this Rule are these following.

$$\begin{array}{r} \textit{l.} \quad \textit{l.} \quad \textit{l.} \quad \textit{s.} \quad \textit{d.} \\ 100 \dots 9 \dots 793 : 12 : 7 \\ 6 \end{array}$$

$$\begin{array}{r} \textit{l.} \ 47 \mid 61 : 15 : 6 \\ \phantom{\textit{l.} \ 47 \mid 61 : 15 : } 20 \end{array}$$

$$\begin{array}{r} \textit{s.} \ 12 \mid 35 \\ \phantom{\textit{s.} \ 12 \mid 35 } 12 \end{array}$$

$$\begin{array}{r} \textit{d.} \ 4 \mid 26 \end{array}$$

$$\begin{array}{r} \textit{l.} \quad \textit{l.} \quad \textit{l.} \quad \textit{s.} \quad \textit{d.} \\ 100 : \dots 8 \dots 43 : 14 : 3 \\ 8 \end{array}$$

$$\begin{array}{r} \textit{l.} \ 3 \mid 49 : 14 : 0 \\ \phantom{\textit{l.} \ 3 \mid 49 : 14 : } 20 \end{array}$$

$$\begin{array}{r} \textit{s.} \ 9 \mid 94 \\ \phantom{\textit{s.} \ 9 \mid 94 } 12 \end{array}$$

$$\begin{array}{r} \textit{d.} \ 11 \mid 28 \end{array}$$

After the same manner may this *following question* and such like be *resolved*, viz. When 100 Ells of Linnen-cloth costs 30*l.* 18*s.* 9*d.* what is the price of 1 Ell? *Answer* 6*s.* 2*d.* 1 *farth.*

Ells. l. s. d. Ell.
100 : 30 : 18 : 9 ... 1
20

Shil. 6 | 18
12

Pence. 2 | 25
4

Farth. 1 | 00

XX. When the given gain of (or allowance for) 100 Integers consists of some number of Pounds not exceeding 10, together with some Aliquot part or parts of a Pound, the operation will be little different from the last mentioned Examples, as may appear by the resolution of the subsequent Question, viz. What must be allowed for 2156 l. 13 s. 4 d. at the rate of 6 l. 15 s. for 100 l.? *Answer* 145 l. 11 s. 6 d. thus found. I first multiply the said 2156 l. 13 s. 4 d. by 6 (the number of Pounds in the given allowance 6 l. 15 s.) after the manner of the last Examples, and subscribe the Product which is 12940 l. under the line as you see; then since 15 s. are equal to $\frac{1}{4}$ l. together with $\frac{1}{4}$ l. I take $\frac{1}{2}$ of 2156 l. 13 s. 4 d. which is 1078 l. 6 s. 8 d. likewise $\frac{1}{4}$ of the said 2156 l. 13 s. 4 d. to wit, 539 l. 3 s. 4 d. and having subscribed these Quotients under the Product first found, and added them all together, I find 14557 l. 10 s. 0 d. for the total Product, with which I proceed as in the former Examples; and so at length the *Answer* is found to be 145 l. 11 s. 6 d. View diligently the Operation.

l. l. l. s. d.
100 .. $6\frac{3}{4}$.. 2156 : 13 : 4
6 $\frac{3}{4}$

12940 : 00 : 0

1078 : 06 : 8

539 : 03 : 4

l. 145 | 57 : 10 : 0
20

s. 11 | 50
12

d. 6 | 00

C H A P. III.

Concerning Exchanges of Coins, Weights, and Measures.

I. **T**HE rate or proportion between *Coins, Weights, &c.* of different kinds being known, either from some good *Author*, or rather by experience; it will not be difficult, to such as understand the *Rule of Three*, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind: But since in some cases the common manner of working may be much contracted, I shall endeavour to shew the most compendious ways to perform this Business.

II. In exchanging of things of different kinds, (whether they be *Coins* or *Weights, &c.*) when two things of different kinds are compared together, the question may be resolved by one single *Rule of Three*, as will be evident by the subsequent Examples; viz.

Quest. 1. How many *Riders* at 21 s. 2½d *sterling* the piece, ought to be received for 251 l. 6 s. 4d ½ of *sterling* Money?

Answer, 237 *Riders*. For the first and third terms in the *Rule of Three*, which arise from this question, being converted into half-pence, the proportion will be this,

$$509 \cdot 1 :: 120633 \cdot 237.$$

Quest. 2. If 100 *Ells* of *Antwerp* make 75 *Yards* of *London*, how many *Yards* of *London*-measure will 27 *Ells* of *Antwerp* make? *Answer* 20¼ *yards*.

$$100 \cdot 75 :: 27 \cdot 20\frac{1}{4}.$$

III. When more than two different *Coins, Weights, Measures, &c.* are compared together, viz. when one kind of *Coin* is compared with a second of another kind; that second with a third; the third with a fourth; the fourth with a fifth, &c. two different cases are ordinarily raised from such comparison, viz.

It may be required to know {
 1. How many pieces of the first *Coin* are equal in value to a given number of pieces of the last *Coin*: Or,
 2. How many pieces of the last *Coin* are equal in value to a given number of pieces of the first kind of *Coin*?

An

An Example of the first Case.

If 35 Ells of *Vienna* make 24 Ells of *Lyons*; 3 Ells of *Lyons* 5 Ells of *Antwerp*; and 100 Ells of *Antwerp* 125 Ells at *Frankfort*; how many Ells of *Vienna* are equal to 50 Ells at *Frankfort*? *Answer*, 35 Ells of *Vienna*.

For the more easy understanding of the resolution of this Question and others of like nature. Let *a* represent an ell at *Vienna*, *b* an ell at *Lyons*, *c* an ell at *Antwerp*, and *d* an ell at *Frankfort*; then may the given terms in the Question be stated in the following order.

$$\begin{array}{l} \text{Suppositions} \left\{ \begin{array}{l} 35 a = 24 b \\ 3 b = 5 c \\ 100 c = 125 d \end{array} \right. \\ \text{The Question} \quad 50 d = ? a \end{array}$$

This order of placing the said given numbers (or terms) being observed, it appears, that if 35 *a* be accounted to stand in the first place; 24 *b* in the second; 3 *b* in the third; 5 *c* in the fourth; 100 *c* in the fifth, &c. then all the terms which stand in odd Places, to wit, in the first, third, fifth, and seventh places, will necessarily fall under the first row or column on the left hand, and all the terms that stand in even Places, to wit, in the second, fourth, and sixth places, will fall under the latter column.

These things premised, all questions which come under Case 1. before-mentioned, may be resolved by this Rule, *viz.*

Rule I.

Multiply all the given Terms that stand in odd places (to wit, in the first column) according to the Rule of continual Multiplication, and reserve the last Product for a Dividend: Again, multiply continually all the terms which stand in even places; so shall the Product be a Divisor, and the Quotient arising from the said Dividend and Divisor will be the answer of the Question.

So in the last mentioned Question, if all the Numbers in the first column, to wit, 35, 3, 100, and 50, be multiplied continually, the Product will be 525000 for a Dividend: Also if all the Numbers in the latter Column, *viz.* 24, 5, and 125 be multiplied continually, the last Product will be 15000 for a Divisor, and the Quotient arising from the said Dividend and Divisor will be 35, which is the Number of Ells of *Vienna* required.

35	24
3	5
100	125
50	

$$525000 : 15000 \quad 525000 (35$$

The reason of the said Rule I. will be manifest by solving the question propounded by three single Rules of Three, thus,

$$I. \quad 24 \text{ } b. \quad 35 \text{ } a :: 3 \text{ } b. \quad \frac{35 \times 3}{24} \text{ } a (= 5 \text{ } c$$

$$II. \quad \frac{5}{1} \text{ } c. \quad \frac{35 \times 3}{24} \text{ } a :: \frac{100}{1} \text{ } c. \quad \frac{35 \times 3 \times 100}{5 \times 24} \text{ } a (= 125 \text{ } d$$

$$III. \quad \frac{125}{1} \text{ } d. \quad \frac{35 \times 3 \times 100}{5 \times 24} \text{ } a :: \frac{50}{1} \text{ } d. \quad \frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24} \text{ } a$$

which fourth Proportion at last found, to wit,

$$\frac{35 \times 3 \times 100 \times 50}{125 \times 5 \times 24}$$

$$125 \times 5 \times 24$$

with the before-mentioned order of placing the Terms propos'd in the question, gives the very Rule I. before express'd in words.

An Example of the latter of the two Cases before-mentioned.

If 10 lb. of *Avoirdupois*-weight at *London* be equal to 9 lb. of *Amsterdam*; 45 lb. at *Amsterdam*, to 49 lb. at *Bruges*; and 98 lb. at *Bruges*, equal to 116 lb. at *Dantzick*; how many lb. of *Dantzick* are equal to 112 lb. of *Avoirdupois*-weight at *London*? *Answer* 129.92 lb. of *Dantzick*.

That the Operation may be the more clear, let *a* represent one pound of *Avoirdupois*-weight; *b* one lb. of *Amsterdam*; *c* one lb. of *Bruges*, and *d* one lb. of *Dantzick*; then let the question be stated after the order in the first Case, viz.

$$\text{Suppositions} \begin{cases} 10 \text{ } a = 9 \text{ } b \\ 45 \text{ } b = 49 \text{ } c \\ 98 \text{ } c = 116 \text{ } d \end{cases}$$

$$\text{The Question} \quad 112 \text{ } a = ? \text{ } d$$

These things premised, all questions which fall under Case 2. before-mentioned, may be resolved by this Rule, viz.

Rule

Rule II.

Multiply all the given Terms which stand in even Places (to wit in the latter Column) and the last odd Term in the first Column according to the Rule of continual Multiplication, and reserve the last Product for a Dividend: Again, multiply continually the rest of the Terms that stand in odd places (to wit in the first Column) for a Divisor; so shall the Quotient arising be the answer of the Question.

Or in this latter Case, if you place the last of the given Terms in the same Column with the even Terms, the Rule for solving questions, that fall under the latter Case, will be this which follows, *viz.*

Multiply continually all the Numbers in the latter Column for a Dividend; also multiply continually all the Numbers in the first Column for a Divisor, so shall the Quotient arising be the Answer of the question. Thus the answer of the last mentioned question will be found 129.92, to wit, 129 $\frac{23}{100}$ lb. of Dantzick, as is evident by the subsequent Operation.

$$\begin{array}{r|l} 10 & 9 \\ 45 & 49 \\ 98 & 116 \\ & 112 \end{array}$$

$$44100 \mid 5729472 \text{ (129.92)}$$

The Reason of the said Rule II. will be manifest by solving the Question proposed by three single Rules of Three, thus,

$$I. \ 9 \text{ b. } 10 \text{ a} :: 45 \text{ b. } \frac{45 \times 10}{9} \text{ (} a = 49 \text{ c.}$$

$$II. \ 49 \text{ --- c. } \frac{45 \times 10}{9} \text{ a} :: \frac{98}{1} \text{ c. } \frac{45 \times 10 \times 98}{49 \times 9} \text{ a (} = 116 \text{ d.}$$

$$III. \ \frac{45 \times 10 \times 98}{49 \times 9} \text{ a. } \frac{116}{1} \text{ d} :: \frac{112}{1} \text{ a } \frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98} \text{ d.}$$

Which fourth Proportional last found, to wit,

$$\frac{49 \times 9 \times 116 \times 112}{45 \times 10 \times 98}$$

— being well viewed and compared with the fore-mention'd Order of placing the Terms given in the question, discovers the very Rule II. before express'd in words.

Note. When the same Numbers happen to be Multipliers

in the Dividend, and also in the Divisor, such Multipliers may be cancelled in both, and thereby much labour will often be spared.

Such as have much practice in calculating *Exchanges*, and exactly know the rate or proportion between two different Weights, or Measures, or Coins, which they would compare together, may by the *Rule of Three* frame Tables of proportions for the more speedy reducing of a given quantity of one kind of Weight, Measure, &c. into a quantity of the same value in another kind of weight, &c. In the expressing of which proportions it will be very convenient, that the first Number or Antecedent of each Proportion be made 1, or unity, and the second Term or Consequent a Decimal, or else a mixt number, whose Fractional part is a Decimal; for then the Coin, Weight, &c. of the one Place (whose term is 1) may be reduced to that of the other place, by help of those Tables and of Multiplication of Decimals without sensible error: For Example, it has been observed by some Ingenious Merchants, that 100 lb. of *Avoirdupois*-weight at *London*, are equal to 89 lb. in *Paris* by the King's Beam, and consequently 1 lb. *Avoirdupois* is equal to $\frac{89}{100}$ lb. or .89 lb. at *Paris*; (for if 100 give 89, then 1 will give .89; therefore any Number of Pounds *Avoirdupois* being multiplied by .89 (with respect to Multiplication of Decimals, explained in the 26th Chapter of the preceeding Book) will produce pounds of *Paris*: Again, If 89 lb. of *Paris* be equal to 100 lb. *Avoirdupois*, then 1 lb. of *Paris* will be near equal to 1.1235 lb. of *Avoirdupois*; therefore any Number of Pounds of *Paris* being multiplied by 1.1235, will produce pounds *Avoirdupois* very near.

Upon this ground I have collected the Proportions in the following Tables, which I would not have any to confide in further than they know them to be agreeable to truth: For I have only derived them from those delivered by Mr. *Lewis Roberts*, Merchant, in his *Map of Commerce*, printed at *London*, Anno 1638, and do herein only aim at the *Instruction* of ingenious *Merchants* and *Factors* in the briefest ways of calculating their *Exchanges*, the Rate or Proportion being truly known; in which Practice, *Decimal Arithmetick* (that has no Enemy but the Ignorant) will be very serviceable.

*A Table for the Reduction of Avoirdupois-Weight
at London, to the Weights of divers Foreign
Cities and remarkable Places.*

		lb.
One Pound of Avoirdupois-weight at London, makes at	Antwerp	.9615
	Amsterdam	.9
	Abbeville	.91
	Ancona	I .282
	Avignon	I .12
	Bourdeaux	I .91
	Burgundy	.91
	Bolonia	I .25
	Bruges	.98
	Calabria	I .3698
	Calais	I .07
	Constanti- nople }	.8474
	Deerp	Loder ;
	Dantzick	.91
	Ferrara	I .16
	Florence	I .3333
	Flanders in general }	I .282
	Geneva	I .06
		.9345

Genoa

One Pound of Avoirdupois-weight at London makes at		lb.
	Genoa }	1 .4084 <i>suttle.</i>
	Hamburgh }	1 .4285 <i>gross.</i>
	Holland }	.92
	Lisbon }	.95
		.881
	Lyons }	1 .07 <i>common-weight.</i>
		.98 <i>silk-weight.</i>
		1 .9 <i>customers-weight.</i>
	Leghorn }	1 .3333
	Milan }	1 .4285
	Mirandola }	1 .3333
	Norimbergh }	.88
	Naples }	1 .4084
	Paris }	.89
	Prague }	.83
	Placentia }	1 .3888
	Rochel }	1 .12
	Rome }	1 .27
	Rouen }	.875 <i>by vicont.</i>
		.9017 <i>com. weight</i>
	Sevil }	1 .08
	Thoulouse }	1 .12
	Turin }	1 .2195
	Venice }	1 .5625 <i>suttle.</i>
		.9433 <i>gross.</i>
	Vienna }	.813

The use of the preceeding Table will be manifest by the subsequent *Example, viz.*

How much Weight at *Dantzick* do 320 lb. *Avoirdupois* make? *Answer* 371.2 lb. Seek in the foregoing Table for *Dantzick*, and right against it you will find 1.16, which shews that 1 lb. *Avoirdupois* is equal to 1.16 lb. at *Dantzick*; therefore multiply 320 by 1.16, so will the Product be 371.2 lb. of *Dantzick*, as by the Operation is manifest.

<i>Avoir.</i>		<i>Dantz.</i>		<i>Avoir.</i>		<i>Dantz.</i>
1	:	1.16	::	320	:	371.2
				1.16		

1920

320

320

371|20

A Table for the Reduction of the Weights of divers foreign Cities and remarkable Places to Avoirdupois-Weight at London,

		lb.
One Pound Weight in	<i>Antwerp</i>	1 .04
	<i>Amsterdam</i>	1 .1111
	<i>Abbeville</i>	1 .0989
	<i>Ancona</i>	.78
	<i>Avignon</i>	.8928
	<i>Bordeaux</i>	1 .0989
	<i>Burgundy</i>	1 .0989
	<i>Bolonia</i>	.8
	<i>Bruges</i>	1 .0204
	<i>Calabria</i>	.73
	<i>Calais</i>	.9345
	<i>Deep</i>	1 .0989
	<i>Dantzick</i>	.862
	<i>Ferrara</i>	.75
	<i>Florence</i>	.78
	<i>Flanders in</i> >	
	general. <	.9433
	<i>Geneva</i>	1 .07
	<i>Genoa</i> } futtle	.71
	} gross	.7

makes at London of Avoirdupois-weight

One

One Pound Weight in		lb.	makes at London of Avoirdupois-weight
	Hamburgh	1 .0865	
	Holland	1 .0526	
	Lisbon	1 .135	
	Lyons		
		common weight	.9345
		silk weight	1 .0204
		custom weight	1 .1111
	Leghorn	.75	
	Milan	.7	
	Mirandola	.75	
	Norimbergh	1 .1363	
	Naples	.71	
	Paris	1 .1235	
	Prague	1 .2048	
	Placentia	.72	
	Rochel	.8928	
	Rome	.7874	
	Rouen	1 .1428	
		by Vicont.	
		com. weight.	1 .1089
	Sevil	.9259	
	Thoulouse	.8928	
	Turin	.82	
	Venice	.64	
		futtle	
		gross	1 .06
	Vienna	1 .23	

The

The Use of the last mentioned Table, will be manifest by this Example, viz.

In 224 lb. weight at *Hamburgh*, how many Pounds *Avoirdupois* ?

Answer, 243,376. lb.

Seek in the Table for *Hamburgh*, and right against it you will find 1,0865, which shews that 1 lb. of *Hamburgh* makes 1.0865 lb. *Avoirdupois* ; therefore if 1,0865 lb. be multiplied by 224, the Product will be Pounds *Avoirdupois*.

$$\begin{array}{r}
 1 \dots 1,0865 \dots 224 \\
 \hline
 224 \\
 \hline
 43460 \\
 21730 \\
 21730 \\
 \hline
 243 \mid 3760
 \end{array}$$

A Table for the Reduction of English Ells to the Measures of divers foreign Cities, and remarkable Places.

One Ell at London makes at	{ Amsterdam		1 .6949	Ells
	{ Antwerp		1 .6666	
	{ Bruges		1 .64	
	{ Arras		1 .65	
	{ Norimberg		1 .74	
	{ Colen		2 .08	
	{ Lisle		1 .66	
	{ Maestricht		1 .57	
	{ Frankfort		2 .0866	
	{ Dantzick		1 .3833	
	{ Vienna		1 .45	Aulns
	{ Paris		.95	
	{ Rouen		1 .03	
	{ Lyons		1 .0166	
	{ Calais		1 .57	
	{ Venice	linnen	1 .8	Braces
		filk	1 .96	
	{ Lucca		2	
	{ Florence		2 .04	
	{ Milan		2 .3	
	{ Leghorn		2	
	{ Madera		2	
	{ Isles		1 .0328	

One

One Ell at London makes at	Sevil	1 .35	}	Vares
	Lisbon	1		
	Castile	1 .3875		
	Andalusia	1 .3625		
	Granada	1 .3625	}	Palms
	Genoa	4 .1083		
	Saragossa	.55		
	Rome	.56		
	Barcelona	.7125	}	Canes
	Valentia	1 .2125		

The Use of the foresaid Table will be manifest by the subsequent Example, viz.

In 325 Ells of *London*, how many Ells at *Antwerp*?

Answ. 541.645 Ells: Seek in the Table for *Antwerp*, and right against it you will find 1.6666, which being multiplied by 325, produces 541.645 Ells of *Antwerp*, as by the Operation is manifest.

$$\begin{array}{r} 1 \dots 1.6666 \dots 325 \\ 325 \end{array}$$

$$\begin{array}{r} 83330 \\ 33332 \\ 49998 \end{array}$$

$$541|6450$$

*A Table for the Reduction of the Measures of
divers foreign Cities, and remarkable Places,
to English Ells.*

One Ell at	Amsterdam		.59	
	Antwerp		.6	
	Bruges		.6097	
	Arras		.606	
	Norimberg		.5747	
	Colen		.4807	
	Lisle		.6024	
	Maestricht		.6369	
	Frankfort		.4792	
	Dantzick		.7228	
	Vienna		.6896	
		makes at London		
One Auln at	Paris		.0526	
	Rouen		.9708	
	Lyons		.9836	
One Brace at	Calais		.6369	
	Venice	{ linnen	.5555	
		{ silk	.5102	
	Lucca		.5	
	Florence		.4901	
	Milan		.4347	
	Leghorn		.5	
	Madera Isles		.9681	

One

One Vane at	{	Sevil			.7407	
		Lisbon				
		Castile			.7207	
		Andalusia			.7339	
		Granada			.7339	
One Palm at Genoa					.2079	
One Cane at	{	Saragossa			.8181	
		Rome			.7857	
		Barcelona			.4035	
		Valentia			.8247	

makes at London

Ells

The Use of the said Table will be manifest by the subsequent Example, viz.

In 730 Aulns at Lyons, how many Ells at London?

Ans^r. 718.028. Seek in the Table for Lyons, and right against it you will find .9836, which being multiplied by 730, produces 718.028 Ells of London, as by the Operation is manifest.

$$\begin{array}{r}
 1 \dots, 9836 \dots 730 \\
 \underline{730} \\
 295080 \\
 68852 \\
 \hline
 718 \mid 0280
 \end{array}$$

Note. That one and the same Kind of Weight or Measure seldom or never alters from its peculiar Quantity, in the Kingdom or Commonwealth, where such Weight or Measure was first established; but one and the same Kind of Money often rises and falls in its Value in foreign Parts: For which cause I have spared the Pains of calculating *Decimal Tables* for Coins; yet to give some Light to such as read modern Relations, and want experimental Knowledge in this Matter, I shall here insert a Table in the same State as I find it in the aforesaid Map of Commerce, and refer the Reader, for further satisfaction, to the Tables in *Rider's Dictionary*, concerning Coins, Weights, and Measures, both antient and modern.

Of

Of Exchanges of London, with divers foreign Cities.

	Pence	Sterl.		
Placentia	64	for	1	Crown
Lyons	64	for	1	Crown
Rome	66	for	1	Ducat
Genoa	65	for	1	Crown
Milan	$64\frac{3}{4}$	for	1	Crown
Venice	50	for	1	Ducat
Florence	$53\frac{1}{2}$	for	1	Ducatoon
Naples	50	for	1	Ducat
Lecchia in Calabria	}	50	for	1 Ducat
Barri				
Palermo	$57\frac{1}{2}$	for	1	Ducat
Messina	$56\frac{1}{2}$	for	1	Ducat
Antwerp & Colon	} 1 l. sterl.	for	$34\frac{1}{2}$	{ shill. flem.
Valentia				
Saragossa	$57\frac{1}{2}$	for	1	Ducat
Barcelona	59	for	1	Ducat
Lisbon	64	for	1	Ducat
Bononia	$53\frac{1}{2}$	for	1	Ducat
Bergamo	$53\frac{1}{3}$	for	1	Ducatoon
Bergamo	52	for	1	Ducatoon
Frankfort	$59\frac{1}{2}$	for	1	Florin
Genoa	83	for	1	Crown

London exchanges in the Denomination of Pence sterling with all other Countries, Antwerp and those neighbouring Provinces of Flanders and Holland excepted, with which it exchanges by the entire Pound of 20 Shillings English (or Sterling.)

C H A P. IV.

Practical Questions about various Things: viz. Tare, Tret, Loss, Gain, Barter, Factorship, and Measuring of Tapestry.

Of Abatements and Allowances in Traffick, viz. 1. Of Tare.

IN the Trade of Merchandize there are in use various Allowances, and Abatements, known by the Names of *Tare*, *Tret*, &c. concerning which I shall give a few Examples, whereby the practical *Arithmetician* will easily see, that there is more Difficulty in the Name than in the Thing; for the Rate, or Proportion agreed upon, in any Allowance or Abatement (be it called by what Name soever) being once known, the Arithmetical Work will quickly be dispatched by the *Rule of Three*, or else by that and some of the former Rules mixtly used, as partly appears by the following Questions.

Gross Weight is composed of the Neat Weight of the Commodity, and also of the Tare, to wit, the Chest, Bag, But, &c. which contains the Commodity.

Quest. 1. A Factor buys 4 Chests of Sugar marked A. B. C. D. The gross Weight of every Chest in Avoirdupois greater Weight is as follows.

	C.	q.	lb.
A.	11	...	19
B.	10	...	20
C.	11	...	13
D.	10	...	17

The total gross Weight

44 ... 1 ... 17

Now supposing the *Tare*, or *Weight* of every Chest when it is empty, to be 37 lb. the *Question* is what neat *Weight* of *Sugar* will remain, when the total *Tare* is subtracted? *Answ.*

43 C. 0 q. 4 lb.

	C.	q.	lb.	
from	44	1	13	<i>The total gross Weight.</i>
Subtr.	1	1	08	<i>The total Tare.</i>

Rem. 43 ... 0 ... 05 The neat Weight of Sugar.

Quest. 2. If from 990 C. 3 qu. 21 lb. gross Weight, Tare is to be subtracted after the Rate of 14 lb. per C. (or 112 lb.) of gross Weight, how many C. neat will remain? Answ. 867 C. 0 q. 7 lb.

1. The *gross Weight* being converted into Pounds by the sixth

sixth Rule of the 7th Chapter of the preceeding Book, will give 110985 lb.

II. Then by the Rule of Three.

$$112 : 14 :: 110985 : 13873 \frac{1}{2}$$

$$\text{or } 8 : 1 :: 110985 : 13873 \frac{1}{2}$$

III. From 110985 the gross Weight :

Subtr. 13873 $\frac{1}{2}$ the total Tare.

	C.	qu.	lb.
Rest neat	97111 $\frac{7}{8}$	= 867	. . 0 . . 7 $\frac{7}{8}$

Note. When the Number of lb. to be abated per C. for Tare is an aliquot Part of 112, as in the last mentioned Example, where 14 = $\frac{1}{8}$ of 112, the Operation may be thus :

C.	C.	C.	q.	lb.	C.	qu.	lb.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
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1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.
1	.	.	.	.	1	.	.

the Question is, what the neat Weight is worth in Money after the Rate of 8 *lb.* 8 *s.* for every C. (or 112 *lb.*? *Answ.* 382½.

I. The gross Weight in *lb.* is 6188 *lb.*

II. 112 : 16 :: 6188 : 884.

or 7 : 1 :: 6188 : 884.

III. 6188 — 884 = 804

IV. 104 : 100 :: 5304 : 5100

V. 112 : 8½ :: 5100 : 382½

Quest. 6. A Merchant has bought Linnen Cloth at 11 *s.* *per* Ell, which proving worse than he expected, he is willing to sell it at such a Price, that he may lose precisely after the Rate of 1 ½ *l.* for every 20 *l.* that he laid out; the Question is to know at what Price he ought to sell the Ell, that the Proportion in the said Loss may be observed? *Answ.* 10 *s.* 1 *d.* *per* Ell.

Of Loss and Gain.

I. 20 — 1½ = 18½

II. 20 : 18½ :: 11 : 10½ *Shillings.*

Otherwise.

I. 20 : 1½ :: 11 : 1½

II. 11 — 1½ = 9½

Quest. 7. If 100 *lb.* Weight of any Commodity cost 30 *s.* at what Price must 1 *lb.* Weight of that Commodity be sold to gain after the Rate of 10 *lb.* for every 100 laid out? *Answ.* 3½ *d.* *per* *lb.* Weight.

I. 100 : 110 :: 30 :: 33

II. 100 : 33 :: 1 : 1½ *s.* (or 3½ *d.*

Quest. 8. A Merchant sells a Parcel of Jewels which cost him 250 *l.* ready Money, for 559 *l.* payable at the end of 6 Months; the Question is (his Security being supposed to be good) what his Gain was worth in ready Money, upon rebate of Interest at the Rate of 6 *l.* for 100 *l.* for a Year? *Answ.* 300 *l.*

559 — 250 = 309

103 : 100 :: 309 . 300

Quest. 9. How much Sugar at 8 *d.* *per* *lb.* Weight may be bought for 20 C. of Tobacco, at 3 *l.* *per* C? *Of Barter.*

Answ. 1800 *lb.* Weight of Sugar.

1 : 3 :: 20 : 60

1 : 3 :: 60 : 1800

Quest. 10. A has 100 Pieces of Silks, which are worth but 3 *l.* *per* Piece in ready Money, yet he barterers them with B at 4 *l.* *per* Piece, and at that Rate takes their Value of B in Wools, at 7 *l.* 10 *s.* *per* C. which are worth but 6 *l.* *per* C. in ready Money; the Question is to know what Quantity of

I i

Wools

Wools pays for the Silks, and which of the two, A or B, is the Gainer, and how much? *Ans.* $53\frac{1}{2}$ C. of Wools pays for the Silks, and A gains 20 l. by the Barter.

$$\text{I. } 7\frac{1}{2} : 1 :: 400 : 53\frac{1}{2}$$

$$\text{II. } \left\{ \begin{array}{l} 1 : 6 :: 53\frac{1}{2} : 320 \\ \text{or } 7\frac{1}{2} : 6 :: 400 : 320 \end{array} \right.$$

So it is evident, that the true Worth of the Wool which B delivered, was 320 l. for which he received only of A the worth of 300 l. in Silks, and therefore B loses 20 l. by the Barter.

Quest. 11. A Merchant delivered to his Factor 600 l. upon condition that if the Factor add to it 250 l. of his own Money, and bestow his Pains in managing the whole Stock, he should then have $\frac{2}{3}$ Parts of the total Gain. The Question is to know what Stock the Factor's Service was estimated at? *Ans.* 150 l.

I. The Factor's Part of the Gain being $\frac{2}{3}$, the Merchant must necessarily have the Remainder, which is $\frac{1}{3}$

$$\text{II. } \frac{1}{3} : \frac{2}{3} :: 600 : 400$$

$$\text{III. } 400 - 250 = 150$$

Quest. 12. A Merchant delivers to his Factor 320 l. and permits him to add to it 64 l. of his own Money, to be employed in Traffick; and by Agreement between them the Factor's Service is estimated equivalent to a certain Stock; which is such, that if the total Gain be divided proportionably according to those three Stocks, the Factor is to receive $\frac{1}{5}$ of the total Gain, in consideration of the said imaginary Stock (being the Value of his Service;) the Question is to know the full Part of the Gain belonging to each, and what Stock the Factor's Service was valued at? *Ans.* The Merchant $\frac{2}{5}$ of the Gain, and the Factor $\frac{1}{5}$, whose Service was valued at 96 l. Stock.

$$\text{I. } 320 + 64 = 384$$

$$\text{II. } \frac{4}{5} : \frac{1}{5} :: 384 : 96$$

$$\text{III. } \begin{array}{r} 320 \\ 64 \\ \hline 96 \end{array}$$

$$\begin{array}{l} \text{I. } 320 : \frac{2}{5} \\ \text{II. } 64 : \frac{1}{5} \\ \text{III. } 96 : \frac{1}{5} \end{array}$$

Quest. 13. If a Piece of Arras-Hangings, in the Form of a long Square, has for its length $6\frac{1}{4}$ Yards *English*, and Breadth 4 Yards; how many square Ells, or Sticks *Flemish* are contained in that Piece, when the

*Of Measuring
of Tapestry.*

the Length of a *Flemish* Ell is equal to $\frac{1}{4}$ Yard *English*? *Answ.* $44\frac{1}{2}$ square Ells or Sticks *Flemish*.

Forasmuch as by supposition, a *Flemish* Ell in length, has such Proportion to an *English* Yard in length, as 3 to 4, and consequently the Square of the one to the Square of the other, as 9 to 16: Therefore in a direct Proportion, as 9 is to 16; so in any given Number of square Yards *English*, to a Number of square Ells *Flemish*, which will take up equal Space with the said square Ells *English*. Also in a direct Proportion, as 16 is to 9, so is any given Number of square Ells *Flemish*, to a Number of square Yards *English*, which will take up an equal Space with the said *Flemish* Ells: Therefore to resolve the aforesaid Question, first find the Number of square Yards *English*, contained in the said Piece of Arras, by multiplying the Length and Breadth in Yards mutually one by the other, then proceed according to the aforesaid Proportion; so the Work will stand thus,

I. $6\frac{1}{4} \times 4 = 25$ square Yards *English*.

II. $9 : 16 :: 25 : 44\frac{1}{2}$ square Ells *Flemish*.

Otherwise.

$6\frac{1}{4}$ Yards *English* in length give by the Rule } $8\frac{1}{4}$ length.
of Three in *Flemish* Ells

Also 4 Yards *English* give in *Flemish* Ells --- $5\frac{1}{4}$ breadth.

Therefore the Product of the said $8\frac{1}{4}$ multiplied by $5\frac{1}{4}$, gives for the superficial Content } $44\frac{1}{2}$
as before

Quest. 14. If a Piece of Tapestry, in the Form of a long Square, be in length $15\frac{1}{4}$ Ells *Flemish*, and in breadth $4\frac{1}{4}$ Ells *Flemish*, how many square Yards *English* are contained in that Piece, when 4 Ells *Flemish* in length are equal to 3 Yards *English*? *Answ.* $37\frac{1}{4}$ square Yards *English*.

I. $15\frac{1}{4} \times 4\frac{1}{4} = 66\frac{1}{4}$.

II. $16 : 9 :: 66\frac{1}{4} : 37\frac{1}{4}$.

CHAP. V.

Concerning the Interest of Money, and the Construction of Tables to that purpose.

I. IN resolving Questions concerning Interest of Money, four Things are to be well observed; to wit, First, the Principal or Money lent for Gain or Interest; Secondly, the Time

for which the said Principal is lent; Thirdly, the Rate or Proportion which the Principal bears to the Sum of the Principal and Interest; And fourthly, the Interest itself: So if 100 *l.* be lent, upon Condition that 106 *l.* shall be repaid at the End of a Year, the said 100 *l.* is called Principal; the Time for which the said Principal is lent, is one Year; the Proportion which the Principal bears to the Sum of the Principal and Interest is such, as 100 has to 106 *l.*; lastly, the Interest itself is 6 *l.*

II. Interest is either Simple, or Compound.

III. Simple Interest is that which arises or is computed from the Principal only: So if 100 *l.* be lent for two Years, the simple Interest thereof, after the Rate of 6 *l.* for 100 *l.* for one Year, will be 12 *l.* viz. 6 *l.* due at the first Year's End, and 6 *l.* due at the second Year's End.

IV. Compound Interest is that which arises from the Principal, and also from the Interest thereof, and therefore it is called Interest upon Interest: So if 100 *l.* be lent and forborn three Years, and the Compound Interest thereof is to be computed, after the Rate of 6 *l.* for 100 *l.* for one Year, there will arise, besides the simple Interest of the Principal for three Years, the Interest of 6 *l.* (due at the first Year's End) for two Years, and the Interest of 6 *l.* (due at the second Year's End) for one Year following.

V. Rebate or Discount of Money is, when a Sum of Money due at any time to come, is satisfied by the Payment of so much present Money, which if it was put forth at a certain Rate of Interest for the said Time, would become equal to the Sum first due: So if 100 *l.* be due at the end of two Years, and is to be satisfied by the Payment of present Money upon Rebate, after the Rate of 6 *l.* *per centum, per annum, simple Interest*, there ought to be so much ready Money paid, which in two Years after the said Rate of Interest would be augmented to 100 *l.* In like manner, if the Rebate or Discount were to be made after any Rate of compound Interest, so much ready Money ought to be paid, as at such Rate of compound Interest, for the Time agreed on, would become equal to the Sum first due. *Examples* of the manner of Computation by Rebate may be seen in the tenth and fourteenth Rules of this Chapter.

VI. In the taking of Interest, or Use-money, for the Loan or Forbearance of Money lent, respect must be had to the Rate limited by Act of Parliament, which lately restrained all Persons from taking more than 6*l.* for the Interest or Use of 100*l.* lent for a Year; but what part of 6*l.* may be taken for the Interest of 100*l.* lent for half a Year, a quarter of a Year, a Month, or any other part of a Year, is not express'd in the Act:

The Foundation upon which the Rules for computing simple Interest are grounded.

In this case, therefore, we must observe Custom and daily Practice; so we shall find that 3*l.* is usually taken for half a Year's Interest of 100*l.* and 30*s.* for a quarter of a Year, &c. by which practice, this following Analogy (which is the Ground or Reason of the common Rules for computing simple Interest) seems to be assumed for a safe Exposition of the Statute, *viz.* That such Proportion as the whole Year (supposed to consist of 365 Days) has to any propounded Space of Time more or less than a Year, such Proportion any Interest (not exceeding the Rate limited by the Act) for any Principal lent for a Year, ought to have to the Interest of the same Principal for the Time agreed upon: This Analogy being granted, the Manner of computing simple Interest, for any Principal lent and forborn any time proposed, will be such as is express'd in the two next Sections.

VII. The Interest or Gain of 100*l.* principal Money forborn for a Year being known, the Interest of any other principal Money for the same Time may be found out by one single Rule of Three; for as 100*l.* Principal is in proportion to the Interest thereof, so is any other Principal to its Interest: So if it be demanded, what 270*l.* will gain in a Year, at the Rate of 6*l.* for 100*l.* for one Year, the Answer will be found to be 16*l.* 4*s.* For,

$$\begin{array}{ccccccc} l. & l. & l. & l. & l. & s. & d. \\ 100 & : 6 & :: 270 & : 16,2 & \text{(or } 16.4.0 \end{array}$$

A second Example. What is the Interest of 175*l.* 18*s.* 11*d.* for a year, at the Rate of 6*l.* for 100*l.* for a Year?
Ans. 10*l.* 11*s.* 1*½**d.* as, by the following Operation (which is performed after the practical Manner, delivered in the nineteenth Rule of the second Chapter of this Appendix) is evident.

<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
100	:	6	:	:	175	:	18 . 11
					(10 . 11 . 1 $\frac{6}{10}$)		
					multiply by 6		

<i>l.</i>		10		55	:	13	:	6
				20				

<i>s.</i>		11		13
				12

<i>d.</i>	...	1		62
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VIII. If the Interest of 100 *l.* Principal for one whole Year, or 365 Days be known, the simple Interest of any other Principal, for any number of Days more or less than 365, may be found out by the following Rule, *viz.*

Multiply these three Numbers according to the Rule of continual Multiplication, to wit, the given Interest of 100 *l.* for a Year, the Principal, whose Interest is required, and the Number of Days prescribed, reserving the last Product for a Dividend: Also multiply 365 by 100, and reserve this Product for a Divisor: Lastly, finish Division, so shall the Quotient be the Interest or Gain sought.

Note here, that the two Principals, to wit 100 *l.* and the other propounded, are supposed to be of one and the same Denomination: Also the Interest required will be of the same Denomination with the given Interest of 100 *l.*

For an Example of this Rule, let it be required to find out the Interest of 400 *l.* for a Week, or 7 Days, at the Rate of 6 *l.* for 100 *l.* for a Year, or 365 Days: First multiplying these three Numbers 6, 400, and 7 continually (*viz.* multiplying 6 by 400, and the Product thence arising by 7) the last Product will be 16800 for a Dividend; also multiplying 365 by 100, the Product is 36500 for a Divisor: Lastly, dividing 16800 by 36500 (after Cyphers at pleasure are added to 16800) the Quotient (according to the fourth Rule of the 27th Chapter of the preceeding Book) will be discovered to be this Decimal .4602, which is equal to 9 *s.* 2 *d.* 1 *farth.* (as will appear by the brief Way of valuing a Decimal Fraction in the fourth Rule of the 26th Chapter.)

The Reason of the above-mentioned Rule for the computing of Interest for Days, will be manifest by this following Way of solving the same Question by two single Rules of Three, *viz.*

I.

$$\text{I. } 100 : 6 :: 400 : \frac{6 \times 400}{100}$$

$$\text{II. } \frac{365}{1} : \frac{6 \times 400}{100} :: \frac{7}{1} : \frac{6 \times 400 \times 7}{365 \times 100}$$

Which fourth Proportional in the latter *Rule of Three*, to wit,

$$\frac{6 \times 400 \times 7}{365 \times 100}$$

being well viewed, the Truth of the Rule before delivered will be manifest.

Hence a vulgar Error in computing Interest is discovered ; for some argue thus, 6 *l.* is the Interest of 100 *l.* for a Year ; therefore 10 *s.* (or $\frac{1}{12}$ of 6 *l.*) is the Interest for a Month, and consequently 2 *s.* 6 *d.* for a Week or seven Days, and so the Interest of 400 *l.* for 7 Days, computed after that manner, would be 10 *s.* which exceeds the Answer found by the preceding Rule by 9 $\frac{1}{2}$ *d.* very near ; which Fallacy has its rise from the taking, (or rather mistaking) of 28 Days for $\frac{1}{12}$ Part of the Number of Days in a Year, when indeed the just $\frac{1}{12}$ Part of 365 Days consists of 30 $\frac{5}{12}$ Days.

And farther, by the Help of this decimal Fraction of a Pound, to wit, .000164383, which is very near the Interest of one Pound for a Day, at the Rate of 6 *per cent. per annum*, (as will appear by the preceding Rule) the Interest of any Principal (supposed to be Pounds or decimal Parts of a Pound) for any Number of Days agreed on, at the said Rate of Interest, may be found out by Multiplication only, *viz.* First multiply the said Decimal, .000164383, by the Principal, whose Interest is required ; then multiply that Product by the Number of days proposed, so will this last Product be the Interest required ; (but in these Multiplications respect must be had to the cutting off of Places in the Products, according to the second and third Rules of the 26th Chapter of the preceding Book ;) for Example, if it be required to find the Interest of 1000 *l.* for 131 Days, at the Rate of 6 *per Cent. per Annum*, the *Ans.* will be found 21,534 +, or 21 *l.* 10 *s.* 8 *d.* + ; for, according to the Rule last given,

$$.000164383 \times 1000 \times 131 = 21,534 +$$

But at another Rate of Interest, a *peculiar* Decimal instead of the said .000164383 (which serves only for 6 *per cent. per annum*) must be found out by the first Rule aforegoing, before.

*Another Rule
for computing
simple Interest
for Days.*

fore the latter Rule can take place, the Reason of which latter Rule does also evidently arise from two single Rules of Three.

IX. When an Annuity payable yearly is in Arrear for any Number of Years, and it is required to know what the same will amount to, simple Interest being computed for every particular yearly Payment, from the time it became due, until the End of the Term of years, the Work will be as in this following Example, *viz.* If an Annuity, or yearly Rent of 134 *l.* 10*s.* 6*d.* be all forborn till the End of 4 Years, what will it then amount to, simple Interest being allowed at the Rate of 6 *per cent. per annum*, for each Year's Rent, from the Time on which it was due, until the End of the said Term of 4 Years? *Ans.* 586 *l.* 10*s.* 6*½d.*

It is evident by the Question, that at the Rate of interest proposed, there must be computed the Interest of 134 *l.* 10*s.* 6*d.* (due at the third Year's End) for one Year (to wit, the fourth Year;) also the Interest of the like Sum due at the 2d Year's End, for 2 Years (to wit, the 3d and 4th Years;) likewise the Interest of the same Sum due at the first Year's End for three Years (to wit, the second, third, and fourth Years;) all which Interest being added to the Sum of the four Year's Rent, the total Sum shews what the said Annuity will amount to at the End of the said Term of 4 years.

Explication.

		<i>l.</i>	<i>s.</i>	<i>d.</i>
The Interest of 134 <i>l.</i> 10 <i>s.</i> 6 <i>d.</i> at 6 <i>per cent. per annum</i> , for	1 is . . .	8	1	5,16
	2 is . . .	16	2	10,32
	3 is . . .	14	4	3,48
The Sum of the 4 Years Rent (to wit, 4 times 134 <i>l.</i> 10 <i>s.</i> 6 <i>d.</i>)	}	is . . . 138 . 2 . 0		
All which added together give the <i>Answer</i> of the Question, to wit,	}	. . . : 586 . 10 . 6,96		

X. When it is required to find out how much ready Money will satisfy a Debt due at the end of any space of time to come, by rebating or discounting at a given Rate of simple Interest, it may be effected by this Rule, *viz.* First, find out the Interest of 100*l.* at the settled Rate of Interest, for the time which

Of Rebate or Discount of Money at simple Interest.

which the ready Money is to be paid before-hand ; then adding the Interest so found to 100 *l.* make always the Sum of that Addition the first Term in a Rule of Three ; 100 *l.* the second Term ; and the Debt proposed to be satisfied the third Term ; lastly, the fourth Proportional found out by the said *Rule of Three*, will be the ready Money which ought to be paid in satisfaction of the Debt propounded.

Example 1. If a Debt of 100 *l.* be payable at the End of a Year to come, how much ready Money will discharge that Debt, by rebating or discounting at the Rate of 6 *per cent. per annum*? *Answ.* 94 *l.* 6 *s.* 9 *d.* 2 *far.* very near ; for by the *Rule of Three*,

$$106 : 100 :: 100 : 94,3396 +$$

That is to say, if 106 *l.* (which is composed of 100 *l.* Principal, and 6 *l.* Interest) proceeds from 100 *l.* Principal forborn for a Year, what Principal forborn for a Year does 100 *l.* (composed of Principal and Interest) proceed from? *Answ.* 94,3396 *l.* +, (or 94 *l.* 6 *s.* 9½ *d.* very near) principal Money: Therefore 94 *l.* 6 *s.* 9½ *d.* in ready Money, is of equal Value with 100 *l.* due at the End of a Year to come ; for if the said 94 *l.* 6 *s.* 9½ *d.* be put forth at Interest for a Year, at the Rate of 6 *per cent. per ann.* it will gain 5 *l.* 13 *s.* 2½ *d.* very near, which together with the said 94 *l.* 6 *s.* 9½ *d.* makes the 100 *l.* the Debt first proposed to be discharged by Rebate.

Example 2. If a 150 *l.* 10 *s.* be payable at the End of 37 Days to come, how much present Money will discharge the said Debt, by rebating after the Rate of 6 *per cent. per ann*? *Answ.* 148 *l.* 14 *s.* 3½ *d.* +, as by the following Operation is manifest.

days *l.* days *l.*

$$\text{I. } 365 : 6 :: 73 : 1,2$$

l. *l.* *l.* *l.*

$$\text{II. } 101,2 : 100 :: 150,5 : 148,7154 +$$

That is to say, First, I seek by a single *Rule of Three* the Interest of 100 *l.* for 73 Days, at the Rate of Interest proposed, saying, if 365 Days (or a Year) gain 6 *l.* what will 73 Days gain? *Answ.* 1,2 *l.* or 1,2 *l.* Then adding the said 1,2 to 100, I say, by a second *Rule of Three*, if 101,2 *l.* Principal and Interest, payable at the End of 73 Days to come, be equivalent to 100 *l.* ready Money, what ready Money is 150 *l.* 10 *s.* (or 150,5) payable at the End of 73 Days to come equivalent unto? So by multiplying and dividing (according to the Rules of decimal Multiplication and Division explained

in Chapters 26 and 27 of the preceeding Book) the Quotient or Answer of the Question will be found 148,7154 +; that is, 148*l.* 14*s.* 3½*d.* +; for the Decimal ,7154 being valued according to the brief Way at the End of the fourth Rule of the 26th Chapter, will by Inspection only be discovered to be 14*s.* 3½*d.* which Rule I shall here once for all, advise the Learner to be well acquainted with.

The Proof.

Seek (by the *Rule of Three*) what the ready Money found as aforesaid will gain, in so much time as it is paid beforehand, at the Rate of Interest propounded: Then having added this Gain to the said ready Money, if the Sum be equal to the Debt first proposed to be satisfied by Rebate, the ready Money was rightly found out. So the last Example will be thus proved.

$$\begin{array}{cccc} l. & l. & l. & l. \\ 100 : 1,2 :: 148,7154 : (1,7845 \end{array}$$

Which fourth Proportional 1,7845 being added to 148,7154, the Sum will be 150.4999 +, which does not want a Farthing of 150*l.* 10*s.* the Debt first proposed.

XI. When it is required to find the present Worth of an Annuity, by rebating or discounting at a given Rate of simple Interest, the Operation will be as in the following Example, *viz.* How much present Money is equivalent to an Annuity or Rent of 100*l.* *per annum*, to continue 5 Years, Rebate being made at the Rate of 6*l.* for 100*l.* for one Year, at simple Interest? *Ans.* 425*l.* 18*s.* 9½*d.* very near.

It is manifest that there must be computed the present Worth of 100*l.* due at the first Year's End, also the present Worth of a 100*l.* due at the second Year's End, and in like manner for the third, fourth, and fifth Years; all which particular present Worths being added together, the Aggregate or Sum will be the total present Worth of the Annuity, to wit,

$$\begin{array}{r} 8286150 \\ \text{In the Example above produced, } 425, \text{ --- } l. \\ 8821267 \end{array}$$

that is, 425*l.* 18*s.* 9½*d.* very near.

The Operation by Decimals (which comes near enough to the Truth) is as follows, *viz.*

l.

	<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>	
1.	106	: 100	: : 100	: 94,33962	+
2.	112	: 100	: : 100	: 89,28571	+
3.	118	: 100	: : 100	: 84,74576	+
4.	124	: 100	: : 100	: 80,64516	+
5.	130	: 100	: : 100	: 76,92307	+

Answ. 425,93932 +

Here by the way, from the manner of resolving the last mentioned *Question*, that *Rule* commonly called *Equation of Payments*, which is insisted on by divers *Arithmetical Writers*, will be found erroneous, which I thus prove.

1. Since that *Rule* aims at the *reducing* of several Days of *Payment*, upon which particular Sums of Money are due, to a mean Time upon which the *Aggregate* or *Total* of these particular Sums ought to be paid, without Damage to the *Debitor* or *Creditor*, there must be *necessarily* some Rate of *Interest* implied: For otherwise, why may not any Day at pleasure be *assigned* for one entire *Payment*?

2. If some Rate of *Interest* be implied, then *Equity* requires that the present *Worth* of the total Sum *payable* at one entire *Payment*, *Rebate* or *Discount* being made according to that Rate of *Interest*, may be equal to the Sum of the present *Worths* of the particular Sums of *Money*, *Rebate* being made at the same Rate of *Interest*.

3. In regard the said *Rule* mentions no particular Rate of *Interest*, it ought to be true at any Rate of *Interest* whatsoever.

4. Let us therefore *examine* the said *Rule*, according to the Rate of 6 *per cent. per annum*, simple *Interest*, by taking the last mentioned *Question* for an *Example*, which (according to the accustomed Manner) will be thus stated, *viz.* If 500 *l.* ought to be paid by five equal yearly *Payments*, to wit, 100 *l.* at each Year's End, what Time ought to be given for the *Payment* of the said 500 *l.* at one entire *Payment*, without Loss either to the *Debitor* or *Creditor*.

5. By proceeding according to the said *Rule* of *Equation of Payments* (which says, if the Sum of the *Products*, arising from the Multiplication of every particular Sum of Money by its respective Time, be divided by the Sum or *Aggregate* of the said particular Sums of Money, the Quotient will be the mean Time to be assigned for one entire *Payment*) there will be found three Years, which Time (according to the said *Rule*) ought to be given for the *Payment* of the whole 500 *l.*

K k 2

6. Now

6. Now if 500*l.* due at the End of three Years to come, be worth as much in present Money, as is the present Worth of an *Annuity* of 100*l.* to continue five Years; then the said *Rule of Equation* is true; otherwise false: but the present Worth of 500*l.* due at the End of three Years to come, Rebate being made at the Rate of 6 *per cent. per ann. simple Interest*, will be found (by the tenth Rule of this Chapter) to be 423*l.* 14*s.* 6*d.* 3*f.* very near; also the present Worth of the said *Annuity*, Rebate being made as before, is found, (as appears by the Resolution of the last mentioned Question) to be 425*l.* 18*s.* 9½*d.* very near; wherefore it is evident that the *Creditor* loses 2*l.* 4*s.* 2¼*d.* very near, by receiving the whole 500*l.* at the three Year's End: Moreover, at 6 *per cent. per ann. compound Interest*, he would lose 1*l.* 8*s.* 6*d.* very near; as will be manifest by the *Tables of compound Interest* hereafter expressed: So that the Loss will be either more or less, according as the Rate of Interest differs: And therefore I conclude the said Rule (as also all other Rules or Resolutions of Questions which have dependance on it,) to be erroneous.

Altho' Questions of this nature seldom come into practice, yet he that will take the Pains, may find out such a mean Time as is required by the said *Rule of Equation of Payments*, at any Rate of simple Interest by this following Rule, *viz.*

First, by the preceeding tenth Rule of this Chapter, find out the present Worth of every particular *Sum* in the *Question* payable at a Time to come, by rebating at the Rate of Interest agreed on; then find in what Time the *Sum* of those present *Worths* will be augmented to the Total of all the particular *Sums* payable at Times to come, according to the first Agreement; so shall the Time found out, be the mean Time for the *Payment* of the whole *Debt*: Thus the mean or equated Time in the last *Example* will be found to be 2,8979, *£c.* Years, (not three Years, as the said *Rule of Equation of Payments* would have it;) for by rebating at 6 *per cent. per ann. simple Interest*, 500*l.* payable at the End of 2,8979, *£c.* Years to come (that is, 2 Years and 328 Days very near) is worth in ready *Money* 425*l.* 18*s.* 9½*d.* very near; and the same ready *Money* is also the present Value of 100*l.* *Annuity* for five Years, at the same Rate of Interest, as before has been manifested. But to return to the Path from which I have made a Digression.

From the preceeding tenth *Rule* of this Chapter, the following *Tables I. and II.* are deduced, whose Construction and Use are afterwards declared.

Years

Table I.		Table II.	
Years	Which shews in decimal Parts of a Pound, the present Worth of one Pound due at the End of any Number of Years to come, not exceeding 7 Years, at the Rate of 6 per centum, per annum, simple Interest.	Years	Which shews in Pounds and decimal Parts of a Pound, the present Worth of one Pound Annuity, to continue any Number of Years not exceeding 7, at the Rate of 6 per centum, per annum, simple Interest.
1	,943396	1	,943396
2	,892857	2	1,836253
3	,847457	3	2,683710
4	,806451	4	3,490162
5	,769230	5	4,259391
6	,733294	6	4,994685
7	,704225	7	5,698910

The Construction of Table I.

The Numbers in the first *Table* which are placed right against the Numbers of Years 1, 2, 3, 4, 5, 6, and 7, are decimal *Fractions*, one Pound of *English* Money being the *Integer*, and are thus found (according to the preceeding tenth Rule of this Chapter,) *viz.*

$$\begin{array}{l}
 106 : 100 :: 1 : ,943396 + \\
 112 : 100 :: 1 : ,892857 + \\
 118 : 100 :: 1 : ,847457 +
 \end{array}$$

whereby it appears, that 1 *l.* due at the End of a Year to come, is worth in ready Money ,943396 +, that is, 18 *s.* 10 *d.* 1 *f.* and somewhat more. Also 1 *l.* due at the End of two Years to come, is worth in ready Money ,892857 +, or 17 *s.* 10 $\frac{1}{4}$ *d.* Rebate being made at the Rate of 6 per cent. per ann. simple Interest; the like is to be understood of the Rest of the Numbers in *Table I.*, which may be continued to more Years, and other *Tables* also of Rebate may be framed upon the same Ground, for Months, or Days, by the ingenious Artist.

The Use of Table I.

The practical Use of the said first *Table* will be manifest by solving this following Question; *viz.* How much ready Mon-

W.

will discharge 345 *l.* 15 *s.* 6 *d.* due at the End of five Years to come, by rebating simple Interest at the Rate of 6 *per cent. per annum*? *Answ.* 265 *l.* 19 *s.* 7½ *d.* which is thus found out; *viz.* In the preceeding *Table I*, right against 5 Years, I find the Decimal ,76923, which shews that 1 *l.* due at the End of five Years to come is worth in ready Money ,76923, (that is, 15 *s.* 4½;) then instead of 15 *s.* 6 *d.* mentioned in the Question proposed, taking the Decimal ,775, which is equal to 15 *s.* 6 *d.* (the same being reduced according to the fifth Rule of the 23d Chapter of the preceeding Book) I say, by the *Rule of Three*,

$$1 : ,76923 :: 345 ,775 : (265 ,9805 +$$

That is to say, if 1 *l.* give ,76923 *l.* what will 345 ,775 *l.* give? *Answ.* 265 ,9805 *l.* for multiplying 345 ,775 by ,76923 according to the second Rule of the 26th Chapter of the preceeding Book, the Product will be 265 ,9805, that is, 265 *l.* 19 *s.* 7½ *d.*

The Construction of Table II.

The Numbers in the second *Table* are found out by the Addition of those in the first, *viz.* the first Number in the latter *Table* is the same with the first Number in the former; the second in the latter is the Sum of the first and second in the former; the third in the latter is the Sum of the first, second, and third in the former, and after that manner the rest are found; (the Reason of which Composition is manifest from the Example of the 11th Rule afore-going;) otherwise the Numbers in *Table II.* may be found more easily thus, *viz.* the first Number in the said *Table II.* is the same with the first Number in *Table I*, the second Number in the latter *Table* is compos'd of the second Number in the former, and the first in the latter; the third Number in the latter *Table* is compos'd of the third Number in the former, and the second in the latter; the fourth in the latter is compos'd of the fourth Number in the former, and the third in the latter; the like is to be understood of the rest of the Numbers in *Table II*, which might be continued to more Years, and fitted to other Rates of Interest; but I shall spare that labour, in regard a more equal Way of finding out the present Worth of an Annuity, agreeable to the accustomed and practical Rates of buying and selling Annuities or Rents for Terms of Years, is grounded upon a Computation of Interest upon Interest, as will hereafter be made manifest; for at simple Interest an Annuity will be overvalued.

The Use of Table II.

The Use of *Table II.* will appear by this following Example; *viz.* What is the present Worth of an Annuity of 100 *l.* *per annum*, payable yearly during the Term of five Years, Discount or Rebate being made at the Rate of 6 *per cent. per ann.* simple Interest? *Answer*, 425 *l.* 18 *s.* 9½ *d.* very near, which is thus found out, *viz.* In the preceeding *Table II.*, right against five Years, I find this Number, 4,259391, which shews that an Annuity of 1 *l.* payable yearly during five Years, is worth in ready Money 4,259391 *l.* (that is, 4 *l.* 5 *s.* 2 *d.* and somewhat more) therefore, I say, by the *Rule of Three*;

$$\begin{array}{cccc} l. & l. & l. & l. \\ 1 & : & 4,259391 & :: 100 : (425,9393 \end{array}$$

That is to say, if 1 *l.* give 4,259391 *l.* what will 100 *l.* give? *Answ.* 425 *l.* 18 *s.* 9½ *d.* very near; for by multiplying 4,259391 by 100, the Product (according to the second Rule of the 26th Chapter of the preceeding Book) is 425,9393, that is, 425 *l.* 18 *s.* 9½ *d.* very near. Which Operation being compared with the Manner of solving the same Question before-mentioned in the 11th Rule of this Chapter, the great Benefit of Tables of this kind in point of Expedition will be apparent.

XII. When it is required to know, what Sum of Money any propounded Principal forborn any Number of Years, will at the End of such Term be augmented to, Interest upon Interest being computed at a given Rate;

Of the forbearance of Money at compound interest.

there must be found a Rank of continual Proportionals, more in number by one than is the Number of Years in the Question; of which Proportionals the first is the Principal assigned, the second must encrease or proceed from the first, the third from the second, &c. in such Manner or Rate, as 106 proceeds from 100, (or as 108 from 100, if the Rate of Interest be 8 *per cent.*) then will the last Proportional be the Answer of the Question: So if 300 *l.* principal Money be put forth at Interest upon Interest, at the Rate of 6 *l.* for 100 *l.* for one Year, and all forborn until the End of 4 Years, there will then be due 378,743088, or 378 *l.* 14 *s.* 10½ *d.* very near, as by the four following *Rules of Three* is manifest.

$$100 : 106 :: \left\{ \begin{array}{l} 300 \\ 318 \\ 337,08 \\ 357,3048 \end{array} \right. : \left\{ \begin{array}{l} 318 \\ 337,08 \\ 357,3048 \\ 378,743088 \end{array} \right.$$

For the said 300 *l.* will at the first Year's End be augmented to 318 *l.* which 318 *l.* being put forth as a Principal for 1 Year, will (at the second Year's End) be augmented to 337,08; again, this 337,08 being put forth as a Principal for 1 Year, will (at the third Year's End) be augmented to 357,3048; in like manner, 357,3048 being put forth as a *Principal* for 1 Year, will (at the fourth Year's End) be augmented to 378,743088, which is the Number required by the Question. And if the Work be well examined, it will appear (as was before declared) that the Principal first assigned, to wit, 300 *l.* and the Numbers resulting successively at the Ends of the several Years are *continual Proportionals*, viz. these five Numbers are so qualified, that if the

300 | 318 | 337,08 | 357,3048 | 378,743088
second be multiplied by itself, the Product will be equal to the Product of the first and third; also if the third be multiplied by it self, the Product will be equal to the Product of the second and fourth; in like manner, if there were more *continual Proportionals* in a *Rank*, if any one Proportional which is placed *between* two next on each side of such a one, be multiplied by itself, the Product will be equal to the Product of those two Extremes, (which is a Property peculiar to continual Proportionals.)

Two Number?
being given to
find a third,
a fourth, a
fifth, &c. in
continual Pro-
portion.

Note here by the way, that if any two Numbers be propounded, suppose 300 and 318, and it be required to find to them a third, a fourth, a fifth, &c. in continual Proportion, multiply the second Proportional 318 by it self, and divide the Product 101124 by the first Proportional 300, so shall the Quotient 337,08 be a third in continual Proportion: In like manner, if you multiply the third Proportional 337,08 by it self, and divide the Product 113622,9264 by the second Proportional, 318, the Quotient 357,3048 will be a fourth in continual Proportion; and after the same manner a fifth, a sixth, or as many as you please may be found out.

From what has been said by way of Explication of the preceding 12th Rule, the following *Table III.* is deduced, the Construction and Use of which is afterwards declared.

TABLE

TABLE III.

Which shews what one Pound will amount to, being ferborn to the End of any Term of Years under 31, compound Interest being computed yearly, at any of these Rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Year.	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	1.08160	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.23216	1.25440
3	1.12486	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100	1.36763	1.40492
4	1.16985	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410	1.51807	1.57351
5	1.21665	1.27628	1.33823	1.40255	1.46932	1.53862	1.61051	1.68505	1.76234
6	1.26531	1.34009	1.41851	1.50073	1.58087	1.67710	1.77156	1.87041	1.97382
7	1.31593	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871	2.07616	2.21068
8	1.36856	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358	2.30453	2.47596
9	1.42331	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794	2.55803	2.77307
10	1.48024	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374	2.83942	3.10584
11	1.53945	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311	3.15175	3.47854
12	1.60103	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842	3.49849	3.89597
13	1.66507	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227	3.88328	4.36349
14	1.73167	1.97993	2.26090	2.57853	2.90719	3.34172	3.79749	4.31044	4.88711
15	1.80094	2.07892	2.39655	2.75903	3.17216	3.64248	4.17724	4.78458	5.47356

A Continuation of the preceding Table III.

Year.	4	5	6	7	8	9	10	11	12
16	1.87298	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497	5.31089	6.13039
17	1.94790	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447	5.89509	6.86604
18	2.02581	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991	6.54355	7.68996
19	2.10684	2.52695	3.02559	3.61652	4.31570	5.14166	6.11590	7.26234	8.61276
20	2.19112	2.65329	3.20713	3.86968	4.66095	5.60441	6.72749	8.06231	9.64629
21	2.27876	2.78596	3.39956	4.14056	5.03383	6.10880	7.40024	8.94916	10.80384
22	2.36991	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027	9.93357	12.10031
23	2.46471	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430	11.02626	13.55234
24	2.56330	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973	12.23915	15.17862
25	2.66583	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470	13.58546	17.00006
26	2.77246	3.55567	4.54938	5.80735	7.39635	9.39915	11.91817	15.07986	19.04007
27	2.88336	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999	16.73864	21.32488
28	2.99870	3.92012	5.11168	6.64883	8.62710	11.16713	14.42099	18.57990	23.88386
29	3.11865	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309	20.62369	26.74993
30	3.24339	4.32194	5.74349	7.61225	10.06255	13.26767	17.44940	22.89229	29.55992

The Construction of the preceeding Table III.

The Numbers 1, 2, 3, 4, &c. to 30, in the first Column on the left Hand signify Years; the Numbers, 4, 5, 6, 7, 8, 9, 10, 11, and 12, placed at the Head of the rest of the Columns, denote Rates of *Interest* for 100 *l* lent for a Year, and the Numbers set in the several Columns under those Rates of *Interest*, are found out by the *Rule of Three* in Decimals, in manner following, viz.

I.	100 : 104 :: 1	: (1,04
II.	100 : 104 :: 1,04	: (1,0816
III.	100 : 104 :: 1,0816	: (1,12486

That is to say, First, if 100 *l*. put forth at *Interest* for a Year be augmented to 104 *l*. at the Year's End, what will 1 *l*. be then augmented to at the same Rate? *Ans.* 1,04 *l*. (that is, 1 *l*. 0 *s*. 9 *d*. 2 *f*. and somewhat more) which 1,04 (or 1,04000, the Cyphers after the 4 being of no value in Decimals) is the first Number in the 2d Column belonging to 4 *per cent*. and is placed right against 1 Year in the first Column.

Secondly, say if 100 *l*. lent for a Year be augmented to 104 *l*. at the Year's End, what will 1,04 *l*. be then encreas'd to at the same Rate? *Ans.* 1,0816 *l*. (that is, 1 *l*. 1 *s*. 7 *d*. 2 *f*. +) which 1,0816 is the second Number in the said Column of 4 *per cent*. and is set right against 2 Years in the first Column.

Thirdly, as 100 is to 104, so is 1,0816 to 1,12486 (or 1 *l*. 2 *s*. 5 *d*. 2 *f*. +) which 1,12486 is the 3d Number in the Column of 4 *per cent*. and is placed right against 3 Years in the first Column. Hence it appears, that 1 *l*. at 4 *per cent*. *per ann*. compound Interest, will at the End of 3 Years be augmented to 1,12486 *l*. (that is, 1 *l*. 2 *s*. 5 *d*. 2 *f*. and somewhat more.)

After the same manner the rest of the Numbers in the second Column, as also in the other Columns, are found out, (*mutatis mutandis*.)

The Use of the preceeding third Table.

Quest. 1. What will 136 *l*. 15 *s*. 6 *d*. be augmented to, being forborn 20 Years, *Interest* upon *Interest* being computed at the Rate of 6 *per cent*. *per annum*? *Ans.* 438 *l*. 13 *s*. 1 *d*. very near, which is thus found out.

First, looking into the 4th Column of the said third Table, to wit, that Column which has the Figure 6 placed at the Head

L 1 2

of

of it, I find right against 20 Years the Number 3,20713, which shews that 1 *l.* being continued 20 Years at 6 *per cent. per annum*, compound *Interest*, and all forborn until the End of the said *Term*, will be augmented to 3,20713 *l.* (that is, 3 *l.* 4 *s.* 1 *d.* 2 *f.* and somewhat more;) therefore after the 15 *s.* 6 *d.* in the *Question* is reduced to the Decimal 5,775 (by the 16th Rule of the 23d *Chapter* of the preceeding Book,) I multiply the said *tabular* Number 3,20713, by 136,775 (the Sum propounded in the *Question*) according to the second Rule of the 26th *Chapter*, so the Product is found to be 438,655, &c. that is, 438 *l.* 13 *s.* 1 *d.* for the Answer of the *Question*. View the Operation here following.

$$\begin{array}{r}
 1 : 3,20713 :: 136,775 : (438,655 \\
 136,775 \\
 \hline
 1603665 \\
 2244991 \\
 2244991 \\
 1924278 \\
 962139 \\
 3,20713 \\
 \hline
 438165520575
 \end{array}$$

Quest 42. If 320 *l.* be forborn eleven Years, at Interest upon Interest at 5 *per cent. per ann.* what will be due at the End of those eleven Years for Principal and Interest? *Ans.* 547 *l.* 6 *s.* 1 *d.* +. For in the third Column of the third Table under the Figure 5 at the Head of the Column, and right against eleven Years, you'll find this Number 1,71033, which shews that 1 *l.* at the End of eleven Years, will, at 5 *per cent. per ann.* compound Interest, be augmented to 1,71033, (that is, 1 *l.* 14 *s.* 2 *d.* 1 *f.* and somewhat more;) wherefore by multiplying the said 1,71033 by 320, (the Number of Pounds proposed in the *Question*) the Product will be 547,305, &c. that is, 547 *l.* 6 *s.* 1 *d.* +, for the Answer of the *Question*. See the following Operation.

$$\begin{array}{r}
 1 : 1,71033 :: 320 : (547,305 + \\
 320 \\
 \hline
 3420660 \\
 513099 \\
 \hline
 547130560
 \end{array}$$

After

After the same manner, the Numbers belonging to any of the other Rates of Interest mentioned in the third Table are to be used.

XIII. When an Annuity payable yearly is in Arrear for any Number of Years, and it is required to know what the same will amount to, compound Interest being computed for each particular Annuity, from the Time it became due until the End of the Term of Years, the Work will be as in the following Example; *viz.* Suppose an Annuity of 300 *l.* payable at yearly Payments be forborn, and all unpaid until the End of four Years; the Question is, what will then be due, compound Interest being computed at the Rate of 6 *per cent. per ann.* for every yearly Payment from the Time it becomes due, to the End of the said Term of four Years? *Answ.* 1312 *l.* 7*s.* 8*d.* very near.

*The Manner
of summing
up Annu-
ties in Ar-
rear, with
Allowances
of Interest
upon Interest*

It is evident by the Question, that there must be computed what 300 *l.* due at the 3d Year's End will be augmented to in one Year (to wit, the 4th Year) at 6 *per cent.* Also what 300 *l.* due at the 2d Year's End will be increased to in two Years, (to wit, the third and fourth Years;) likewise what 300 *l.* due at the first Year's End will be augmented to, in the three following Years, (to wit, the second, third, and fourth Years) all which Sums being added to 300 *l.* (the Payment due at the End of the fourth Year, which is incapable of any Improvement,) the Aggregate or Sum will be the total Money in Arrear at the End of the fourth Year, to wit, 1312 $\frac{3848}{1000}$ *l.* as may appear by the following Operation, *viz.*

The last Payment of the Annuity due at the End of the fourth Year is	}	300
Again, the 300 <i>l.</i> due at the third Year's End, will in one Year, after the Rate of 6 <i>per cent.</i> be augmented to	}	318
Also 300 <i>l.</i> due at the second Year's End will in two Years, at the Rate of 6 <i>per cent. per annum</i> , compound Interest, be increased to, (as appears by the first Example of the 12th Rule aforegoing)	}	337,08
In like manner 300 <i>l.</i> due at the first Year's End, will in three Years be augmented to	}	357,3048

The Sum due at four Year's End is ——— 1312,3848

The

The Invention of the Numbers before-mentioned being well examined, it will appear, that if an Annuity or Rent payable at yearly Payments be improved to the utmost at Interest upon Interest, and all forborn or respited to the End of certain Years, the Total then due will be the Sum of a Rank of continual Proportionals as many in number, as there are yearly Payments, the first of which Proportionals is the first (or any one) Year's Rent: And the second Proportional proceeds from the first in the same Rate as 106 proceeds from 100, if the Rate of Interest be 6 *per cent.* (or as 108 proceeds from 100, if the Rate of Interest be 8 *per cent.* &c.) and so likewise the third from the second, the fourth from the third, &c. (after the Manner of the Operation in the first Example of the twelfth Rule of this Chapter.)

Otherwise.

Find a Principal which may have such Proportion to 300, as 100 has to 6, and say by the *Rule of Three*,

$$6 : 100 :: 300 : 5000.$$

That is to say, as 6 *l.* Interest has 100 *l.* for a Principal, so 300 *l.* Interest has 5000 *l.* for a Principal; then seek what 5000 *l.* will be augmented to, being forborn four Years at 6 *per cent. per ann.* compound Interest, (after the manner of the first *Example* of the 12th Rule foregoing:) So you'll find 6312,3848, from which subtracting the said Principal 5000 *l.* the Remainder (as before) is 1312,3848 *l.* being the Sum which 300 *l.* Annuity will be augmented to at the End of four Years, according to the said Rate of Interest, the Annuity being payable at yearly Payments.

The Reason of the latter Rule.

If a Principal be put forth at Interest upon Interest payable by yearly Payments, and all be forborn until the End of certain Years, the Total then due is equal to the Aggregate or Sum of these three Numbers; to wit, the said Principal first put forth, the Sum of the annual simple Interests of that Principal, and the utmost Improvement of those simple Interests, by computing Interest upon Interest: Therefore, if from the said Aggregate the first Principal be subtracted, the Remainder must necessarily consist of the Sum of the annual simple Interests,

Interests, (which are in the Nature of an Annuity,) and the utmost Improvement of those simple Interests (or Annuity) by computing Interest upon Interest.

The Construction of the following Table IV.

Upon the aforesaid Grounds, the following *Table IV.* is calculated, to shew what one Pound Annuity, payable at yearly Payments, and forborn any Number of Years under 31, will amount to, by computing Interest upon Interest, at any of the Rates expressed at the Head of the said Table.

But the same *Table* may be more easily composed, by the Addition of the Numbers in the preceeding *Table III.* in this manner, *viz.* the first Number in every one of those Columns in the following *Table IV.* at the Head of which are placed the Numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, signifying Rates of Interest *per cent.* is 1 or Unity; the second Number in each of these Columns in the latter *Table* is composed of 1 or unity, and the first Number in the respective Columns of the said *preceeding Table III.*

Also the third Number in every of the said Columns of this *latter Table* is composed of 1, and the Sum of the first and second *Numbers* of the respective Columns of the former *Table*, and in that order the Rest are found out: or more easily thus: The third Number in the latter *Table* is composed of the second Number in the latter, and of the second in the former; the fourth Number in the latter is composed of the third in the latter, and of the third in the former, &c. But you are to observe, that according to either of these Ways of composing the fourth *Table* by Addition, the Numbers in the preceeding *Table III.* ought to be continued to more Places than are there expressed; to prevent Errour, which may happen by the adding of defective decimal Fractions.

TABLE

TABLE IV.

Which shews what one Pound Annuity, payable by yearly Payments, and forborn any Number of Years under 31 will amount to, at the End of the Term, compound Interest being computed at any of these Rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

No.	4	5	6	7	8	9	10	11	12
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	4.24646	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100	4.70973	4.77932
5	5.41632	5.52563	5.63709	5.75073	5.86660	5.98471	6.10510	6.22780	6.35284
6	6.63297	6.80190	6.97531	7.15329	7.33592	7.52333	7.71561	7.91285	8.11518
7	7.89829	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21422	9.64910	9.89746	10.25980	10.63662	11.02847	11.43588	11.85943	12.29969
9	10.58279	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947	14.16397	14.77565
10	12.00610	12.57789	12.18079	13.81644	14.48656	15.19292	15.93742	16.72200	17.54873
11	13.48635	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116	19.56142	20.65458
12	15.02580	15.91712	16.86994	17.88845	18.97712	20.14071	21.38428	22.71318	24.13313
13	16.62683	17.71297	18.88213	20.14064	21.49529	22.95338	24.52271	26.21163	28.02910
14	18.29191	19.59863	21.01506	22.55048	24.21492	26.01918	27.97498	30.09491	32.39260
15	20.02358	21.57856	23.27596	25.12002	27.15211	29.36091	31.77248	34.40535	37.27971

A Continuation of the preceding Table VI.

Ye	4	5	6	7	8	9	10	11	12
16	21.82452	23.65749	25.67252	27.88505	30.32428	33.00339	35.94972	39.18993	42.75328
17	23.69751	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917	50.39593	55.74971
19	27.67120	30.53900	33.75999	37.37896	41.44626	46.01845	51.15909	56.93948	63.43968
20	29.77807	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499	64.20283	72.05244
21	31.96920	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249	72.26514	81.69873
22	34.24796	38.50521	43.39228	49.00573	55.45675	62.87333	71.40274	81.21430	92.50258
23	36.61788	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302	91.14788	104.60289
24	39.08260	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732	102.17415	118.15524
25	41.64590	47.72709	54.86450	63.24903	73.10593	84.70089	98.34705	114.41330	133.33387
26	44.31174	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176	127.99877	150.33393
27	47.08421	54.66912	63.70576	74.48382	87.35076	102.72313	121.09994	143.07863	169.37400
28	49.96758	58.40258	68.52810	89.69769	95.33882	112.96821	134.20993	159.81728	190.69888
29	52.96628	62.32271	73.63979	87.34652	103.95593	124.13535	143.63092	178.39718	214.58275
30	56.08403	66.43384	79.05818	94.46078	113.28321	136.30753	164.49402	199.02087	241.33268

M m

The

The Use of the preceeding Table IV.

The Use of the said fourth Table will be manifest by the Manner of solving this Question, *viz.* if an Annuity of 200 *l.* payable by yearly Payments for fifteen Years, be all forborn or unpaid until the End of the said Term, what will it then amount to, upon a Computation of Interest upon Interest, at the Rate of 6 *per cent. per annum*? *Answ.* 465 *l.* 10 *s.* 4 *d.* 2 *f.* very near, as by the following Operation is evident: For in the Column belonging to 6 *per cent.* (to wit, that Column which has the Figure 6 placed at the Head of it) right against 15 Years, you will find 23.27596, which shews that an Annuity of 1 *l.* payable at yearly Payments for 15 Years, will at the End of the said Term (compound Interest being computed at 6 *per cent. per annum*) amount to 23.27596 *l.* (or 23 *l.* 5 *s.* 6 *d.* +) Therefore multiplying the said Tabular Number 23.27596 by 20, (because the Annuity propounded is 20 *l.*) the Product will be 465.519 +, that is, 465 *l.* 10 *s.* 4 *d.* 2 *f.* which is the Answer of the Question. View the following Operation.

$$\begin{array}{r} 1 : 23.27596 :: 20 : (465.519 + \\ \quad \quad \quad 20 \\ \hline 465 | 51920 \end{array}$$

In the same Manner, the Numbers in the other Columns are to be used.

On Rebate at compound Interest. XIV. When a Sum of Money is due at a Time to come, and it is required to know what it is worth in ready Money, Rebate being made at a given Rate of compound Interest, the Work

will not be much different from the 12th Rule of this Chapter, *viz.* there must be found a Series or Rank of continual Proportionals more in number by one, than is the Number of Years in the Question; of which Proportionals, the first is the Money proposed to be rebated, the second must decrease or lessen from the first, the third from the second, &c. in such Manner or Rate as 100 decreases from 106 (or as 100 from 108, if the Rate of Interest be 8 *per cent.*) then will the last Proportional be the Answer of the Question: So if 378 $\frac{741088}{1000000}$ *l.* be due at the End of four Years wholly to come,

come, it will be found to be worth in ready Money 300 *l.* Rebate being made at compound Interest at 6 *per cent.* as by the four following *Rules of Three* is manifest, which may be proved by the preceeding 12th Rule, where it will appear that 300 *l.* being forborn four Years, will at the said Rate of compound Interest be augmented to 378.743088 *l.*

$$\begin{array}{rcl}
 & \text{\textit{l.}} & \text{\textit{l.}} \\
 106 : 100 :: \left\{ \begin{array}{l} 378.743088 : 357.3048 \\ 357.3048 : 337.08 \\ 337.08 : 318 \\ 318 : 300 \end{array} \right.
 \end{array}$$

Upon this Ground the following *Table V.* is calculated, to shew what one Pound due at the End of any Number of Years to come, is worth in present Money, Rebate being made, at the Rates of compound Interest, mentioned in the said *Table*; by the Help of which and of *Multiplication*, Questions of Rebate for any Sum proposed may be performed without considerable Errour.

M m 2

TABLE

TABLE V.

Which shews what one Pound payable at the End of any Term of Years to come under 31, is worth in ready Money; Discount or Rebate being yearly computed at any of these Rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum. compound Interest.

Year.	4	5	6	7	8	9	10	11	12
1	.961538	.952381	.943396	.934579	.925925	.917431	.909090	.900900	.892857
2	.924556	.907029	.889996	.873438	.857338	.841680	.826446	.811622	.797193
3	.888996	.863837	.839619	.816297	.793832	.772183	.751314	.731191	.711780
4	.854802	.822702	.792093	.762895	.735029	.708425	.683013	.658731	.635518
5	.821927	.783526	.747258	.712986	.680583	.649931	.620921	.593451	.567426
6	.790314	.740215	.704960	.666342	.630169	.596267	.564474	.534640	.506631
7	.759917	.710681	.665057	.622749	.583490	.547034	.513158	.481658	.452349
8	.730690	.676839	.627412	.582009	.540268	.501866	.466507	.433926	.403883
9	.702586	.644608	.591898	.543933	.500248	.460427	.424097	.390924	.360610
10	.675564	.613913	.558391	.508343	.463193	.422410	.385543	.352184	.321973
11	.649580	.584679	.526787	.475092	.428882	.387532	.350494	.317283	.287476
12	.624596	.556837	.496989	.444012	.397113	.355534	.318630	.285840	.256675
13	.600573	.530321	.468839	.414964	.367697	.326178	.289064	.257514	.229174
14	.577474	.505067	.442300	.387817	.340461	.299246	.263331	.231994	.204619
15	.555264	.481017	.417165	.362446	.315241	.274538	.239392	.209004	.182696

A Continuation of the preceding Table V.

Year.	4	5	6	7	8	9	10	11	12
16	.533208	.458111	.393646	.338734	.291860	.251869	.217629	.188292	.163121
17	.513373	.436206	.371364	.316574	.270269	.231073	.197844	.169632	.145644
18	.493628	.415520	.350343	.295864	.250249	.212993	.179858	.152822	.130039
19	.474642	.395733	.330512	.276508	.231712	.194489	.163508	.137677	.116106
20	.456386	.376889	.311804	.258419	.214548	.178430	.148643	.124034	.103666
21	.438833	.358942	.294155	.241513	.198655	.163698	.135130	.111742	.092556
22	.421955	.341849	.277505	.225713	.183940	.150181	.122840	.100668	.082642
23	.405726	.325571	.261797	.210947	.170115	.137781	.111678	.090692	.073787
24	.390121	.310067	.246978	.197146	.157699	.126405	.101525	.081705	.065882
25	.375116	.295302	.232998	.184249	.146018	.115967	.092296	.073608	.058823
26	.360689	.281240	.219810	.172195	.135201	.106392	.083905	.066313	.052520
27	.346816	.267848	.207367	.160930	.125186	.097607	.076277	.059742	.046893
28	.333477	.255093	.195630	.150402	.115913	.089548	.069343	.053821	.041869
29	.320651	.242946	.184556	.140562	.107327	.082154	.063039	.048487	.037383
30	.308318	.231377	.174110	.131367	.099377	.075371	.057308	.043682	.033377

The

The Construction of the preceeding Table V.

The Numbers 1, 2, 3, 4, &c. to 30, (in the first Column on the left Hand) signify Years; the Numbers 4, 5, 6, 7, 8, 9, 10, 11, and 12, placed at the Head of the rest of the Columns, express Rates of Interest for 100 *l.* lent for a Year, and the Numbers set in the several Columns under those Rates of Interest are found out by the *Rule of Three* in Decimals, in manner following, *viz.*

I.	104 : 100 :: 1	:	(,9615384615, &c.
II.	104 : 100 :: ,9615384615 +	:	(,9245562, &c.
III.	104 : 100 :: ,9245562, &c.	:	(,888996 +

That is to say, first, if 104 decrease to 100, or if 104 *l.* payable at the End of a Year to come be worth 100 *l.* in ready Money, what ready Money is 1 *l.* due at the End of a Year to come, worth? *Answ.* ,9615384615 + (or 19 *s.* 2 *d.* 3 *f.* very near: So that ,961538 is the first Decimal in the second Column belonging to 4 *per cent.* in *Table V.* and is placed right against 1 Year in the first Column.

Secondly, say in like manner, if 104 decrease to 100, what will ,9615384615, &c. the Decimal found by the first *Rule of Three* decrease to? *Answ.* ,9245562, &c. the first 6 places whereof to wit, ,924556 are the second Decimal in the said Column of 4 *per cent.* which is set right against two Years.

Thirdly, as 104 is to 100; so is ,9245562, &c. (the Decimal found by the 2d *Rule of Three*) to ,888996 + (or 17 *s.* 9 *d.* 1 *f.* +) which is the third Decimal in the Column of 4 *per cent.* Hence it appears, that 1 *l.* due at the End of 3 Years to come, is worth ,888996 + (or 17 *s.* 9 *d.* 1 *f.* and some what more) in ready Money, Rebate being made at the Rate of 4 *per cent. per ann.* compound Interest.

After the same manner, the rest of the Decimal Fractions in the said second Column, as also in the other Columns, are found out, (*mutatis mutandis.*)

The Use of the preceeding Table V.

To exemplify the said fifth Table, let it be required to find out how much ready Money will discharge a Debt of 350 *l.* payable at the End of 7 Years to come, by rebating at the Rate of 7 *per cent. per ann.* compound Interest? *Answ.* 221 *l.* 13 *s.* 11 *d.* 3 *f.* very near. For in the fifth Column, at the Head

Head of which is placed 7, signifying 7 *per cent.* right against 7 Years I find .622749, which shews, that 1 *l.* due at the End of 7 Years to come, is worth in present Money .622749 decimal Parts of a Pound, Rebate being made at the said Rate of compound Interest. Therefore multiplying the said Tabular Number .622749 by the said 356 *l.* the Debt proposed, the Product (according to the second Rule of the 26th Chapter) will be 221.698, *£*c. that is, 221 *l.* 13 *s.* 11 *d.* 3 *f.* which is the Answer of the Question. See the subsequent Operation.

$$1 : .622749 :: 356 : (221.698 + 356)$$

$$\begin{array}{r} 3736494 \\ 3113745 \\ 1868247 \\ \hline 221698644 \end{array}$$

In the same Manner, the Numbers in the other Columns are to be used.

XV. The finding out the present Worth of an Annuity is grounded upon this Foundation; to wit, if the present Money which is paid for the Purchase of an Annuity, to continue any Term of Years, be put forth at any Rate of compound Interest, and all forborn until the End of the said Term, and that the total Money then due be put into one Scale: Also if the total Sum of the utmost Improvements of the annual Payments of the Annuity, put forth at the same Rate of compound Interest, from the Time those annual Payments become due, until the End of the Term, be put in the other Scale, the Scales will be even, *viz.* the said two total Sums of Money must be equal one to the other.

To find the present worth of Annuities, by a Computation of compound Interest.

Now to find out such a present Worth of an Annuity, there are diverse Ways, some of which I shall here explain by Examples:

First therefore, let it be required to find the present Worth of an Annuity of 378.73088 *l.* to continue three Years, *compound Interest* being computed at 6 *per cent. per ann.* ? *Ans.* 1012.3848 *l.*

It is evident by the Question, that there must be computed (after the Manner of the Example upon the 14th Rule foregoing)

going) the present Worth of 378.743088 l. due at the first Year's End, also the present Worth of the like Sum due at the second Year's End, and in like manner for the third Year; all which particular present Values being added together, the Aggregate or Sum will be the total present Worth of the Annuity proposed, *viz.*

378.743088 l. payable at the End of one Year is	}	l.
worth in ready Money (as is evident by the fourteenth Rule afore-going) . . .		
Also the like Sum payable at the End of two Years to come, is worth in ready Money	}	337.08
Again, the like Sum payable at the End of three Years to come, is worth in ready Money . . .		
		318.

Therefore the total present Worth of an Annuity of 378.743088 l. to continue 3 Years is 1012.3848

Otherwise.

Find a Principal which may be in such Proportion to the propounded Annuity 378.743088 l. as 100 is to 6, which will be exactly 6312.3848 l. for

$$6 : 100 :: 378.743088 : 6312.3848.$$

Then supposing this Principal so found to be a Sum due at the End of three Years to come, find what it will be worth in ready Money, by diminishing it according to the 14th Rule of this Chapter, so will you find 5300 l. for the ready Money equivalent to the said 6312.3848 l. due at the End of three Years, which ready Money 5300 l. being subtracted from the said 6312.3848 l. leaves (as before) 1012.3848 l. for the present Worth of the said Annuity of 378.543088 l. to continue three Years, compound Interest being allowed a 6 per cent. *per annum.*

The Reason of the latter Rule.

It will not be difficult to apprehend, that if 6312.3848 l. ready Money be put forth as a Principal at Interest upon Interest, it will at three Years End be augmented to an Aggregate or Sum composed of these three Numbers; to wit, the said Principal 6312.3848 , the Sum of the annual simple Interests of that Principal, and the utmost Improvement of those simple Interests, by Interest upon Interest: And because (by the Operation afore-going) 5300 l. ready Money part of the said ready Money 6312.3848 l. will at three Years End be augmented to 6312.3848 l. Part of the said Aggregate; therefore 1012.3848 l.

the

the Complement or remaining Part of the said ready Money 6312.3848 *l.* must necessarily be augmented to the Complement or remaining Part of the said Aggregate, which remaining Part last mentioned is composed of the Sum of the aforesaid simple Interests, and of their utmost Improvement at Interest upon Interest; that is, the said Remainder is the utmost Improvement of an Annuity of 378.743088 *l.* to continue three Years, compound Interest being allowed at 6 *per cent. per annum.*

The Construction of the following Table VI.

Upon the aforesaid Grounds the following Table VI. is calculated, to shew how much ready Money an *Annuity* of one Pound to continue any Number of Years under 31, and payable at yearly Payments, is worth, upon a *Computation* of compound Interest at any of the Rates *per centum*, mentioned at the Head of the said Table. But the said Table VI, may more easily be compos'd by means of the preceeding Table V, in this manner; *viz.* the first Number in every of the *Columns* (except the *Column* of Years) in the following Table VI, is the same with the first Number in the like *Columns* respectively in the preceeding Table V; the second Number in each of the said *Columns* of the sixth Table, is the Sum of the first and second Numbers in the respective *Columns* of the fifth Table; the third Number in the said *Columns* of the sixth Table, is the Sum of the first, second, and third Numbers in the respective *Columns* of the first Table: Or yet more easily thus, the third Number in the sixth Table, is composed of the third in the fifth Table, and of the second in the sixth; the fourth Number in the sixth Table is composed of the fourth in the fifth, and of the third in the sixth; the like is to be understood of the Rest. But you are to observe, that according to this Method of composing the sixth Table by Addition, the Numbers of the fifth Table must be continued to more Places than are there express'd, to prevent Error arising by the Addition of defective Decimal Fractions.

TABLE VI.

Which shews the present Worth of one Pound Annuity, to continue any Term of Years under 31, and payable by yearly Payments, compound Interest being computed at any of these Rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Year.	4	5	6	7	8	9	10	11	12.
1	.96153	.95238	.94339	.93457	.92592	.91743	.90909	.90090	.89285
2	1.88609	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553	1.71252	1.69005
3	2.77509	2.72324	2.67301	2.62431	2.57709	2.53129	2.48685	2.44371	2.40183
4	3.62989	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986	3.10244	3.03734
5	4.45182	4.32947	4.21236	4.10019	3.99270	3.88965	3.79078	3.69589	3.60477
6	5.24213	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526	4.23053	4.11140
7	6.00205	5.78637	5.58233	5.38928	5.20636	5.03295	4.86841	4.71219	4.56375
8	6.73274	6.46321	6.20979	5.97129	5.74663	5.53481	5.33492	5.14612	4.96763
9	7.43533	7.10782	6.80169	6.51523	6.24688	5.99524	5.75901	5.53704	5.32824
10	8.11089	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456	5.88923	5.65022
11	8.76047	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20651	5.93769
12	9.38507	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369	6.49235	6.19437
13	9.98964	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335	6.74987	6.42354
14	10.56312	9.89864	9.29498	8.74546	8.24423	7.78614	7.36668	6.98186	6.62816
15	11.11838	10.37965	9.71224	9.10791	8.55947	8.06068	7.60608	7.19087	6.81086

A Continuation of the preceding Table VI.

Re	4	5	6	7	8	9	10	11	12
16	11.65229	10.83776	10.10598	9.44664	8.85136	8.31255	7.82371	7.37916	6.97398
17	12.16566	11.27406	10.47725	9.76322	9.12163	8.54364	8.02155	7.54879	7.11962
18	12.65929	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141	7.70161	7.24996
19	13.13393	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492	7.83929	7.36577
20	13.59032	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356	7.96332	7.46944
21	14.02915	12.82115	11.76407	10.83557	10.01680	9.29224	8.64869	8.07507	7.56200
22	14.45111	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154	8.17574	7.64464
23	14.85683	13.48857	12.30337	11.27218	10.37105	9.58020	8.88322	8.26643	7.71843
24	15.24696	13.79864	12.55035	11.46933	10.52875	9.70661	8.98474	8.34813	7.78431
25	15.62207	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704	8.42174	7.84313
26	15.98276	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094	8.48805	7.89505
27	16.32958	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722	8.54780	7.94255
28	16.66305	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656	8.60162	7.98442
29	16.98371	15.14107	13.59071	12.27767	11.15840	10.19828	9.36960	8.65011	8.02180
30	17.29202	15.39244	13.76482	12.40004	11.25778	10.27365	9.42691	8.69379	8.05518

The Use of the preceeding Table VI.

The Use of the said sixth Table will appear by the Manner of solving these two subsequent Questions, *viz.*

Quest. 1. What is the present Worth of an Annuity or Rent of 56 *l. per annum*, payable by yearly Payments for 21 Years, accounting Interest upon Interest at the Rate of 6 *per cent. per annum*? *Answ.* 658 *l.* 15 *s.* 9 *d.* very near, thus found out: In the fourth Column of the preceeding Table VI, under the Figure 6 at the Head, and right against 21 Years, I find 11.76407, which shews that an Annuity of 1 *l.* payable by yearly Payments for 21 Years, is worth in present Money 11.76407 *l.* or 11 *l.* 15 *s.* 3 *d.* 1 *f.* (and somewhat more) Interest upon Interest being computed on both sides at the Rate of 6 *per cent. per ann.*; therefore multiplying the said Tabular Number 11.76407 by 56, (56 because the Annuity proposed is 56 Pounds) the Product (according to the second Rule of the 26th Chapter of the preceeding Book) will be found to be 658.787, *£*7c. that is, 658 *l.* 15 *s.* 9 *d.* very near: Wherefore I conclude that the Answer of the Question is 658 *l.* 15 *s.* 9 *d.* View the following Operation.

$$\begin{array}{r}
 1 : 76407 :: 56 : (658,787 + \\
 \quad \quad \quad 56 \\
 \hline
 7015842 \\
 5882035 \\
 \hline
 65878792
 \end{array}$$

Quest. 2. What is the present Worth of an Annual Rent of 45 *l.* payable by yearly Payments for 21 Years, Interest upon Interest being computed at 10 *per cent. per annum*? *Answ.* 389 *l.* 3 *s.* 10 *d.* very near; for in the Column of 10 *per cent.* in the said sixth Table, right against 21 Years, and under 10, at the Head I find this Number 8.64869; which shews that at 10 *per cent.* compound Interest, an Annuity or Rent of 1 *l.* payable by yearly Payments for 21 Years, is worth in ready Money 8.64869 *l.* that is, 8 *l.* 12 *s.* 11 *d.* 3 *f.* therefore multiplying the said Tabular Number 8.64869, by 45, (the Rent propounded) the Product will be 389.191 +, that is, 389 *l.* 3 *s.* 10 *d.* very near, which is the Answer of the Question.

$$1 : 8.64869 :: 45 : (389,191 + 45)$$

$$\begin{array}{r} 4324345 \\ 3459476 \\ \hline 38919105 \end{array}$$

In the same Manner, the Numbers in the other Columns of Table VI. are to be used.

But farther, the Numbers of the said sixth Table will at first sight shew, how many Year's Purchase an Annuity to continue any Number of Years under 31 is worth, to be sold for present Money, compound Interest being computed on both sides, at any of the said Rates, 4, 5, 6, 7, 8, 9, 10, 11, and 12 *per cent.* So if you

To find how many Years Purchase an Annuity or a Lease for Years is worth.

desire to know how many Year's Purchase an Annuity issuing out of Lands for 21 Years, to begin presently, is worth, if it were to be sold for ready Money, when the current Rate of Interest is 6 *per cent.* Seek in the first Column of Table VI for 21 Years, and carry your Eye from thence equidistant to the Head-line of the Table, till you come under 6, which (as before has been said) signifies 6 *per cent.* So in the fourth Column you will find 11.76407, whereof you need only consider 11.76, which shews that the said Annuity is worth 11 Year's Purchase, (or 11 times one Year's Rent whatever it be) and 76 Parts of one Year's Purchase divided into 100 Parts, or $11 \frac{3}{4}$ Year's Purchase and little more. The same Annuity, when Money was at 8 *per cent.* was worth 10 Year's Purchase, and about $\frac{1}{10}$ Part of a Year's Purchase more, as the Number in the Column of 8 *per cent.* right against 21 Years will discover.

In like manner, supposing 10 *per cent.* to be a fit Rate to be allowed in the Valuation of Leases of Houses, the Lease of a House for 21 Years will be found by the said Table to be worth 8 Year's Purchase, and $\frac{64}{100}$ Parts of a Year's Purchase, or 8 Year's Purchase and an half, and half a quarter of a Year's Purchase, and somewhat more: But note here, that in valuing Leases, the Rate *per centum* is to be set higher or lower, according to the Goodness of the Thing leased, and the Certainty or Uncertainty of the Rent.

XVII. When

XVI. When a Sum of Money is proposed, and it is required to know what Annuity (to continue any Number of Years, and according to any given Rate) that Sum will buy; you may presuppose at Pleasure an Annuity for the Term of Years propounded, and find the Value of that Annuity in ready Money (according to the fifteenth Rule afore-going) at the Rate assigned; then will the Proportion be as follows:

Of the Purchase of Annuities at compound Interest.

As the Value found is in Proportion, to the supposed Annuity; so is the Sum of Money propounded, to the Annuity required.

So if it be required to find what Annuity, to begin presently, and to continue three Years, 500 *l.* in present Money will purchase, compound Interest being computed at 6 *per cent. per annum*: The Answer will be, 187 *l.* 1 *s.* 1 *d.* very near.

For presupposing an Annuity at Pleasure, to wit, 378,743⁰⁸⁸ *l.* payable yearly for 3 Years, the Value of it in present Money will (by the fifteenth Rule of this Chapter) be found to be 1012,3848 *l.* Therefore by the *Rule of Proportion*, say,

$$1012,3848 : 378,743088 :: 500 : 187,054.$$

That is to say, if 1012,3848 *l.* in ready Money will buy an Annuity of 378,743⁰⁸⁸ *l.* (to continue three Years then 500 *l.* in present Money will purchase an Annuity (to continue the same Term of Years, and at the same Rate of Interest) of 187,054, *£c.* that is, 187 *l.* 1 *s.* 1 *d.* very near

The Construction of the following Table VII.

Upon this Ground the following *Table VII.* is calculated, to shew what Annuity (to continue any Term of Years under 31, and at any Rate of Interest mentioned at the Head of that *Table*) one Pound will purchase; by which *Table*, and by the Help of *Multiplication*, Questions concerning the *Purchase of Annuities, Rents, or Pensions*, by any Sum of ready Money propos'd, may be resolved without considerable Errour. But a more ready Way to make the said *Table VII.* may be this following, *viz.*

Since

Since it is evident by the Construction of the third *Table* afore-going, that one Pound ready Money is equivalent to 1.06 *l.* payable at the End of a Year to come, at the Rate of 6 *per cent. per annum*; therefore this 1.06 is to be the first Number in the Column intitl'd 6 *per cent.* in the subsequent *Table VII.* Again, the present Value of one Pound Annuity to continue two Years at the said Rate, will be found by the preceeding *Table VI.* to be near 1,83339 *l.* Therefore by the Rule of Proportion, say,

$$1,83339 : 1 :: 1 : ,54543, \text{ \&c.}$$

That is, if 1,83339 *l.* ready Money will purchase an Annuity of 1 *l.* to continue two Years; what Annuity to continue the same time will 1 *l.* in present Money purchase? *Answ.* an Annuity of ,54543 *l.* that is, 10 *s.* 11 *d.* very near, to continue two Years; therefore the said Decimal ,54543 *l.* is to be placed as the second Number in the fourth Column of the subsequent *Table VII.* Hence it follows, that if 1 or Unity be divided by every one of the Numbers in all the Columns of *Table VI.* except the first Column of Years, the Quotients will give the respective Numbers to be placed in the like Columns of the following *Table VII.* in which Operation it will be requisite that the Numbers in the preceeding *Table VI.* be continued to more places than are there express'd, to prevent Error that will arise by the adding of defective *Decimals.*

TABLE

TABLE VII.

Which shows what Annuity, payable by yearly Payments, to continue any Term of Years under 31, one Pound will purchase, compound Interest being computed at any of these Rates, to wit, 4, 5, 6, 7, 8, 9, 10, 11, and 12 per cent. per annum.

Year.	4	5	6	7	8	9	10	11	12
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	.53019	.53780	.54543	.55309	.56076	.56846	.57619	.58393	.59169
3	.36034	.36720	.37411	.38105	.38803	.39505	.40211	.40921	.41634
4	.27549	.28209	.28859	.29519	.30192	.30866	.31547	.32232	.32923
5	.22462	.23097	.23739	.24389	.25045	.25709	.26379	.27057	.27740
6	.19076	.19701	.20336	.20979	.21631	.22291	.22960	.23637	.24322
7	.16660	.17281	.17913	.18555	.19207	.19869	.20545	.21221	.21911
8	.14852	.15472	.16103	.16746	.17401	.18067	.18744	.19432	.20130
9	.13449	.14069	.14702	.15348	.16007	.16679	.17364	.18060	.18767
10	.12329	.12950	.13586	.14237	.14902	.15582	.16274	.16980	.17698
11	.11414	.12038	.12079	.13335	.14907	.14694	.15396	.16112	.16841
12	.10655	.11282	.11927	.12590	.13269	.13965	.14676	.15402	.16143
13	.10010	.10645	.10296	.11965	.12652	.13356	.14077	.14815	.15567
14	.09466	.10102	.10758	.11434	.12129	.12843	.13574	.14322	.15087
15	.08994	.09634	.10296	.10979	.11682	.12405	.13147	.13906	.14682

A Continuation of the preceding Table VII.

Year.	4	5	6	7	8	9	10	11	12
16	.08581	.09226	.09895	.10585	.11298	.12029	.12781	.13551	.14339
17	.08219	.08869	.09544	.10242	.10962	.11704	.12466	.13247	.14056
18	.07899	.08554	.09235	.09941	.10670	.11421	.12192	.12984	.13793
19	.07613	.08274	.08962	.09675	.10412	.11173	.11954	.12756	.13576
20	.07358	.08024	.08718	.09436	.10184	.10954	.11745	.12557	.13387
21	.07128	.07799	.08500	.09228	.09983	.10761	.11562	.12383	.13224
22	.06919	.07597	.08304	.09040	.09803	.10590	.11400	.12231	.13081
23	.06739	.07413	.08127	.08871	.09642	.10438	.11257	.12097	.12955
24	.06655	.07247	.07967	.08718	.09497	.10302	.11126	.11978	.12846
25	.06401	.07095	.07822	.08581	.09367	.10180	.11016	.11874	.12749
26	.06256	.06956	.07690	.08455	.09250	.10071	.10915	.11781	.12665
27	.06123	.06829	.07569	.08342	.09144	.09973	.10825	.11698	.12590
28	.06001	.06712	.07459	.08239	.09048	.09885	.10745	.11625	.12524
29	.05887	.06604	.07357	.08144	.08961	.09805	.10672	.11565	.12465
30	.05783	.06496	.07264	.08058	.08882	.09733	.10607	.11502	.12414

The Use of the preceeding Table VII.

Quest. What Annuity or yearly Rent issuing out of Lands, to begin presently, and to continue 14 Years, will 320 *l.* purchase, compound Interest being reckoned on both sides, at the Rate of 6 *per cent. per annum*? *Ans.* 34 *l.* 8 *s.* 6 *d.* very near, which is thus found out, *viz.* In the 4th Column of the aforegoing *Table VII.* under 6 at the Head of that Column, and right against 14 Years you'll find this Decimal .10758, which shews that 1 *l.* ready Money will purchase an Annuity of .10758 *l.* (that is 2 *s.* 1 *d.* 2 *f.* +;) therefore multiplying the said Decimal .10758 by the said 320, the Product (according to the 2d Rule of the 26th Chapter of the preceeding Book) will be found to be 34.425, &c. that is 34 *l.* 8 *s.* 6 *d.* very near, which is the *Answer* of the *Question*.

$$\begin{array}{r}
 1 : 10758 :: 320 : (34.425 + \\
 \quad \quad \quad 320 \\
 \hline
 \quad \quad 215160 \\
 \quad 32274 \\
 \hline
 34.42560
 \end{array}$$

In like manner, if 10 *per cent.* be thought a fit Rate of Interest to be allowed in purchasing Leases of Houses; 500 *l.* will buy a present yearly Rent of 63 *l.* 18 *s.* 1 *d.* payable for 16 Years out of a House. For under 10 at the Head of the 8th Column, and right against 16 Years (in the preceeding *Table VII.*) you'll find this Decimal .12781, which being multiplied by 500, (the Number of Pounds proposed to purchase the Lease,) the Product will be found to be 63.90500, that is, 63 *l.* 18 *s.* 1 *d.* +, as by the subsequent Operation is manifest.

$$\begin{array}{r}
 1 : .12781 :: 500 : (63.905 \\
 \quad \quad \quad 500 \\
 \hline
 \quad 63.90500
 \end{array}$$

XVII. Upon the same Foundations which have been laid in the

the 12, 13, 14, 15, and 16th Rules of this Chapter, for the making of Tables which respect yearly Payments; Tables may be made for half-yearly and quarterly Payments, the Interest of 100 *l.* for $\frac{1}{2}$ Year, and likewise for $\frac{1}{4}$ Year being first agreed upon: For if we suppose that at the Rate of 6% for 100 *l.* for a Year, the Interest of 100 *l.* for $\frac{1}{2}$ Year is 3 *l.* the Numbers 100 and 103 are to be used in the same manner to calculate *Tables* for half-yearly Payments, as the Numbers 100 and 106 have been before made use of to form *Tables* for yearly Payments. But if at the Rate of 6 per cent. per annum, the Interest of 100 *l.* for $\frac{1}{2}$ Year ought to be such, that being added to the said Principal 100 *l.* and the Whole put forth at Interest for the next half Year, at the said Rate, the Sum then due (to wit, at the Year's End) must exactly amount to 106 *l.* In this case, a Geometrical mean proportional Number between the Extremes 100 and 106 must be sought, which mean will (by the following 18th Rule) be found to be near 102.956301 +; and then the Numbers 100 and 102.956301, &c. are to be used instead of the Numbers 100 and 106 in manner aforesaid. In the like case, if it be supposed that at the Rate of 6 per cent. per ann. the Interest of 100 *l.* for $\frac{1}{4}$ Year is 1 *l.* 10 s. or 1.5 *l.* the Numbers 100 and 101.5 are to be used for the calculating of *Tables* for quarterly Payments, after the same manner as the Numbers 100 and 106 for yearly Payments. But if at the Rate of 6 per cent. per ann. the Interest of 100 *l.* for $\frac{1}{4}$ Year ought to be such, that being added to the said 100 *l.* and the Whole put forth at the same Rate of Interest for the next $\frac{1}{4}$ of a Year, and in that manner for the third and fourth Quarters, and that the Sum due at the Year's End must exactly amount to 106 *l.* In this case, a Series or Rank of five Numbers in Geometrical Proportion continued must be considered, viz. the Principal 100 *l.* (which is the lesser of the two extreme Proportionals;) the three Sums (composed of Principal and Interests) due at the End of the first, second, and third Quarters of the Year, (which are the three mean Proportionals) and 106 *l.* due at the Year's End, (which is the greater of the two extreme Proportionals;) now between the said Extremes 100 and 106, the first, (to wit, the least) of the said three mean Proportionals is to be sought, which (by the following 20th Rule of this Chapter) will be found to be near 101.4673 +. And then the Numbers 100 and 101.4673, &c. are to be made use of instead of the Numbers 100 and 106 in manner aforesaid.

*The making
of Tables for
half-yearly
and quarterly
Payments.*

To find a Geometrical mean proportional Number, between two Numbers given.

XVIII. Two Numbers being given, to find a Geometrical mean Proportional between them; multiply the two given Numbers one by the other, and extract the square Root of the Product, so is such square Root the mean Proportional sought: For Example, if 8 and 18 are two Numbers given, and it is required to find a mean Number geometrically Proportional between them, multiply 18 by 8, so is the Product 144, whose square Root is 12 for the mean Proportional sought; so that 8, 12, and 18, are three Numbers in Geometrical Proportion continued, viz. As 8 is in proportion to 12, so is 12 to 18. In like manner, a Geometrical mean Proportional between the Extremes 100 and 106, will be found near 102.956301 +.

XIX. Two Numbers being given, to find the first of two Geometrical mean proportional Numbers between the Extremes given; multiply the Square of the lesser Extreme by the greater, and extract the Cube-root of the Product, so is such Cube-root the lesser of the two mean Proportionals required: For Example, if 8 and 27 are assigned for two Extremes, the lesser mean will be found 12; for according to the Rule, the Square of 8 the lesser Extreme is 64, which being multiplied by 27 (the greater Extreme) produces 1728, whose Cube-root is 12 the lesser Mean sought: Then may the greater Mean be found more easily by the Rule of Three, for $8 : 12 :: 12 : 18$, so that 12 and 18 are two Means, geometrically proportional between the Extremes 8 and 27, viz. these four Numbers are in Geometrical Proportion continued, to wit, 8, 12, 18 and 27.

XX. Two Numbers being given, to find the first of three Geometrical mean Proportionals between the Extremes given; multiply the Cube of the lesser Extreme by the greater, and extract the Biquadrate Root of the Product, so is such Biquadrate Root the first, to wit (the least) of the three mean Proportionals required: For Example, if 2 and 32 are two Extremes given, the first and least of three Geometrical mean Proportionals will be found to be 4: For (according to the Rule) the Cube of 2 (the lesser Extreme given) is 8, which being multiplied by 32 the greater Extreme, produces

duces 256, the Biquadrate Root of which being extracted (according to the 29th Rule of the 33d Chapter of the preceeding Treatise) gives 4 for the first and least of the three Means sought; the other Means may be easily found by the Rule of Three, for

$$2 : 4 :: 4 : 8 :: 8 : 16 :: 16 : 32.$$

So that these five Numbers will appear to be in Geometrical Proportion continued, to wit,

$$2 \quad . \quad 4 \quad . \quad 8 \quad . \quad 16 \quad . \quad 32$$

In like manner, the first and least of three *Geometrical mean Proportionals* between the Extremes 100 and 106, will be found to be near 101.4673, &c. Thus I have shewed the most easy Methods (raised from clear Grounds) to make Tables for the Resolution of the usual Questions, which depend upon the Computation of Interest, by the Help of *Multiplication* only.

Questions to exercise the preceeding Tables, with their Use in solving Questions of the same Nature, when the Number of Years exceeds 30.

Quest. 1. If the Lease of an House be worth 153 *l. Fines*, and 16 *l. yearly Rent*, payable yearly for 21 Years, and the *Lessee* be desirous to bring down the *Fine* to 50 *l.* and so to pay the more *Rent*, the Question is, what *Rent* the Tenant shall pay, accounting compound Interest at the Rate of 10 *per cent. per annum*? *Answ.* 27 *l.* 18 *s.* 1 $\frac{1}{4}$ *d.* very near.

First, find the difference between the *Fines*, which is 103 *l.* Then after the manner of the Examples of the Use of the preceeding *Table VII*, seek what *Annuity* or *Rent* to continue 21 Years, 103 *l.* ready Money will purchase at 10 *per cent.* so will you find 11 *l.* 18 *s.* 1 $\frac{1}{4}$ *d.* which being added to the old *Rent* 6 *l.* gives 27 *l.* 18 *s.* 1 $\frac{1}{4}$ *d.* which the Tenant must pay, to the End that the *Fine* may be diminished to 50 *l.*

Quest. 2. There is a Lease of certain Lands to be let for 14 Years for 250 *l. Fines*, and 44 *l. Rent per annum*, payable yearly; but the Tenant is desirous to pay less *Rent*, viz. 20 Pounds *per ann.* and to give a greater *Fine*; the Question is, what *Fine* ought to be paid to bring down the *Rent* to 20 *l. per annum*, accounting compound Interest, at the Rate of 6 *per cent. per annum*? *Answ.* 473 *l.* 1 *s.* 7 *d.*

First find the difference between the *Rents*, which is 24 Pounds *per annum*. Then by means of the preceeding *Table VI.*

VI. seek what Annuity or Rent 24*l.* *per ann.* to continue 14 Years, is worth in ready Money at 6 *per cent. per annum*, so you will find 223 *l.* 1*s.* 7*d.* which being added to the first Fins 250 Pounds, gives 473 *l.* 1*s.* 7*d.* which the Tenant must pay, to the End that the Rent may be brought down to 20*l.* *per annum*.

Quest. 3. There is a Lease of certain Lands worth 32*l.* *per annum*, more than the Rent paid to the Lord for it, of which Lease seven Years are yet in being, and the Lessee is desirous to take a Lease in *Reversion* for 21 Years, to begin when his old Lease is expired, the Question is, What Sum of Money is to be paid for this Lease in *Reversion*, accounting compound Interest at the Rate of 6 *per cent. per an.*? *Ans.* 250 *l.* 7*s.* 2*d.* +

First, by adding the 7 Years of the Lease in being to the 21 Years you would have in *Reversion* after those 7 are expired, the Sum is 28. Then by the preceeding Table VI.

The present Worth of 1 <i>l.</i> Annuity, for 28 Years at 6 <i>per cent.</i> compound Interest, is	} 13.40610
Likewise the present Worth of 1 <i>l.</i> Annuity for 7 Years -- -- --	} 5.58233
Therefore the Difference of those present Worths, will be the present Value of 1 <i>l.</i> Annuity for 21 Years in <i>Reversion</i> after seven Years	} 7.82383
Which multiplied by 32 (the yearly Rent pro- pounded) gives the Answer of the Question	} 250.36256

Otherwise thus.

First, by the Help of the said Table VI. find how much 32*l.* yearly Rent for 21 Years is worth in ready Money, as if the 21 Years were to begin presently, at the Rate of 6 *per cent.* which ready Money will be found 376.45024 *l.* Then by Table V. find what 376.45024 *l.* due at the End of 7 Years to come, is worth in ready Money, so will it be 250 *l.* 7*s.* 2*d.* which agrees with the Answer before found.

Quest. 4. One would bestow 630 *l.* to purchase a present yearly Rent or Annuity of 60 *l.* to be paid by yearly Payments; the Question is to know how many Years the said Annuity must continue, compound Interest at 6 *per cent. per ann.*, being allow'd on both sides? *Ans.* 17 Years. and 23 days, very near.

First, I divide 630 by 60, the Quotient is 10.5, which shews that 10 Years Purchase and a half are given for the Annuity; then

then searching for 10.5 in *Table VI.* in the Column of 6 *per cent.* I find it not exactly, but the nearest less than it is 10.47725, standing right against 17 Years and the next greater than 10.5 is 10.82760 which is placed against 18 Years: Whence I infer that the Annuity must continue 17 Years and more, yet less than 18 Years. Now the proportional Part of a Year to be added to 17 Years, may be found out near enough for use, thus, *viz.* subtract the said lesser tabular number 10.47725 from the greater 10.82760, so the Remainder will be found .35035: Also subtracting the said 10.47725 from 10.5 (the Quotient first found) the Remainder will be .02275; then say by the Rule of Three in Decimals, as .35035 the greater remainder is to .02275 the lesser; so is 1 Year (the difference between 17 and 18 Years) to .0649 Parts of a Year, or 23 days + (as will appear by the fourth *Rule* of the 26th *Chapter* of the preceeding *Book*;) therefore the Number of Years sought by the Question is 17 Years, 23 Days.

Quest. 5. If an Annuity of 96 *l.* payable by yearly Payments for 14 Years be sold for 826 *l.* what Rate of Interest *per cent.* is implied in that Bargain? *Ans.* 7 *l.* 5 *s.* 7½ *d.* near.

First, dividing 826 by 96, the Quotient is 8.60416, which shews how many Year's Purchase was given for the Annuity; then searching for 8.60416 in *Table VI.* in a right Line passing from 14 Years, equi-distant to the Head-line of the *Table*, I find it not exactly, but the nearest less than it is 8.24423 (which stands in the Column of 8 *per cent.* and the nearest greater is 8.74546 (which stands in the Column of 7 *per cent.*) Whence I infer, that the Rate of Interest required is between 7 and 8 *per cent.* and the proportional Part of 1 *l.* to be added to 7 *l.* may be found out near enough for practice thus, *viz.* subtract the said lesser tabular Number 8.24423 from the greater 8.74546, the Remainder will be .50123. Also subtract 8.60416, (the Quotient first found, which falls between the said tabular Numbers,) from the said greater tabular Number 8.74546, the Remainder will be 14130: Then say by the *Rule of Three* in Decimals, as 50123 the greater Remainder, (or difference between two two tabular numbers) is to 14130 the lesser Remainder, so is 1 *l.* the difference between 7 *per cent.* and 8 *per cent.* to 2819, 83 *c.* or 5 *s.* 7 *d.* 2 *f.* which added to 7 *l.* gives 7 *l.* 5 *s.* 7 *d.* 2 *f.* which is near the Rate of Interest *per cent.* required.

Quest. 6. If a Year's Rent, or one Year's Purchase, be paid as a *Fine*, for renewing or adding 7 Years to 14 yet to come of an old Lease for 21 Years, and accordingly a new Lease be taken
for

for 21 Years, to begin presently (which proportion is ordinarily observed by *Bishops, Deans, and Chapters, Heads and Fellows of Colleges* in letting Leases of their Lands) what Rate of Interest *per cent.* is implied in that Agreement? *Ans.* 11*l.* 11*s.* 8*d.* 1*f.* and somewhat more.

To solve this Question, first I search in the preceeding *Table VI.* to find out two Numbers so seated in some one Column of Interest, that one of them may stand right against 14 Years, and the other against 21 Years, and so qualified that the Difference between them may be exactly 1 or Unity; but not finding any two Numbers precisely answering those Conditions, I take those Numbers that come nearest, which will be found in the Columns of 11 and 12 *per cent.*; for the difference between the Numbers 6.98186 and 8.07507, which stand in the Column of 11 *per cent.* right against 14 Years and 21 Years, is 1.09321, which exceeds 1 (that is one Years Purchase) by .09321; also the difference between 6.62816 and 7.56200 which stand in the Column of 12 *per cent.* right against 14 Years and 21 Years, is .93384, which wants .06616 of 1: Therefore I divide 1 *l.* (the Difference between 11*l.* and 12*l.* *per cent.*) into two Parts, in such proportion one to the other, as the said Decimals .09321 and .06616 are one to the other; so I find the said parts of 1 *l.* to be near .5848 and .4151; or 11*s.* 8*d.* 1*f.* + and 8*s.* 3*d.* 2*f.* +; the former of which being added to 11 *per centum.* or the latter being subtracted from 12 *l. per cent.* gives 11.5848 *l.* or 11*l.* 11*s.* 8*d.* 1*f.* +, which is very near the rate of Interest required by the Question.

Quest. 7. What is the present Worth of 1 *l. per ann.* payable yearly for 10 Years, compound Interest being computed at the Rate of 11.5848 *l. per cent.*? *Answer* 5*l.* 15*s.* 0*d.* very near, which is found out by means of the preceeding *Table VI.* in this manner, *viz.*

The tabular Number for 10 Years at 11 <i>l. per</i>	}	5.88923
<i>centum,</i> is ---		
The tabular Number for 10 Years at 12 <i>l. per</i>	}	5.65022
<i>centum,</i> is ---		
Their Difference is ---		0.23901

Then say by the *Rule of Three* in Decimals, as 1 *l.* the (difference between 11 and 12 *per cent.*) is to .5848 *l.* (to wit, the Decimal by which the given Rate in the Question exceeds 11 *per cent.*) so is .23901 (the Difference found out as above)

to .13977 +, which .13977 +, being subtracted from 5.88923, (the greater of the two tabular Numbers above-mentioned,) there will remain 5.74946, or 5 *l.* 15 *s.* 0 *d.* which is near the present Worth of one Pound yearly Rent, to continue 10 Years, at the proposed Rate of 11.5848 *l.* *per cent.*

After the same manner the present Worth of 1 *l.* yearly Rent, payable for 21 Years, at the same Rate of Interest, will be found to be 7.77503 *l.* or 7 *l.* 15 *s.* 6 *d.* very near, from which, if you subtract 5.74946, (being the afore-mentioned present Worth of 1 *l.* yearly Rent for 10 Years) there will remain 2.02557, or 2 *l.* 0 *s.* 6 *d.* which is near the present Worth of a Lease of 1 *l.* Rent *per annum*, for 11 Years in Reversion, to begin after 10 Years yet to come in a Lease are expired: Hence it is evident, that if a Tenant to a College has 10 Years yet to come in a Lease, at 1 *l.* Rent *per annum*, and desires to have 11 Years renewed or added to those 10, and so takes a new Lease for 21 Years, to begin presently at the same Rent, he must give 2 *l.* 0 *s.* 6 *d.* or two Year's Purchase and $\frac{1}{2}$ Part of a Year's Purchase, very near, (according to the fundamental Proportion before assumed in the sixth Question.) The like may be done for any other Term of Years under 30, by the Help of the said *Table VI.*

But yet by a *Table* calculated purposely for the said Rate of 11.5848 *l.* *per cent.* (according to the fifteenth *Rule* of this *Chapter*) Questions of the same Kind with the two last may be more easily answered; and therefore (for that they come often in practice) I shall here insert such a *Table*, as I find it ready calculated to my Hand by Dr. *Newton*, in his *Scale of Interest* lately published, which *Table* is to be used in every respect like to the preceding *Table VI.* and will be very ready and useful, for the proportioning of Fines, in the renewing of Leases held from Cathedral Churches and Colleges, as will be manifest by the manner of solving the two following Questions.

Concerning
the renewing
of a College
Lease of
Lands.

Quest. 8. If a College-Tenant has 7 Years to come or unspent in a Lease of Lands for 21 Years, at 1 *l.* yearly Rent, and desires to have 14 Years renewed or added to those 7 Years, and so to take a new Lease for 21 Years to begin presently, what must he pay for a Fine? *Ans.* 3 *l.* 3 *s.* 0 *d.*

The Rule for finding out the Answer of the Question proposed, and such like, is this, *viz.*

From 7.77507 (being the Number which answers to 21 Years in this *Table VIII.*) always subtract the tabular Number which belongs to the Number of Years to come or unspent in the old Lease; so the Remainder will shew what Fine must be paid for the Years to be renewed or added, to make those unspent Years in the old Lease to be 21 Years compleat again, at 1 *l.* yearly Rent.

So to solve the Question proposed.

TABLE VIII.
Shewing the present Worth of one Pound Annuity for any Number of Years under 22, at the Rate of 11. 11 s. 8 d. 3 $\frac{1}{4}$ f. per cent. compound Interest.

Years	present Worth.
1	0.90034
2	1.69938
3	2.41922
4	3.06438
5	3.64262
6	4.16088
7	4.62540
8	5.04176
9	5.41496
10	5.74948
11	6.04934
12	6.31819
13	6.55907
14	6.77507
15	6.96868
16	7.14226
17	7.29786
18	7.43737
19	7.56243
20	7.67455
21	7.77507

From the present Worth of 1 *l.* yearly Rent for 21 Years, which is 7.77507
 Subtract the present Worth of the same Rent for 7 Years, (that were unspent in the old Lease) 4.62540
 And there will remain the Fine sought, to wit--- 3.14967

That is to say, 3.14967 *l.* or 3 *l.* 3 *s.* 0 *d.* very near, must be paid as a Fine, for renewing or adding 14 Years to 7 Years, that were unspent in the old Lease, the yearly Rent being 1 *l.* Also the said 3.14967 shews, that such a Renewal is worth 3 Year's Purchase, and near $\frac{1}{4}$ Parts of a Year's Purchase (whatever the Rent be.)

Quest.

Quest. 9. If a Tenant that has 17 Years yet to come in a Lease of Lands held of a College for 21 Years, at 50 *l.* yearly Rent, be desirous to renew 4 Years, and so make those 17 Years to be 21 Years compleat again at the same Rent, what must he give for a Fine? *Answ.* 23 *l.* 17 *s.* 2 *d.* 1 *f.* For according to the Rule before given,

From the present Worth of 1 <i>l.</i> yearly Rent for } 21 Years	7.77507
Subtract the present Worth of the same Rent for } 17 Years (that were unspent in the old Lease.)	7.29786
And there will remain	0.47721
Which multiplied by the Rent	50

The Product will be the Fine sought, to wit, 23 *l.* }
17 *s.* 2 *d.* 1 *f.* } 23.86050

Questions of this Nature may be readily solved without the Loss of one sixteenth Part of a Year's Purchase, by the Help of the following *Table IX*, which I have drawn from the foregoing *Table VIII*, for the Benefit of such as do not understand decimal Fractions: For example, if a College-Tenant desires to have 10 Years added to 11 Years that are to come or unspent in a Lease of Lands, that he may have a new Lease for the Term of 21 Years to begin presently, the following *Table IX*. shews that he must give for a Fine 1 Year's Purchase, and 2 Quarters of a Year's Purchase, and 3 Quarters of a Quarter of a Year's Purchase, *viz.* one Year's Rent, and half a Year's Rent, and three Quarters of a Quarter of a Year's Rent: Supposing then the Rent to be 48 *l. per. ann.* the Fine may be computed thus:

	<i>l.</i>	<i>s.</i>	<i>d.</i>
One Year's Rent is	48	00	00
Half a Year's Rent is	24	00	00
Three Quarters of a Quarter of a Year's Rent is--	9	00	00
The Sum is the Fine required	81	00	00

Whence it appears that the Tenant must give 81 *l.* as a Fine, for adding 10 Years to 11 Years that were unexpired in his old Lease, to the end he may have a new Lease for 21 Years in being.

In like manner the following *Table IX*. shews that the Fine for renewing or adding 7 Years to 14 Years that are unspent in a Lease of Lands, to the End there may be a new Lease for 21 Years in being, is valued at one Year's Purchase precisely, which is the fundamental Proportion assumed in calculating the foregoing *Table VIII*, as before was said.

TABLE IX.

The Fine for renewing or adding

Years-----		Years-----		Year's Purchase.		Quarters of a Year.
1 to 20	}		is valued at	0	:	0 : 1
2 to 19				0	:	0 : 3
3 to 18				0	:	1 : 1
4 to 17				0	:	1 : 3
5 to 16				0	:	2 : 2
6 to 15	}		is valued at	0	:	3 : 0
7 to 14				1	:	0 : 0
8 to 13				1	:	0 : 3
9 to 12				1	:	1 : 3
10 to 11				1	:	2 : 3
11 to 10	}		is valued at	2	:	0 : 0
12 to 9				2	:	1 : 1
13 to 8				2	:	2 : 3
14 to 7				3	:	0 : 2
15 to 6				3	:	2 : 1
16 to 5	}		is valued at	4	:	0 : 2
17 to 4				4	:	2 : 3
18 to 3				5	:	1 : 1
19 to 2				6	:	0 : 1
20 to 1				6	:	3 : 2

The

The like may be done for renewing any other Term of Years under 21, at any Rent proposed.

But because it may sometimes happen, that the Number of Years in Questions belonging to the preceeding 3, 4, 5, 6, and 7 Tables may exceed 30, I shall by the five following Questions shew, how by means of those Tables the Answer to any Question of that Nature may be found out near the Truth, when the Term of Years is above 30.

*Of finding
out Tabular
Numbers for
any Term of
Years above
30.*

Quest. 10. If 340 *l.* be put forth at 4 *per cent.* compound Interest, and both Principal and Interest is forborn until the End of 45 Years, what will then be due? *Answ.* 1986 *l.* very near.

To resolve this Question and the like, observe this Rule, *viz.* First make choice of such Numbers of Years in Table III, that if they be added together, will make the Number of Years proposed in the Question, as 17 and 28, or 15 and 30, each of which Pairs make 45; then looking into Table III. in the Column belonging to 4 *per cent.* you'll find, right against 17 and 28 Years, these Numbers, 1.94790 and 2.99870, which being multiplied one by the other, will produce 5.84116 $\frac{1}{2}$, or 5 *l.* 16 *s.* 10 *d.* which is the Encrease of 1 *l.* forborn 45 Years at 4 *per cent.* compound Interest; therefore multiplying the said 5.84116 by 340, the Product will give 1985.994, *£c.* or 1986 *l.* very near, for the Answer of the Question.

The Reason of the said Rule will be manifest by this Theorem, *viz.* If there be a Rank of Numbers in Geometrical Proportion continued, beginning with 1 or Unity, as 1, 2, 4, 8, 16, 32, 64, 128, *£c.* Also if the first Term 1 be cast away, and over or under all the Rest of the Terms there be placed another Rank of Numbers, beginning at 1 and proceeding according to the natural Order of Numbers, as 1, 2, 3, 4, 5, 6, 7, *£c.* which may be called the *Indices* of those in the first Rank, after the first Term 1 is cast away: I say, if any two of these remaining Geometrical Proportionals be multiplied one by the other, the Product will be a Proportional correspondent to that *Index*, which is equal to the Sum of the *Indices* answering to the two Proportionals that were multiplied one by the other.

Proport. 2 . 4 . 8 . 16 . 32 . 64 . 128

Indices. 1 . 2 . 3 . 4 . 5 . 6 . 7

So if 4 and 32, which are the second and fifth Proportionals in the upper Rank, be multiplied one by the other, the Product

is

is 128, which will be the seventh Proportional, because the Sum of the *Indices* 2 and 5, which answer to the said 4 and 32, is 7. In like manner, because the Sum of the *Indices* 3 and 4 is 7, therefore if the third and fourth Proportionals, to wit, 8 and 16, be multiplied one by the other, the Product will also give the seventh Proportional 128. Now since the Numbers in every one of the Columns, except the first Column of Years, in the preceeding *Table III*, are continual Proportionals whose first Term is 1, but it is excluded out of the said Columns, as appears by the Construction of that *Table*, and for that the Number of Years 1, 2, 3, 4, 5, &c. are placed as *Indices*, shewing the Order or Seat of those Proportionals inserted in the Columns, therefore the Rule before given for continuing that *Table* to any Number of Years is manifest.

Quest. 11. If one Pound be due or payable 50 Years hence, what is it worth in ready Money, by rebating at 5 *per cent. per annum*, compound Interest? *Ans.* .08720, £*c.* or 1*s.* 9*d.* +, which is found out by the Help of *Table V*, in the same manner as the Answer to the last Question; (respect being had to the second and third Rules of the 26th Chapter of the preceeding Book, concerning the Multiplication of decimal Fractions.)

Quest. 12. If an Annuity of one Pound payable yearly for 40 Years, be all forborn until the End of that Term, what will it then amount to, compound Interest being computed at 5 *per cent. per annum*? *Ans.* 120*l.* 16*s.* 0*d.* thus found out: First, according to the second way of calculating the fourth *Table* in the thirteenth *Section* of this *Chapter*, find out a Principal, which may have such proportion to the proposed Annuity 1*l.* as 100*l.* has to 5*l.*; saying, if 5*l.* Interest has 100*l.* for a Principal, what Principal must 1*l.* Interest have? *Ans.* 20*l.* Secondly, seek (after the manner of the preceeding tenth Question) what 20*l.* will be augmented to being forborn 40 Years, at the Rate of 5 *per cent. per ann.* compound Interest, so you'll find 140.798 +, from which subtracting the said Principal 20*l.* the Remainder will be 120.798 +, or 120*l.* 6*s.* which is the Answer of the Question.

Quest. 13. If an Annuity of one Pound payable yearly for 37 Years is to be sold for present Money, what is it worth, compound Interest being computed on both sides at 6 *per cent. per annum*? *Ans.* 14*l.* 14*s.* 9*d.* which is found out thus: First, according to the second Method of calculating the sixth *Table* in the fifteenth *Section* of this *Chapter*, find out a Principal

Principal in such proportion to one Pound (the proposed Annuity) as 100 is to 6, so will such Principal be found 16.66666 +; then after the manner of the preceeding eleventh Question, find out the ready Money which is equivalent to 16.66666, due 37 Years hence, so will such ready Money be found to be 1.92988 +, (or 1 *l.* 18 *s.* 7 *d.*) which being subtracted from the said Principal 16.66666, the Remainder will be 14.73678 +, or 14 *l.* 14 *s.* 9 *d.* which is the Answer of the Question propounded.

Quest. 14. What Annuity payable by yearly Payments to continue 37 Years will one Pound purchase at 6 *per cent. per annum*, compound Interest? *Answ.* 1 *s.* 4 *d.* near, which is discovered thus: First, find out the present Worth of one Pound Annuity to continue 37 Years, which present Worth (by the last Question) will be found 14.73678 *l.* Then say by the *Rule of Three*, if 14.73678 *l.* will purchase an Annuity of one Pound, (to continue 37 Years) what Annuity to continue the same Term will 1 *l.* purchase? *Answ.* .06785 +, or 1 *s.* 4 *d.* which is the Answer of the Question proposed.

CHAP. VI.

A Demonstration of the Rule of Three, or Rule of Proportion.

I. **F**OUR Numbers are said to be Proportional, when the first contains the second, so often as the third contains the fourth; likewise when the first is such Part of the second, as the third is of the fourth; so these Numbers following are called Proportionals, *viz.*

$$\begin{array}{l} 4 \times 6 : 6 :: 4 \times 9 : 9 \\ \frac{2}{3} \times 12 : 12 :: -\frac{2}{3} \times 15 : 15 \end{array}$$

That is to say, 4 times 6 (or 24) is said to have such Proportion to 6, as 4 times 9 (or 36) has to 9. In like manner, $\frac{2}{3}$ of 12 (or 8) bears such Proportion to 12; as $\frac{2}{3}$ of 15 (or 10) does to 15.

II. When four Numbers are Proportionals, the Product arising from the Multiplication of the two Extremes is equal to the Product of the two Means.

Demon-

Demonstration.

By the preceeding Definition 1, these four Numbers are Proportionals, viz.

$$\begin{array}{l} 4 \times 6 : 6 :: 4 \times 9 : 9 \\ B \times C : C :: B \times D : D \end{array}$$

The Product of the two Extremes is--

The Product of the two Means is --

$$\text{But } \left\{ \begin{array}{l} 4 \times 6 \times 9 \\ B \times C \times D \end{array} \right\} = \left\{ \begin{array}{l} 6 \times 4 \times 9 \\ C \times B \times D \end{array} \right\}$$

Therefore the Proposition is manifest.

Likewise,

By the preceeding Definition these four Numbers are Proportionals, viz.

$$\frac{2}{3} \times 12 : 12 :: \frac{2}{3} \times 15 : 15$$

The Product of the two Extremes is --

The Product of the two Means is --

$$\text{But } \frac{2}{3} \times 12 \times 15 = 12 \times \frac{2}{3} \times 15$$

Wherefore the Proposition is every way proved.

III. From the last Proposition arises the *Rule of Proportion*, commonly called the *Rule of Three*, or *Golden Rule*, which teaches by three Numbers given to find a fourth Proportional Number in this manner, viz. *Multiply the second and third Numbers mutually one by the other, and divide the Product by the first Number ; so the Quotient will be the fourth proportional Number sought, in a direct Proportion.* This Rule has been fully exemplified in the 8th Chapter of the preceeding Book, and the Truth of the said Rule may be thus demonstrated, viz. Let there be three Numbers given to find a fourth in direct Proportion, viz. if 24 gives 6, what will 36 give ? Or as 24 is in proportion to 6, so is 36 to a fourth proportional Number sought for, which fourth Proportional (whatsoever

it be) we may suppose to be Q , and then these four Numbers will be Proportionals, *viz.*

$$24 : 6 :: 36 : Q$$

Therefore by the second Proposition of this Chapter,

$$24 \times Q = 6 \times 36$$

And because if equal plain Numbers be severally divided by one and the same Number, the Quotients will necessarily be equal between themselves; therefore

$$Q = \frac{6 \times 36}{24}$$

Whereby it is manifest, that the fourth proportional Number is equal to the Quotient, that arises by dividing the Product of the Multiplication of the second and third Proportionals by the first, which was to be proved.

Note, That every *Rule of Three inverse* may be made a *Rule of Three direct*, by making the third Term the first, and by proceeding forward to the other two Terms; therefore one and the same Demonstration serves for both Rules.

CHAP. VII.

A Demonstration of the Double Rule of Fellowship.

THE *Double Rule of Fellowship* (commonly called the *Rule of Fellowship with Time*) presupposes two Things, *viz.* 1. That the particular Stocks of Merchants in Company, have continued unequal Spaces of Time in the common Stock. 2. That at the End of their Partnership, the total Gain or Loss is to be divided among them, in such manner, that their Shares

Q may

may have such Proportion between themselves, as those Sums of Interest-money have one to another, which at any Rate *per centum*, simple Interest only being computed, might be gained by the particular Stocks, within the respective Times of their continuance in the common Stock: Now for the effecting of such a proportional Partition, the said *Double Rule of Fellowship* gives this Direction, *viz.* Divide the total Gain or Loss into such Parts, which have the same Proportion one to the other, as is between the Products arising from the Multiplication of each particular Stock by its correspondent Time.

For *Example*, Suppose two Merchants, A and B, to be Partners in Traffick, for a certain Time first agreed on between them, and that A permits his Stock of 100 *l.* to be employed in their joint Traffick three Months, and that B forbears his Stock of 50 *l.* eight Months; I say, (according to the said *Rule of Fellowship* with Time) whatever the total Gain or Loss be, that Part of it which belongs to A must have such Proportion to the Gain or Loss of B, as 100×3 (or 300) has to 50×8 (or 400.) This Rule has been fully exemplified in the 13th Chapter of the preceeding Book, and the Truth thereof, taking the two premised Suppositions for granted, may be thus demonstrated.

1. Supposing 100 *l.* (the Stock of A) to gain in three Months any certain Sum of Money, as two Pounds; I seek how much 50 *l.* (the Stock of B) will gain in the same Time, and at the

said Rate: So I find $\frac{2 \times 50}{100}$ *l.* for,

$$100 : 2 :: 50 : \frac{2 \times 50}{100}$$

2. Having found what 50 *l.* will gain in three Months, I seek how much the said 50 *l.* will gain in

eight Months at the same Rate, and so I find $\frac{2 \times 50 \times 8}{100 \times 3}$ *l.*

$$\text{for } \frac{3}{2} : \frac{2 \times 50}{100} :: \frac{8}{1} : \frac{2 \times 50 \times 8}{100 \times 3}$$

3. Thus

3. Thus it appears, that if 100 *l.* in three Months gains 2 *l.* then 50 *l.* in eight Months will gain at the

rate of $\frac{2 \times 50 \times 8}{100 \times 3}$; so that the Proportion of the Gain of A to the Gain of B is,

$$\text{As } 2 \text{ is to } \frac{2 \times 50 \times 8}{100 \times 3}.$$

4. If both the Terms (to wit, the *Antecedent* and *Consequent*) of the said Proportion be severally multiplied by the said *Denominator* 100×3 , the Products will be in the same Proportion with the Numbers of Terms multiplied, (by 17 *e.* 7 *Euclid.*) viz. the Gain of A will be to the Gain of B,

$$\text{As } 1 \times 100 \times 3 \text{ is to } 2 \times 50 \times 8$$

5. Lastly, Because 2 (the supposititious Gain first assumed) is a Multiplier as well in the *Antecedent* as in the *Consequent* of the last mentioned Proportion, it may be expunged out of both, and so the Gain of A will be to the Gain of B in this Proportion, (which was to be proved) to wit,

$$\text{As } 100 \times 3 \text{ is to } 50 \times 8$$

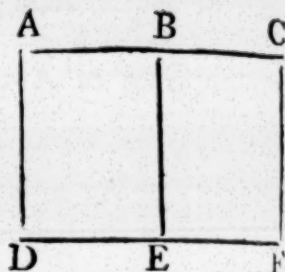
CHAP. VIII.

A Demonstration of the Rule of Alligation alternate, and the Use of the said Rule in the Composition of Medicines.

I. **I**N order to the Demonstration of the said Rule, I shall premise this *Lemma*, viz. If the Difference of any two Numbers given, be multiplied by a Number assigned, the Product will be equal to the Difference between the Products which arise from the Multiplication of those two Numbers severally by the Number assigned.

Suppositions.

Two Lines or } $AC = 10$
 Numbers given. } $BC = 4$
 Their difference. $AB = 10 - 4$
 A Multiplier } $AD = 5$
 assigned.



Which Suppositions, and the *Diagram* being well viewed, the Truth of the said *Lemma* will be evident, viz.

$$AB \times AD = AC \times AD - BC \times BE \quad (AD)$$

$$10 - 4 \times 5 = 10 \times 5 - 4 \times 5.$$

II. To add the more Light to the following *Demonstration* of the Rule of *Alligation alternate*, I shall propound a Question which properly belongs to the said Rule, viz. Suppose a *Vintner* having *French Wines* at 5 *d.* the *Quart*, and at 10 *d.* the *Quart*, would make a Mixture of them in such a manner, that he may sell the mixt Quantity at 7 *d.* the *Quart*, and so make as much Money of the Mixture, as if he should sell each Quantity of Wine at its own Price; the Question is to know what Proportion the Quantities of both sorts of Wine in the Mixture must bear one to the other. Here, according to the *Rule of Alligation alternate*, I take the Differences between the mean Price assigned for the Mixture, and the two other given Prices, and place those Differences alternately, viz. the Difference between 7 and 10 being 3, I write 3 against 5; likewise 2 being the Difference between 7 and 5, I write 2 against 10; so I conclude, that the Quantity to be taken of that sort of Wine of 10 *d.* the *Quart*, must have such Proportion to the Quantity of 5 *d.* the *Quart*, as 2 to 3. That is to say, if 2 *Quarts* at 10 *d.* the *Quart*, be mixed with 3 *Quarts* at 5 *d.* the *Quart*, the total Mixture 5 *Quarts* being sold at 7 *d.* the *Quart*, will yield as much Money as the said 3 *Quarts* at 5 *d.* the *Quart*, together with the said 2 *Quarts* at 10 *d.* the *Quart*; as is evident by the subsequent Work.

$$\begin{array}{r} 10 \ 2 \\ 7 \} \ 5 \ 3 \\ \hline 5 \end{array}$$

Quarts

	Quarts	Pence	Quarts	Pence
I	1	:	5	:
II	1	:	10	:
III	$15 \div 20 = 7 \times 5 = 35$			

From the Premises it appears, that when two Things are given to be mixt in such manner as the *Rule of Alligation alternate* requires, the Proposition to be demonstrated will be this, namely,

Three Numbers A B C being given in such sort, that A is less than B, but greater than C, if the Difference between A and B be multiplied by C, and the Difference between A and C be multiplied by B, the Sum of those Products will be equal to the Product arising from the Multiplication of A, by the Sum of the said Differences.

Demonstration.

	<i>mean Price</i>	<i>extreme Prices</i>	<i>alternate Differences</i>	<i>Products</i>
	A	B C	A—C B—A	BA—BC CB—CA
	{			
		$ B-C $	$ BA-CA = B-C \times A$	

The Difference between B and A is B—A, which multiplied by C, produces (as is evident by the *Lemma* afore-going in the first Section of this Chapter) CB—CA. Also the Difference between A and C is A—C, which multiplied by B, produces BA—BC. Then the Sum of those two Products is BA—CA, (for + CB and —CB expunge one the other,) which Sum is manifestly the same with the Product arising from the Multiplication of A the mean Price by B—C the Sum of the aforefaid Differences (to wit, the Sum of A—C and B—A) for + A and — A expunge one another.

When

When more than two Things of different Prices are given to be mixt as aforesaid, the *Demonstration* will not be otherwise. For if the Sum of every two Products arising from the Multiplication of two alternate Differences by their respective Prices, be equal to the Product of the mean Price multiplied by the Sum of the said Differences; the Sum of all the said Products will also be equal to the Product of the mean Price multiplied by the Sum of all the Differences; as will clearly appear by view of the subsequent Work.

$$\begin{array}{rcc}
 \text{Products of the mean} & & \text{Products of alternate} \\
 \text{Price multiplied by the} & & \text{Differences multiplied} \\
 \text{several Sums of alter-} & & \text{by their respective Pri-} \\
 \text{nate Differences.} & & \text{ces.} \\
 \left. \begin{array}{l} \text{If } D + E = F \times G \\ \text{and } H + K = F \times M \end{array} \right\} & & \left. \begin{array}{l} \text{Products of alternate} \\ \text{Differences multiplied} \\ \text{by their respective Pri-} \\ \text{ces.} \end{array} \right\} \\
 \text{Then } D + E + H + K = F \times G + M
 \end{array}$$

And farther, because if equal Numbers be severally divided by one and the same Number, the Quotients will be equal between themselves, therefore from the Premises this *Corollary* will arise.

C O R O L L A R Y.

In the *Rule of Alligation alternate*, if the Aggregate of the Products arising from the Multiplication of the several alternate Differences by their respective Prices, be divided by the Sum of the said Differences, the Quotient will be equal to the mean Price. This may be a Proof of any Example of the said *Rule of Alligation*.

Of the Composition of Medicines.

I. Medicines and Simples in respect of their Qualities are considered in some of these five Ways, *viz.* either as they are hot, or cold, moist, or dry, or as they are temperate; so that such Simples or Medicines as work heat in our Bodies are said to be hot, such cold as are the Cause of Coldness, &c.

See more of this in Mr. J. Dee's Mathematical Preface; also Tom. II. of P. Herigon; and Master Moor's Arithmetick.

II. The mean or middle between the extreme Qualities of *Heat* and *Coldness*, also between *Dryness* and *Moisture*, is called *Temperate*, or the *Temperature*; from which every one of the said Qualities, *hot*, *cold*, *moist*, and *dry*, differs in four *Degrees*; so that a Medicine or Simple is said to be either *temperate*, or else *hot*, *cold*, *moist*, or *dry*, in the first, second, third or fourth Degree.

III. If the Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed as you see from A to B, the Differences between 5 (the middle Number) and the superiour Numbers 6, 7, 8, 9, will be 1, 2, 3, 4, which may represent the 4 Degrees of the Qualities hot and dry; likewise the Differences between 5 and the inferiour Numbers 4, 3, 2, 1, will be 1, 2, 3, 4, which may represent the 4 Degrees of the Qualities cold and moist, the Temperature represented by 0, being the Mean or Middle from whence the said Degrees do swerve.

Ind.	Degr.	
B 9	4	Qualities hot and dry.
8	3	
7	2	
6	1	
5	0	Temperature.
4	1	Qualities cold and moist.
3	2	
2	3	
A 1	4	
Ind.	Degr.	

IV. Since

IV. Since the *Rule of Alligation alternate* requires, that of two Things miscible, the one must exceed the Mean proposed, and the other be less; therefore the Questions of *Alligation* in this kind are to be worked with the Numbers in the aforesaid Column A B: For by them the Degrees and Qualities are discovered, being placed as you see in the Column adjacent to A B, and for distinction sake, those Numbers in the said Column A B, may be called the *Indices* or *Exponents* of the *Degrees*; which *Indices* are to be used in the same Manner as the Prices of Merchandizes in the Question of *Alligation alternate* in the fourteenth Chapter of the preceeding Book, and therefore those Examples may be compared with these.

P R O P. I.

Having divers Simples whose Qualities are known, to make a Composition or Mixture of them, in such manner that the Quality of the Medicine may be some Mean among the Qualities of the Simples, and the Quantity of it any Quantity assigned.

Example 1. An *Apothecary* has four Sorts of Simples, A, B, C, D, whose Qualities are as follow, viz. A is hot in the fourth Degree, B is hot in the second, C is temperate, and D is cold in the third Degree; the Question is to know what Quantities of each of them ought to be taken, to make a Medicine, whose Quantity may be 12 Ounces, and the Quality in the first Degree of Heat? Seek in the aforesaid Column A B, for the *Indices* or *Exponents* of the Qualities of the Simples given, viz. for A which is hot in the fourth Degree, take 9; for B which is hot in the second, take 7; for C which is temperate, take 5; and for D which is cold in the third Degree, take 2: That done, rank those Numbers in the same manner as the Prices of Merchandizes in the Questions of the 14th Chapter, viz. descend from the highest Degree of Heat to the Temperature, and so proceed downwards to the Degrees of Cold, setting 6 the *Index* or *Exponent* of the mean Quality propounded, which is 1 Degree of Heat, as common to them all: Then, by crooked Lines or otherwise, connect two such *Indices*, whereof one may be greater than the Mean, and the other less, and proceeding according to the Rule of the 14th Chapter, you'll find that to make a Medicine of 9 Ounces, and the Quality resulting to be in the first Degree of Heat, you must take 1 Ounce of A (being

ing that Simple which was hot in 4°. 4 Ounces of B, 3 Ounces of C, and 1 Ounce of D, as will be manifest by the Proof.

Degr.	Oun.	Simp.	The Proof.
6 { 9	1	A	9 × 1 = 9
7	4	B	7 × 4 = 28
5	3	C	5 × 3 = 15
2	1	D	2 × 1 = 2
	9		9)54(6

Lastly, by the *Rule of Proportion* you may encrease the Medicine to the Quantity of 12 Ounces, and yet the Quality to continue in the first Degree of Heat, according to the following Operation.

Oun.	Oun.	Oun.	Oun.	
9	1	12	$1\frac{1}{3}$	of A
9	4	12	$5\frac{1}{3}$	of B
9	3	12	4	of C
9	1	12	$1\frac{1}{3}$	of D

The Quantity assigned 12 Ounces.

By the other Connexions of the Qualities, other Quantities of every Simple would arise; but that hath been sufficiently manifested in the Questions of the fourteenth Chapter.

Example 2. Suppose there are five *Simples*, A, B, C, D, E, whose Qualities are as follow, viz. A is hot in 3°. B is hot in 2°. C is hot in 1°. D is cold in 1°. E is cold in 3°. and it is required to mix 4 Ounces of B, with such Quantities of the Rest, that the Quality of the *Medicine* may be temperate?

Degr.	Oun.	Simp.	The Proof.
8	1	A	$8 \times 1 = 8$
7	3	B	$7 \times 3 = 21$
6	1	C	$6 \times 1 = 6$
4	3+1	D	$4 \times 4 = 16$
2	2	E	$2 \times 2 = 4$
	11		11) 55 (5

Proceed as before, so you'll find that to make a *Medicine* of 11 Ounces, and the Quality of the *Form* resulting to be Temperate, you must take 1 Ounce of A, 3 Ounces of B, 1 Ounce of C, 4 Ounces of D, and 2 Ounces of E: Then since the Quantity of B, in the Composition propos'd, is limited, viz. 4 Ounces; find Numbers which may be in such Proportion to 4 (the Quantity of B assigned) as the Numbers 1, 1, 4, 2 (the Quantities of A, C, D, E, in the aforesaid Composition of 11 Ounces) are to 3 (the Quantity of B in the said Composition) in manner following.

Oun.	Oun.	Oun.	Oun.	
3	1	4	$1\frac{1}{2}$	of A
3	1	4	$1\frac{1}{3}$	of C
3	4	4	$5\frac{1}{2}$	of D
3	2	4	$2\frac{2}{3}$	of E

} to be mixed with 4 Ounces of B.

P R O P. II.

A *Medicine* being compounded of several *Simples*, whose Qualities and Quantities are known, to find the Degree of the *Form* resulting, viz. the exact Temperament of the *Medicine*.

Example 1. Suppose a *Medicine* to be compounded of two *Simples*, viz. 6 Ounces of B hot in 4°. and three Ounces of C hot in 3°. and it is required to find the Temperament of the *Medicine*, viz. the Degree and Quality resulting from such mixture? Seek in the aforesaid Column A B for the *Indices*, of the respective Degrees and Qualities of the *Simples* given, and dispose them

them orderly in Ranks right against their respective Quantities ; then multiply each *Index* by its respective Quantity, and divide the Sum of the Products by the Sum of the Quantities : So will the Quotient be the *Index* of the Degree, and Quality of the *Medicine*.

<i>Ind.</i>	<i>Qun.</i>	<i>Prod.</i>
9 :	6 =	54
8 :	3 =	24
	9	78 ($8\frac{2}{3}$)

So in the said Example, the Quotient will be found $8\frac{2}{3}$, which is the *Index* of $3\frac{2}{3}$ Degrees of Heat, and therefore the said *Medicine* is hot in $3\frac{2}{3}$ Degrees.

Forasmuch as any two Quantities miscible according to the *Rule of Alligation alternate*, are in such Proportion one to the other, as the respective alternate Differences between the mean Quality of the Mixture, and the Qualities correspondent to the said Quantities ; the Demonstration of the aforesaid *Rule* will be manifest by the *Corollary* afore-going in this Chapter.

Example 2. Suppose a *Medicine* to be compounded of 4 *Simples*, whose Qualities and Quantities are known, *viz.* 2 Ounces of A hot in 3° . 3 Ounces of B hot in 2° . 4 Ounces of C temperate, and 5 Ounces of D cold in 4° . and let it be required to find the mean Quality resulting from such Mixture. According to the aforesaid *Rule*, I multiply each *Index* by its respective Quantity, and divide the Sum of the Products by the Sum of the Quantities, so the Quotient is $4\frac{3}{7}$, which is the *Index* of $\frac{4}{7}$ Degrees of Cold (for the Difference between 5 the *Index* of the Temperature, and $4\frac{3}{7}$ the *Index* found, is $\frac{4}{7}$ Degrees of Cold) which is the Quality of the said *Medicine*.

<i>Ind.</i>	<i>Qun.</i>	<i>Prod.</i>
8	2	= 16
7	3	= 21
5	4	= 20
1	5	= 5
14)		62 ($4\frac{3}{7}$)
		R r 2

Example

Example 3. Suppose a Medicine to be compounded of several *Simples*, whose Qualities and Quantities are as follow, *viz.* 4 Ounces of a *Simple* which is cold in 2° . and moist in 1° ; 5 Ounces hot in 3° . and (in respect of dryness and moisture) temperate; 3 Ounces hot in 2° . and dry in 2° ; 6 Ounces hot in 1° . and moist in 4° ; 4 Ounces cold in 3° . and moist in 1° ; the Question is to know the Temper resulting?

In the Resolution of this Question there must be two distinct Operations, each of them like that in the last Example, *viz.*

1. Find in the same manner as before, the Degree and Quality resulting from the Commixture of the Qualities hot and cold; so you'll discover $5\frac{7}{2}$ which is the Index of $\frac{7}{2}$ Degrees of Heat (for the Difference between 5, the Index of the Temperature, and $5\frac{7}{2}$ the Index found, is $\frac{7}{2}$ Degrees of Heat.

<i>Ind.</i>	<i>Oun.</i>	<i>Prod.</i>		<i>Ind.</i>	<i>Oun.</i>	<i>Prod.</i>			
3	×	4	==	12	4	×	4	==	16
8	×	5	==	40	5	×	5	==	25
7	×	3	==	21	7	×	3	==	21
6	×	6	==	36	1	×	6	==	6
2	×	4	==	8	3	×	4	==	12
22)	117	($5\frac{7}{2}$)		22)	80	($3\frac{7}{2}$)			

2. Seek in the same manner, the Temper resulting from the Mixture of the Qualities dry and moist; and you'll find $3\frac{7}{11}$, which is the Index of $\frac{4}{11}$ Degree of moisture; so the Quality of the said Medicine is $\frac{4}{11}$ Degree of Heat, and $\frac{7}{2}$ Degree of Moisture, as by the Operation is manifest.

P R O P. III.

To augment or diminish a Medicine in Quality, according to any Degree assigned.

Suppose a Medicine to be compounded as follows, *viz.* 1 Dram of a *Simple* hot in 4° . 2 Drams hot in 3° . 2 Drams hot in 2° . 1 Dram hot in 1° . 1 Dram cold in 1° . and 1 Dram cold in 2° . Then will the Quality of the said Medicine be in $1\frac{1}{2}$ Degree of Heat (as is manifest by the second Proposition.) Now let

let it be required to augment the said Medicine in quality, *viz.* to add such a Quantity of some one of the Ingredients (or some other Simple) which may raise the Quality of the Medicine $\frac{1}{2}$ Degree; so that the Temperament of the Medicine after it is encreased in Quantity, may be in 2° . of Heat. Make choice of such a Simple, the Index of whose Quality may exceed the Index of the Quality assigned, *viz.* take that Simple which is hot in 3° . whose Index is 8, then proceed according to the 1 Example of the first Proposition; so will you find that if 1 Dram of the aforesaid Medicine be mixed with $\frac{1}{2}$ Dram of that Simple which is hot in 3° . the Temper resulting from such Mixture will be in 2° . of heat.

Lastly, by the *Rule of Three*, say, if 1 Dram require $\frac{1}{2}$ Dram, what shall 8 Drams (the Quantity of the Medicine first given) require?

Ans. 4 Drams: So that if 4 Drams of a Simple which is hot in 3° . be mixed with 8 Drams of a Medicine hot in $1\frac{1}{2}$ Degree, the Temper resulting will be in 2° . of heat, as by the Operation is manifest.

$$\begin{array}{rcl}
 & \text{Drams.} & \\
 & \text{Ind.} & \\
 7 \left\{ \begin{array}{l} 6\frac{1}{2} \\ 8 \end{array} \right. & | & \begin{array}{l} 1 \\ \frac{1}{2} \end{array} \\
 1 : \frac{1}{2} :: 8 : 4
 \end{array}$$

The Proof.

$$\begin{array}{rcl}
 & \text{Drams.} & \text{Prod.} \\
 \text{Ind.} & & \\
 6\frac{1}{2} \times 8 & = & 52 \\
 8 \times 4 & = & 32 \\
 \hline
 12) 84 & (7
 \end{array}$$

If it be required to diminish a Medicine in quality, you are to make choice of such a Simple, the Index of whose Quality may be less than the Index of the Quality assigned, and then to proceed as before.

Here

Here observe, that if in Questions of this nature, the Quantities of the Simples be express'd by Weights of diverse Denominations, they must be reduced to that Weight which is of the lowest Denomination in the Question, according to the sixth Rule of the seventh Chapter of the preceeding Book.

The augmenting or diminishing of a Medicine, in respect of Quantity: Also the finding of the Value of any Quantity of a Medicine, the Prices of the Ingredients being known, will be familiar to such as understand the Rule of Proportion, and therefore I shall not insist upon them.

CHAP. IX.

A Demonstration of the common Rule of False by two Positions.

I. **W**HAT the ordinary *double Rule of False* is, and how to be used in resolving such Questions as cannot readily be applied to any of the other Rules of *Arithmetick*, has been fully declared in the 15th and 31st Chapters of the preceeding Book; it remains to shew what Kind of Operation is presupposed before the said Rule can be applied to the Resolution of a Question, and then to demonstrate the Truth of the Rule itself.

II. In the said *Rule of False*, look what Operation the Question requires to be performed with the Number sought and some given Number or Numbers, the same kind of Operation in every respect is to be made with each of the two feigned Numbers (commonly called Positions) and the said given Number or Numbers; which threefold Process being finished (whether it be by any one, or all of these Rules, to wit, *Addition*, *Subtraction*, *Multiplication*, and *Division*) there will arise three remarkable Numbers or Results, to wit, one proceeding from the true Number sought. and two others resulting from the two feigned Numbers: Then from these three Results, the Errours are collected, which are nothing else but the Differences between the true Result, and each of the two false Results.

III After the said Errours or Differences are discovered, the *Rule of False* will be of no force, unless this Analogy or Proportionality arise, namely, the first Error must have the same

Proportion to the second, as the Difference between the Number sought, and the first feigned Number, has to the Difference between the said Number sought and the second feigned Number; here therefore it may be demanded, What kind of Operation will produce the said Analogy? To this I answer, when the Question requires the Number sought to be increased, lessened, multiplied, or divided by some given Number, or the Number arising from such Operation to be increased, lessened, multiplied, or divided by some given Number; in any of those Cases, the aforesaid Analogy will necessarily arise, as I shall here manifest in all the said Cases. First therefore, I say, when to each of three Numbers (namely, the Number sought by the *Rule of False*, and the two feigned Numbers) one and the same Number is added, the said Analogy will ensue; for in this case the Difference between the first Sum and the second, will be equal to the Difference between the first and second of the said three Numbers; likewise the Difference between the first Sum and the third, will be equal to the Difference between the first Number and the third, which may be proved in manner following.

Suppositions.

Let there be three Numbers, to wit,

$$\begin{array}{c} A . B . C \\ 11 . 7 . 5 \end{array}$$

Suppose also that the first Number A is greater than either of the Numbers B and C.

Suppose also, some Number as D (3) to be added to each of the said three Numbers, then will the three Sums be,

$$\begin{array}{r|l} A + D & 15 \\ B + D & 10 \\ C + D & 8 \end{array}$$

The Proposition to be demonstrated is, That the Difference between the first Sum and the second, is equal to the Difference between the first Number and the second; also that the Difference between the first Sum and the third, is equal to the Difference between the first Number and the third.

Demon-

Demonstration.

The Difference between the first Number and the second is,

$$A - B$$

The Difference between the first Sum and the second is,

$$A + D - B - D$$

But the latter Difference is manifestly equal to the former, (for $+D$ and $-D$ expunge one the other, ;) to wit,

$$A + D - B - D = A - B$$

Therefore the first Part of the Proposition is proved.

Again, the Difference between the first Number and the third is,

$$A - C$$

The Difference between the first Sum and the third is,

$$A + D - C - D$$

But the latter Difference is manifestly equal to the former, for $+D$ and $-D$ expunge one the other, viz.

$$A + D - C - D = A - C$$

Wherefore the Proposition is fully proved.

The like Property might be demonstrated after the same manner, when one and the same Number is subtracted from three Numbers severally.

Secondly, when three Numbers (namely the Number sought by the *Rule of False* and the two feigned Numbers) are severally multiplied by one and the same Number ; the afore-mentioned Analogy will likewise ensue, as may be thus proved.

Suppositions.

Let there be three Numbers, to wit,

$$\begin{array}{ccc} A & . & B & . & C \\ 3 & . & 5 & . & 8 \end{array}$$

Suppose also that the first Number A is less than either of the Numbers B and C.

Again, Suppose each of those three Numbers to be multiplied by one and the same Number as D (4) and the three Products to be these,

$$\begin{array}{l|l} DA & 12 \\ DB & 20 \\ DC & 32 \end{array}$$

The Proposition to be demonstrated is, That the Difference between the first Product and the second, has such Proportion to the Difference between the first Product and the third, as the Difference between the first Number and the second, has to the Difference between the first Number and the third, *viz.*

$$\begin{array}{ccccccc} DB-DA & : & DC-DA & : : & B-A & : & C-A \\ 8 & : & 20 & : : & 2 & : & 5 \end{array}$$

Demonstration.

Forasmuch as (by the 17th *Prop.* of the seventh Book of *Euclid's Elem.*) if a Number (D) multiplying two Numbers (B-A and C-A) produces other Numbers (DB-DA and DC-DA) the Numbers produced by the Multiplication will be in the same Proportion as the Numbers multiplied are ; therefore

$$DB-DA : DC-DA : : B-A : C-A$$

which was to be demonstrated.

Likewise when three Numbers are divided by one and the same Number, the *Demonstration* will not be otherwise ; and because by the second *Section* of this *Chapter*, the Errours in the *Rule of False* are the Differences between the true Result and the two false Results : Therefore from the preceeding *Demonstrations* it is evident, that the afore-mentioned Analogy or Proportionality (namely, when the first Errour has such Proportion to the second, as the Difference between the Number sought and the first feigned Number has to the Difference be-

S f

tween

tween the said Number sought and the second feigned Number) will succeed from such Operation, as is before declared in the Beginning of the third *Section* of this Chapter.

IV. Now to discern what Kind of Operation will not produce the said Analogy, observe this Note, viz. when a Question requires some given Number to be divided by the Number sought, or any part of it; also when the Number sought or some part thereof is to be squared, cubed, &c. likewise when some parts of the Number sought are to be multiplied one by the other: I say, from such Operations the afore-mentioned Analogy will not arise, and in those Cases, the *ordinary Rule of False* will be useless; as may partly appear by the two following Examples, viz. *What Number is that, by which if 360 be divided, the Quotient will be 24?* Here if two Positions or feigned Numbers be taken, and 360 be divided by each of them, the Errours will not be in the same Proportion with the Differences between the true Number sought and the two feigned Numbers, and therefore the *Rule of False* will be used in vain: Yet if it be asked what Number is that, which being multiplied by 24, the Product will be 360, the Answer to this latter Question is the same with the Answer to the former, and may be found by the *Rule of False*; but such kind of Interpretations and Inferences are not always obvious; and therefore, since the preparative Work of the *Rule of False* (after the Number is taken by guess for the Number sought) proceeds gradually from one Condition in the Question to another, it will for the most Part be easy to determine whether the ordinary *Rule of False* will take place or not, by comparing the Conditions of a Question with the Note before given.

Another Example. A certain Person being demanded what Number of Years he had lived, answered, If $\frac{1}{4}$ of that Number were multiplied by $\frac{1}{4}$ of the same Number, the Product would shew the Number, or his Age: Here it will be in vain to search the Number sought (which is 40) by the *Rule of False*; for the afore-mentioned Analogy or Proportionality will not succeed, and the Question cannot easily be resolved without *Algebra*.

Now from this Supposition, that after the preparative Work of the *Rule of False* is finished, the Errours will be in such Proportion as aforesaid, I shall make it manifest that the *Rule of False* will discover the Number sought.

V. In the Rule of two false Positions there are three Cases, viz. the Errours are either both Excesses and noted with +, or else both

both Defects and noted with —, or lastly, one of the Errours is noted with +, and the other with —.

In the two first Cases the Rule is this: Multiply the Positions or feigned Numbers by the altern Errours, *viz.* the first Position by the second Errour, the second Position by the first Errour, and reserve those Products; then dividing the Difference of the said Products by the Difference of the said Errours, the Quotient will be the Number sought by the Question.

The Demonstration of the said Rule here follows.

Case I. When the Errours are both Excesses, and noted with +.

Suppositions.

1. Let some Number unknown and sought for by } *the Rule of False* be represented by ---- } A
2. Let the first Position (or feigned Number) be ---- B
3. And the second feigned Number ---- C
4. Suppose also that B is greater than C, and each of them greater than A.
5. But farther, suppose the Errour of the first Position to be ---- } F
6. And the Errour of the second Position to be --- G
7. Suppose also that this Analogy be found in the said Numbers, *viz.*

$$B-A : C-A :: F : G$$

8. The Proposition to be demonstrated,

$$A = \frac{F C - G B}{F - G}$$

Demonstration.

9. Forasmuch as by Supposition in 7°.

$$B-A : C-A :: F : G$$

10. Therefore by comparing the Rectangle of the Extremes to the Rectangle of the Means,

$$G B - G A = F C - F A$$

11. And by equal Addition of F A

$$S f 2$$

$$F A$$

$$F A + G B - G A = F C$$

12. Again, forasmuch as by Supposition in 4°,

$$B > C$$

13. And consequently out of 4° and 12°,

$$B - A > C - A$$

14. Therefore out of 9° and 13°,

$$F > G$$

15. Therefore

$$F A > G A$$

16. Therefore

$$F A - G A > 0$$

17. Therefore by equal Subtraction of $G B$ from the Equation in 11°,

$$F A - G A = F C - G B$$

18. Wherefore by dividing both Parts of the last Equation by $F = G$, equal Quotients will arise, viz.

$$A = \frac{F C - G B}{F - G}$$

which was to be demonstrated.

Case II. When the Errors are both Defects, and noted with -.

Suppositions.

1. Let some Number unknown and sought for by the Rule of False be represented by $\left. \begin{array}{l} \text{----} \end{array} \right\} A$
2. Let the first Position (or feigned Number) be $\text{---} B$
3. And the second Position $\text{----} C$
4. Suppose also that B is less than C , and each of them less than A .
5. But farther, suppose the Error of the first Position to be $\left. \begin{array}{l} \text{----} \end{array} \right\} F$
6. And the Error of the second Position $\text{----} G$
7. Suppose also that this Analogy be found in the said Numbers, viz.

$$A - B : A - C :: F : G$$

8. The Proposition to be demonstrated,

FC

$$A = \frac{F C - G B}{F - G}$$

Demonstration.

9. Forasmuch as by Supposition in 7^o,

$$A - B : A - C :: F : G$$

10. Therefore by comparing the Rectangle of the Means to the Rectangle of the Extremes,

$$F A - F C = G A - G B$$

11. And by equal Addition of F C

$$F A = F C + G A - G B$$

12. Again, forasmuch as by Supposition in 4^o,

$$B < C$$

13. And consequently out of 4^o, and 12^o,

$$A - B < A - C$$

14. Therefore out of 9^o and 13^o,

$$F < G$$

15. Therefore

$$F A < G A$$

16. Therefore

$$F A - G A < 0$$

17. Therefore by equal Subtraction of G A from the Equation of 11^o,

$$F A - G A = F C - G B$$

18. Wherefore by dividing both Parts of the last Equation by F G, equal Quotients will arise, viz.

$$A = \frac{F C - G B}{F - G}$$

which was to be demonstrated.

Case III. When one of the Errours is an Excess (to wit, noted by +) and the other a Defect (noted by -)

In this third Case the *Rule of False* is this, viz.

Multiply the Positions by the altern Errours, to wit, the first Position by the second Errour, also the second Position by the first Errour, and reserve those Products; then dividing the Sum of the said Products by the Sum of the said Errours, the Quotient will be the Number sought by the Question.

The

The Demonstration of this latter Rule here follows.

Suppositions.

1. Let some Number unknown and sought for by the *Rule of False* be represented by ----- } A
2. Let the first Position be ----- } B
3. And the second Position ----- } C
4. Suppose also that B is greater than C, and also greater than A, and that C is less than A.
5. Moreover, suppose the Error of the first Position to be F
6. And the Error of the second Position to be ----- G
7. Suppose also that this Analogy be found in the said Numbers, viz.

$$B - A : A - C :: F : G$$

8. The Proposition to be demonstrated.

$$A = \frac{GB + FC}{F + G}$$

Demonstration.

9. Forasmuch as by Supposition in 7°,

$$B - A : A - C :: F : G$$
10. Therefore by comparing the Rectangle of the Means to the Rectangle of the Extremes.

$$FA - FC = GB - GA$$
11. And by equal Addition of FC and GA to the last Equation, this will arise,

$$FA + GA = GB + FC$$
12. Wherefore by dividing both Parts of the last Equation by $F + G$, equal Quotients will arise, viz.

$$A = \frac{GB + FC}{F + G}$$

which was to be demonstrated.

The learned *Herigonius* (in cap. 13. Tom. II. of his *Cursus Mathematicus*) has delivered another Way of resolving the *Rule of False*, namely by the two following Rules, viz.

When

When the Signs of the Errours are alike.

Rule I. As the Sum of the Errours is to the first Errour, so is the Difference of the supposed Numbers to a fourth Proportional, which being added to the first supposed Number, when the said first Supposition is less than the second, or subtracted from it when it exceeds the second; the Sum or Remainder will be the true Number sought.

When the Signs of the Errours are unlike.

Rule II. As the Difference of the Errours is to the first Errour, so is the Difference of the supposed Numbers to a fourth Proportional, which being added to the first supposed Number, when the Signs are —, or subtracted from it when the Signs are +; the Sum or Remainder will be the Number sought.

Both these Rules the said *Herignius* demonstrates geometrically by Lines, upon supposition of the *Analogy* or *Proportionality* before-mentioned in the third Section of this Chapter, and the same may likewise be easily demonstrated according to the preceeding Method by Letters.

C H A P. X.

A Collection of pleasant and subtil Questions, to exercise all the Parts of Vulgar Arithmetick: To which are also added various practical Questions about the Mensuration of Superficial Figures and Solids.

QUEST. 1. If a Wedge of Gold weighing $17\frac{3}{4}$ lb. of Troy-weight be worth $679\frac{5}{7}$ lb. Sterling, what is the Value of $1\frac{1}{2}$ Grain of that Gold?

Examples of the Rule of Three mixtly used with other Rules.

Ans. 2 Pence.

I. $1\frac{1}{2}$ (or $\frac{3}{2}$) of $\frac{1}{24}$ of $\frac{1}{6}$ of $\frac{1}{12} = \frac{1}{48}$

II. $42\frac{2}{7} : 47\frac{8}{7} :: \frac{1}{48} : \frac{1}{12}$

Quest.

Quest. 2. A Man dying gave to his eldest Son $\frac{3}{4}$ of $\frac{1}{4}$ of his Estate, to his second Son $\frac{1}{2}$ of $\frac{1}{2}$ of his Estate, and when they had counted their Portions, the one had 40 *l.* more than the other; the Remainder of the Estate was given to the Wife and younger Children; the Question is, What was the Portion of the eldest Son, also of the second, and how much did belong to the Wife and younger Children.

Ans. The eldest Son's Portion 100 *l.* the second Son's Portion 60 *l.* and 40 *l.* for the Wife and younger Children.

The Fractions being reduced, it will be manifest, that the eldest Son had $\frac{3}{16}$, and the second $\frac{1}{4}$; also the Difference of the said Fractions $\frac{1}{16}$, then say,

$$\frac{3}{16} : 40 :: \frac{1}{4} : 60$$

			<i>l.</i>
The second Son's Portion	-----	-----	60
The Difference of their Portions	-----	----	40
The eldest Son's Portion	-----	-----	100

$$\frac{3}{16} : 40 :: \frac{1}{4} : 60$$

Lastly, $600 - 160 = 440$ for the Wife and younger Children.

Quest. 3. A young Man receives 66 $\frac{3}{4}$ *l.* which was $\frac{3}{4}$ of $\frac{1}{2}$ of his eldest Brother's Portion, and $3\frac{1}{2}$ times his eldest Brother's Portion, was $1\frac{1}{4}$ times his Father's Estate, the Question is, What was the Father's Estate? *Ans.* 560 *l.*

$$\begin{aligned} \frac{3}{4} : 66\frac{3}{4} &:: 1 : 200 \\ 200 \times 3\frac{1}{2} &= 700 \\ 1\frac{1}{4} : 700 &:: 1 : 560 \end{aligned}$$

Quest. 4. If A can finish a Work in 20 Days, and B in 30 Days; in what time will the Work be finished by A and B working together? *Ans.* 12 Days.

First find what Quantity of the Work will be done by each Workman in one and the same time; then it will be, as the Sum of those Quantities is in Proportion to the said time, so is 1 or the whole Work to the time in which such Work will be finished by both Workmen working together.

Days

<i>Days</i>	<i>Work</i>	<i>Days</i>	<i>Work.</i>
30	: 1	: 20	: $\frac{2}{5}$
			add 1
			sum $1\frac{2}{5}$

Hence it appears that A and B working together 20 Days, will finish that Work once, together with $\frac{2}{5}$ of the same Work; therefore say again by the *Rule of Three*.

<i>Work</i>	<i>Days</i>	<i>Work</i>	<i>Days.</i>
$1\frac{2}{5}$	: 20	: 1	: 12

Quest. 5.

*Æreus adsto leo, tubuli mihi lumina bina,
Osque etiam, dextri sic quoque planta pedis :
Binis dextro oculo, ternis lacus iste diebus
Impletur lævo, sed pede bis geminis :
Ori sufficiunt sex horæ. Dic simul ergo.
Quo spatio os, oculi, pesque replere valent ?*

The Sense is this. A brazen Lyon being placed in an artificial Fountain, conveys Water into a Cistern by two Streams issuing from his Eyes, also by one from his Mouth, and by another at the Bottom of his right Foot. Now the Pipes through which these Streams pass, are of different Capacities, in such sort, that by the right Eye set open alone, the rest of the Streams being stopt, the Cistern will be filled in two Days (the length of a Day being supposed to be 12 Hours;) by the left Eye alone in three Days; by the Foot alone in four Days; and by the Mouth alone in six Hours. The Question is, to find in what time the Cistern will be filled, if all those Streams be set open at once? *Answer,* $\frac{12}{7}$ Days.

T t

Days

Days		Cist.		Days		Cist.
2	:	1	::	3	:	$1\frac{1}{2}$
4	:	1	::	3	:	$0\frac{3}{4}$
$\frac{1}{2}$	:	1	::	3	:	6
						add 1
						$9\frac{1}{4}$

The Sum is $9\frac{1}{4}$ Cisterns that will be filled in three Days by all the four Streams running together : Then say by the Rule of Three.

Cist.		Days		Cist.		Days
$9\frac{1}{4}$	:	3	::	1	:	$\frac{1}{2}\frac{2}{7}$

Quest. 6. A Cistern in a certain Conduit is supplied with Water by one Pipe of such bigness, that if the Cock A at the End of the Pipe be set open, the Cistern will be filled in $\frac{1}{2}$ Hour : But at the Bottom of the Cistern two other Cocks B and C are placed, whose Capacities are such, that by the Cock B set open alone, (all the rest being stopt) the Cistern supposed to be full, will be emptied in $1\frac{1}{2}$ Hour : Also by the Cock C set open alone, the Cistern will be emptied in $2\frac{1}{2}$ Hour : Now because more Water will be infused by the Cock A, than can be expelled by both the Cocks B and C in one and the same time ; the Question is, to find in what time the Cistern will be filled, if all the said three Cocks be set open at once ? *Answer,* $1\frac{2}{3}$ Hour.

After the manner of the fourth Question of this Chapter, find how many times the Cistern will be emptied in one and the same Space of Time, by the Cocks B and C running together ; also how much of the Cistern will be filled by A in the same time ; then will the Difference shew how much of the Cistern is gained by the filling Cock in the said time : Lastly, as the Cisterns or Parts gained are in proportion to the correspondent Time ; so is the whole Cistern, to the Time wherein it will be gained or filled.

	<i>Hou.</i>	<i>Cist.</i>	<i>Hou.</i>	<i>Cist.</i>	
I.	$2\frac{1}{2}$	:	1	::	$1\frac{3}{4}$: $2\frac{1}{2}$
					<i>add 1</i>
					<i>sum 1 $2\frac{1}{2}$</i>
					<i>emptied by</i>
					$\left. \begin{matrix} C \\ B \\ B \& C \end{matrix} \right\}$
II.	$\frac{1}{2}$	:	1	::	$1\frac{3}{4}$: $(2\frac{6}{7} \text{ filled by A})$
					$\frac{1\frac{1}{2}}{1\frac{3}{4}}$ gained by A
					$\left. \begin{matrix} C \\ B \\ B \& C \end{matrix} \right\} \text{ in } 1\frac{3}{4} \text{ Hou.}$
III.	<i>Cist.</i>	<i>Hou.</i>	<i>Cist.</i>	<i>Hou.</i>	
	$1\frac{1}{4}$	:	$1\frac{3}{4}$	::	1 : $1\frac{9}{11}$

Quest. 7. Suppose a Dog, a Wolf, and a Lyon were to devour a Sheep, and that the Dog could eat up the Sheep in an Hour, the Wolf in $\frac{3}{4}$ Hour, and the Lyon in $\frac{1}{2}$ Hour : Now if the Lyon begin to eat $\frac{1}{8}$ Hour before the other two, and afterwards, all three eat together, the Question is, in what time the Sheep would be devoured? *Ans.* $1\frac{1}{8}\frac{1}{4}$ Hour.

	<i>Hou.</i>	<i>Sh.</i>	<i>Hou.</i>	<i>Sh.</i>	
I.	If $\frac{1}{2}$	:	1	::	$\frac{1}{8}$: $\frac{1}{4}$

Thus it appears that $\frac{1}{4}$ of the Sheep would be eaten by the Lyon, before the Dog and Wolf began to eat.

II. Proceed according to the fourth Question, so will you find the remaining $\frac{3}{4}$ to be eaten by them all in $\frac{2}{3}\frac{1}{2}$ Hour, which added to $\frac{1}{8}$ gives $1\frac{1}{8}\frac{1}{4}$ Hour, in which time the Sheep would be devoured.

Quest. 8. If 120 $\frac{1}{3}$ l. is to be distributed among three Persons, A, B, C, in such sort, that as often as A takes 5, B shall take 4; and as often as B takes 3, C shall take 2; what will be the Share of each of them?

Ans. A 51 $\frac{4}{7}$ l. B 41 $\frac{2}{3}$ l. C 27 $\frac{1}{3}$ l.

Find three Numbers which may express the Proportions of their Shares, by the *Rule of Three*, or (to avoid Fractions) thus,

$$\begin{array}{r} 5 \dots\dots\dots 4 \\ 3 \dots\dots\dots 2 \\ \hline \end{array}$$

$$15 : 12 : 8$$

Thus found

$$5 \times 3 = 15$$

$$3 \times 4 = 12$$

$$4 \times 2 = 8$$

$$35 : 120\frac{1}{2} :: \left\{ \begin{array}{l} 15 : 51\frac{4}{7} \\ 12 : 41\frac{9}{5} \\ 8 : 27\frac{5}{3} \end{array} \right.$$

Quest. 9. A Governour of a certain Garrison, being desirous to know how much Money the Port or Passage of the Garrison did amount to in certain Months, made choice of a loyal Servant, giving him Orders to receive of every Coachman passing with a Coach 4*d.* of every Horseman 2*d.* and of every Footman $\frac{1}{2}$ *d.* Now at the Year's End, the Servant making his Account to the Governour, gives him 94*l.* 15*s.* 10*d.* and lets him know, that as often as 5 passed with Coaches, 9 passed on Horseback; and as often as 6 passed on Horseback, 10 passed on Foot; the Question is, how many Coaches, Horse-men, and Footmen passed? *Answ.* 2500 Coaches, 4500 Horse-men, 7500 Footmen.

Find three proportional Numbers after the manner of the 8th Question, which will be 5, 9, 13, then proceed as follows;

$$\begin{array}{r} d. \\ 5 \text{ Coaches} \text{ --- } 20 \\ 9 \text{ Horsemen} \text{ --- } 18 \\ 15 \text{ Footmen} \text{ --- } 7\frac{1}{2} \end{array}$$

$$\text{If } 45\frac{1}{2} : 22750 :: \left\{ \begin{array}{l} 5 : 2500 \\ 9 : 4500 \\ 15 : 7500 \end{array} \right.$$

Quest. 10. A Factor would exchange 780*l.* Sterling for double Ducats, Dollars, and French Crowns, the Ducats at 7*s.* 6*d.* the Piece, the Dollars at 4*s.* 4*d.* and the French Crowns at 6*s.* the Piece, to be in such Proportion, that $\frac{1}{2}$ of the Number of Ducats may be equal to $\frac{1}{3}$ of the Number of Dollars; and $\frac{1}{4}$ of the Dollars equal to $\frac{3}{12}$ of the Crowns; the Question is, how many Pieces of every Coin he shall receive for his 780 Pounds?

Answ. 600 Ducats, 900 Dollars, 1200 Crowns.

Find

Find three proportional Numbers (after the manner of the eighth Question) which will be 6, 4, 3.

$$\begin{array}{ccccccc} \frac{1}{2} & \dots & \dots & \dots & \dots & \dots & \frac{1}{3} \\ \frac{1}{4} & \dots & \dots & \dots & \dots & \dots & \frac{1}{6} \\ \hline \frac{1}{2} & : & \frac{1}{4} & : & \frac{1}{6} \\ 6 & : & 4 & : & 3 \end{array}$$

Thus it appears, that six times the Number of Ducats must be equal to four times the Number of Dollars, also equal unto three times the Number of Crowns. Then make choice of three Numbers to answer those Proportions, such as are these, 2, 3, 4, (for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which Numbers proceed as follows,

$$\begin{array}{l} l. \\ 2 \text{ Ducats} \text{ --- } \frac{3}{4} \\ 3 \text{ Dollars} \text{ -- } \frac{1}{2} \\ 4 \text{ Crowns} \text{ -- } 1\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{l} l. \\ \text{say if } 2\frac{3}{4} : 780 : : \end{array} \quad \begin{array}{l} l. \\ \frac{3}{4} : 225 \\ \frac{1}{2} : 195 \\ 1\frac{1}{3} : 360 \end{array}$$

$$\begin{array}{l} l. \quad \text{Ducat} \quad l. \\ \frac{7}{8} : 1 : : 225 : 600 \text{ Ducats} \\ \text{Doll.} \\ \frac{1}{2} : 1 : : 195 : 900 \text{ Dollars} \\ \text{Crown} \\ \frac{7}{10} : 1 : : 360 : 1200 \text{ Crowns} \end{array}$$

Quest. 11. Twenty Knights, 30 Merchants, 24 Lawyers, and 24 Citizens, spent at a Dinner 64 Pounds, which Sum was divided among them in such manner, that 4 Knights paid as much as 5 Merchants, 10 Merchants as much as 16 Lawyers, and 8 Lawyers as much as 12 Citizens; the Question is, to know the Sum of Money paid by all the Knights, also by the Merchants, Lawyers, and Citizens.

Answer. The 20 Knights paid 20 Pounds, the 30 Merchants 24 Pounds, the 24 Lawyers 12 Pounds, and the 24 Citizens 8 Pounds.

Find four Numbers to express the Proportions of their Payments by the *Rule of Three*, or (to avoid Fractions) in manner following; so will the proportional Numbers be 4, 5, 8, 12; viz. 4 Knights paid as much as 5 Merchants, or 8 Lawyers, or 12 Citizens.

$$\begin{array}{rccccccc}
 4 & . & . & . & . & . & . & . & . & . & 5 \\
 10 & . & . & . & . & . & . & . & . & . & 16 \\
 8 & . & . & . & . & . & . & . & . & . & 12 \\
 \hline
 320 & : & 400 & : & 640 & : & 560 \\
 4 & & 5 & & 8 & & 12 \\
 \hline
 \end{array}$$

Thus found

$$\begin{array}{l}
 4 \times 10 \times 8 = 320 \\
 10 \times 8 \times 5 = 400 \\
 8 \times 5 \times 16 = 640 \\
 5 \times 16 \times 12 = 960
 \end{array}$$

Then presupposing that a Knight is to pay 4 s. proceed as follows, viz.

	l.
20 Knights	4
30 Merchants . .	$4\frac{4}{5}$
24 Lawyers . . .	$2\frac{2}{3}$
24 Citizens . . .	$1\frac{1}{3}$

$$\text{say, if } 12\frac{4}{5} : 64 :: \left\{ \begin{array}{l} 4 : 20 \\ 4\frac{4}{5} : 24 \\ 2\frac{2}{3} : 12 \\ 1\frac{1}{3} : 8 \end{array} \right.$$

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Quest. 12. A certain Man with his Wife did usually drink out a Vessel of Beer in 12 Days, and the Husband found by often Experience, that his Wife being absent, he drank it out in 20 Days; the Question is, in how many Days the Wife alone could drink it out? *Ans.* 30 Days.

Note. It is to be supposed that the Husband, in 12 of the 20 Days in which he drank alone, did drink as much as in the 12 Days wherein he drank with his Wife; hence it follows, that in the remaining 8 of the said 20 Days, he drank as much as his Wife did in 12 Days. Therefore by the *Rule of Three* say, If 8 gives 12, what 20? *Ans.* 30. View the following Form of the Work.

$$\begin{array}{r}
 \text{From } 20 \\
 \text{Subtract } 12 \\
 \hline
 \end{array}$$

$$\text{Then if } 8 : 12 :: 20 : 30$$

Quest. 13. If a House be to be built by three Carpenters, A, B, C, working in such sort, that A alone will finish it in 30 Days,

Days, B in 40 Days, and A, B, C, together in 15 Days, in what time could C alone build the House? *Answ.* 120 Days.

I. After the manner of the fourth Question, (find in what time A and B working together will finish the House ;) *Answ.* 17 $\frac{1}{7}$ Days.

$$\begin{array}{cccc} \text{Days} & \text{Work} & \text{Days} & \text{Work} \\ 40 & : & 1 & :: 30 & : & \frac{3}{4} \\ & & & & & \text{add } 1 \end{array}$$

sum 1 $\frac{3}{4}$

$$\begin{array}{cccc} \text{Work} & \text{Days} & \text{Work} & \text{Days} \\ 1\frac{3}{4} & : & 30 & :: 1 & : & 17\frac{1}{7} \end{array}$$

II. Supposing the Work of A and B to be performed by one Person, as D, the House will be built by D in 17 $\frac{1}{7}$ Days, but by D and C together in 15 Days: Then find (according to the 12th Question) in what time C will build the same. *Answ.* 120 Days.

$$\begin{array}{r} \text{From } 17\frac{1}{7} \\ \text{Subtract } 15 \\ \hline \end{array}$$

$$\text{Then if } 2\frac{1}{7} : 15 :: 17\frac{1}{7} : 120$$

The Proof may be worked according to the fourth or fifth Questions.

Quest. 14. Two Travellers, A and B, perform a Journey to one and the same Place in this manner. *viz.* A travels 14 Miles every Day, and had travelled 8 Days before B began; upon the ninth Day B sets forward, and travels 22 Miles every Day; the Question is, to find in what time B shall overtake A? *Answ.* at the End of 14 Days.

I. Find how many Miles A had travelled before B set forward? *Answ.* 112 Miles.

$$\begin{array}{cccc} \text{Day} & \text{Miles} & \text{Days} & \text{Miles} \\ 1 & : & 14 & :: 8 & : & 112 \end{array}$$

II. Seek how many Miles B gains of A in a Day: *Answ.* 8 Miles; For,

$$22 - 14 = 8$$

$$\begin{array}{cccc} \text{Miles} & \text{Day} & \text{Miles} & \text{Days} \\ 8 & : & 1 & :: 112 & : & 14 \end{array}$$

III. If

Quest. 15. There is an Island which is 36 Miles in compass: Now if at the same time, and from the same place, two Footmen, A and B, set forward to travel round about the said Island, and follow one another in such manner, that A travels every Day

Day 9 Miles, and B 7 Miles; the Question is to find in what space of Time they'll meet again, also how many Miles, and how many times about the Island each Footman will then have travelled?

Ans. They'll meet at the End of 18 Days from their first parting; and then A will have travelled 162 Miles (or $4\frac{1}{2}$ times the Compass of the Island;) and B will have travelled 126 Miles (or $3\frac{1}{2}$ the Compass of the Island.)

		Miles			
From	9				
Subtract	7				
	2	:	1	:	36 : 18
mult.	18				
by	9				
36)	162	($4\frac{1}{2}$	mult.	18	
			by	7	
36)	126	($3\frac{1}{2}$			

Quest. 16. Two Footmen, A and B, depart at the same time from *London* towards *York*, travelling at this rate, viz. A goes 8 Miles every Day, B goes 1 Mile the first Day, 2 Miles the second Day, 3 Miles the third Day, and in that Progression he goes forward, travelling in every following Day 1 Mile more than in the preceeding Day; the Question is to know in how many Days B will overtake A?

Ans. In 15 Days.

To resolve this and such like Questions, double 8 (the Number of Miles which A travels daily) which make 16, from which subtract 1, the Remainder is 15, the Number of Days sought.

Quest. 17. If *Exeter* be distant from *London* 140 Miles, and that at the same time one Footman A departed from *London* towards *Exeter*, travelling every Day 8 Miles; another B from *Exeter* towards *London*, travelling every Day 6 Miles, the Question is, in how many Days they'll meet one another, and how many Miles each Footman will have then travelled?

Answer. They'll meet at the End of 10 Days, and then A will have travelled 80 Miles, and B 60 Miles.

add	8	Miles travelled daily by A
	6	Miles travelled daily by B
sum	14	Miles which A and B together did travel daily.

M. Da. Miles. Da.

14 : 1 :: 140 : 10. in which time A and B
will meet each other.

10 × 8 = 80 Miles travelled by A.

10 × 6 = 60 Miles travelled by B.

Quest. 18. A certain Footman A sets out from *London* towards *Lincoln*, and at the same time another Footman B departs from *Lincoln* towards *London*; also A travels every Day $2\frac{1}{2}$ Miles more than B. Now supposing those two Cities to be 100 Miles distant one from the other, and that those two Footmen do meet one another at the End of 8 Days after the Beginning of their Journeys; the Question is, How many Miles each will have then travelled, as also how many Miles each travelled daily?

Answer. A 60 Miles, B 40 Miles: Also A travelled $7\frac{1}{2}$ Miles every Day, and B 5 Miles.

<i>Day</i>	<i>Miles</i>	<i>Days</i>	<i>Miles</i>
1	: $2\frac{1}{2}$	:: 8	: 20

Hence it appears, that at the time of their meeting, A had travelled 20 Miles more than B, which 20 Miles being subtracted from 100 Miles leave 80 Miles, whereof the Half is 40 Miles which B had travelled, therefore A had travelled 60 Miles.

Now to find how many Miles each travelled daily, say,

<i>Days</i>	<i>Miles</i>	<i>Day</i>	<i>Miles</i>
8	: 40	:: 1	: 5

Miles

Therefore $\left\{ \begin{matrix} A \\ B \end{matrix} \right\}$ travelled $\left\{ \begin{matrix} 7\frac{1}{2} \\ 5 \end{matrix} \right\}$ daily.

Quest. 19. There is an Island which is 134 Miles in compass; now at the same time, and from the same Place, two Footmen A and B begin a Journey round about the said Island, but they travel towards contrary Parts, at this rate, *viz.* A travels 11 Miles in every 2 Days, and B 17 Miles in 3 Days; the Question is to find in what space of time A and B will meet one another; and how many Miles each will then have travelled?

U u

Answer,

Answer, They'll meet at the end of 12 Days, and then A will have travelled 66 Miles, and B 68 Miles.

After the manner of the fourth Question of this Chapter, the time sought will be found 12 Days.

Days Miles Days Miles.

2 : 11 :: 3 : $16\frac{1}{2}$

add 17

Days Miles Days
 $33\frac{1}{2}$: 3 :: 134 : 12

The Miles travelled by each will be found in this manner,

Days Miles Days

2 : 11 :: 12 : 66 Miles travelled by A.

3 : 17 :: 12 : 68 Miles travelled by B.

Quest. 20. If a Clock has two Indices (or Hands,) one of which (to wit A) is carried twice round the whole Circumference of the Dial in one Day; and the other (B) once in 30 Days, and that both at once shewing the same Point begin to be moved; the Question is, in what time they will be again conjoined?

Answer, $\frac{30}{59}$ Day, or $\frac{12}{59}$ Hour.

Day Circum. Days Circum.

1 : 2 :: 30 : 60

subtract 1

59

Hence it appears, that in 30 Days A will have run through 60 Circumferences, and B one Circumference only in the same time; so that A gains of B 59 Circumferences in 30 Days, say therefore

Circum. Days Circum. Day

59 : 30 :: 1 : $\frac{30}{59}$

Quest. 21. If 6 lb. of Sugar be equal in value to 7 lb. of Raisins; 5 lb. of Raisins to 2 lb. of Almonds; 3 lb. of Almonds to 5 lb. of Currans; 2 lb. of Currans to 18 d. how many Pence are the Value of 3 lb. of Sugar? *Ans.* 21 d.

multi. these cont.	{	6 S. = 7 R.	}	multi. these cont.
		5 R. = 2 A.		
		3 A. = 5 C.		
		2 C. = 18 d.		
		? d. = 3 S.		
180) 3780 (21				

Quest.

Quest. 22. If 3 Dozen pair of Gloves be equal in value to 2 pieces of Ribbon; 3 pieces of Ribbon to 7 Dozen of Points; 6 Dozen of Points to 2 Yards of Flanders-lace; and 3 Yards of Flanders-lace to 81 Shillings; how many Dozen pair of Gloves may be bought for 28 Shillings?

Answ. 2 Dozen pair of Gloves.

$$\begin{array}{rcl}
 \left. \begin{array}{l} 3 \text{ G.} = 2 \text{ R.} \\ 3 \text{ R.} = 7 \text{ P.} \\ 6 \text{ P.} = 2 \text{ L.} \\ 3 \text{ L.} = 81 \text{ S.} \\ 28 \text{ S.} = ? \text{ G.} \end{array} \right\} \begin{array}{l} \text{mult. these cont.} \\ \text{mult. these cont.} \end{array} & & 2268) 4536 (2 \\
 \hline & & 4536 \qquad 2268
 \end{array}$$

Quest. 23. Suppose a *Grayhound* to be coursing a *Hare*, in such sort, that the *Hare* takes five Leaps for every four Leaps of the *Grayhound*, and that the *Hare* is one hundred of her own Leaps distant from the *Grayhound*; now if three of the *Grayhound's* Leaps be equal to four of the *Hare's*; the Question is, to know how many Leaps the *Grayhound* must take before he obtain his Prey?

Answ. 1200 Leaps.

I. If $3 : 4 :: 4 : 5\frac{1}{3}$

Thus it appears, that 4 of the *Grayhound's* Leaps, are equal to $5\frac{1}{3}$ of the *Hare's* Leaps; and because by the Question the *Grayhound* takes 4 Leaps for every 5 of the *Hare's*, therefore the *Grayhound* in every four of his Leaps, gains $\frac{1}{5}$ of one of the *Hare's* Leaps; therefore say by the *Rule of Three*,

II. If $\frac{1}{5} : 4 :: 100 : 1200$

Quest. 24. There is a certain Room whose Basis is a long Square, which is in Circuit $50\frac{1}{2}$ Feet, and the Height of the Walls, or Sides of the Room is $8\frac{1}{4}$ Feet; all which Walls of the Room, except a space taken out for a Window in form of a long Square, whose height is five Feet, and breadth four Feet, are to be furnished with Hangings of Ell-broad Stuff at 3 s. 4 d. the Yard; the Question is to know how much Money the Stuff will cost.

U u 2

Answ.

Answ. 5 l. 17 s. $6\frac{2}{3}$ d.

$$50\frac{1}{2} \times 8\frac{1}{4} = 416\frac{5}{8} \text{ square Feet.}$$

$$5 \times 4 = 20 \text{ subtract}$$

$$396\frac{5}{8}$$

$$3\frac{3}{4} \times 3 = 11\frac{1}{4} \text{ square Feet in one Yard of Stuff.}$$

$$\text{Feet} \quad d. \quad \text{Feet} \quad d.$$

$$\text{If } 11\frac{1}{4} : 40 :: 396\frac{5}{8} : 1410\frac{2}{3}.$$

Quest. 25. There is a certain Walk which is a long Square, whose length is 40 Yards, and breadth 7 Yards, to be paved with Stones, every one of which being in form of a long Square, is 28 Inches in length, and 24 Inches in breadth; the Question is to know how many such Stones will be requisite to pave the said Walk?

Answ. 540.

Inches Inches

$$1440 \times 252 = 362880 \text{ square Inches.}$$

$$28 \times 24 = 672 \text{ square Inches.}$$

$$672 : 1 :: 362880 : 540 \text{ Stones.}$$

Quest. 26. Suppose a piece of *Tapestry* to be $5\frac{3}{8}$ Yards *Englsh* in length, and $3\frac{7}{8}$ Yards in breadth; the Question is, how many square Ells *Flemish* are contained in that piece of *Tapestry*, when the length of 1 Ell *Flemish* is equal to $\frac{3}{4}$ of a Yard *Englsh*.

Answ. $37\frac{4}{5}$ square Ells *Flemish*.

$$5\frac{3}{8} \times 3\frac{7}{8} = 13\frac{3}{8} \text{ square Yards.}$$

Then because $\frac{9}{16}$ of a square Yard is equal to 1 Ell Square of *Flemish* Measure (for $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$) say,

$$\text{If } \frac{9}{16} : 1 :: 13\frac{3}{8} : 37\frac{4}{5}.$$

Quest. 27. A Workman has performed a piece of Tiling, bearing the Form of a long Square, whose length is 273 Feet, 7 Inches, and breadth 21 Feet 5 Inches; now when Tiles are sold at the Rate of 11 s. $10\frac{3}{4}$ d. for 1000 Tiles, and every square of Tiling consisting of 10 Feet, as well in length as in breadth, takes up a 1000 Tiles, what does the said piece of Tiling amount to?

Answ. 34 l. 17 s. $0\frac{4001}{7600}$ d.

$$\text{I. } 273\frac{7}{12} \times 21\frac{1}{2} = 843\frac{731}{447} \text{ square Feet.}$$

d.

d.

$$\text{II. } 100 : 142\frac{3}{4} :: 843\frac{731}{447} : 8364\frac{4001}{7600}.$$

Quest. 28. A Merchant would bestow 200 l. in Cloves, Mace,

Mace, and Nutmegs, the Cloves being at 5 s. the Pound, the Mace at 11 s. the Pound, and the Nutmegs at 6 s. the Pound; now he would have of every sort an equal Quantity, the Question is, how many Pounds he may have of each sort?

Answ. 200 lb.

s.
5
11
6
—

22 : 1 : : 4400 : 200

The Proof.

lb.	s.		l.
200 at	5	amounts to	----- 50
200 at	11	amounts to	----- 110
200 at	6	amounts to	----- 60
			----- 220

Quest. 29. A Factor is to receive a Sum of Money, and is offered Dollars at 4 s. 4 d. which are worth but 4 s. 3 d. or French Crowns at 6 s. 1½ d. which are worth but 6 s. the Question is by which Coin he shall sustain the least Loss?

Answ. By the Dollars.

d. d. d. d.
52 : 1 : : 73½ : 1¼.

That is, in receiving the Dollars every 6 s. 1½ d. loses 1¼ d. but in receiving the Crowns 6 s. 1½ d. loses 1½ d. which is a greater Loss than 1¼ d.

Quest. 30. A Butcher agrees with a Grafter for the feeding of 20 Oxen, during the Space of 12 equal Months; but at 2 Months End, the Butcher adds 5 Oxen more, and 6½ Months after that, he added 10 Oxen more, and then it was agreed between them, that the Grafter shall feed them all, so long time as will be equivalent to the keeping of the first twenty during 12 Months; the Question is, how long time he shall feed them all, after the putting in of the last ten?

Answ. One Month.

Consider, that as he receives more Oxen to feed, he ought to keep them all the less time; therefore work as the Question imports by the *Rule of Three Inverse*.

Mon.

		Mon.	Oxen			Mon.	Oxen
		12	20				
		2	5				
Oxen							
If 20	:	10	::	25	:	(8	25
						6 $\frac{1}{7}$	10

If 25 : 1 $\frac{3}{7}$:: 35 : (1 Mon.

Examples of the Rule of Fellowship. *Quest. 31.* Two Merchants, viz. A and B, have entered Company; A puts in 500 *l.* and at 4 Months End takes out a certain Sum, leaving the Remainder to continue 8 Months longer: B puts in 250 *l.* and at five Months End puts in 300 *l.* more, and then his whole Sum continues 7 Months longer. Now at the making up of their Account, A finds that he has gained 106 $\frac{2}{7}$ Pounds, and B gained 133 $\frac{1}{7}$ Pounds; the Question is to know, How much A took out of the Bank at 4 Months End?

Ans. 240 *l.*

$$250 \times 5 = 1250$$

$$\text{add } 300$$

$$550 \times 7 = 3850$$

5100

$$133\frac{1}{7} : 5100 :: 106\frac{2}{7} : 4080$$

$$500 \times 4 = 2000 \text{ (subtract)}$$

$$8) 2080 \text{ (260)}$$

$$\text{Lastly, } 500 - 260 = 240 \text{ taken out by A.}$$

The Proof.

$$500 \times 4 = 2000$$

Subtract 240

$$260 \times 8 = 2080$$

4080

Quest. 32. Five Merchants, viz. A, B, C, D, and E, have gained 2025 *l.* which they divide in such sort, that $\frac{1}{2}$ of the Share of A is equal severally to $\frac{1}{4}$ of the Share of B, $\frac{1}{5}$ of C, $\frac{1}{6}$ of D, $\frac{1}{8}$ of E. The Question is, What was the Share of every Merchant?

Ans.

Answ. A 162 *l.* B 324 *l.* C 405 *l.* D 486 *l.* E 648 *l.*

Divide a Number at pleasure into several Parts, which may be in such Proportion as the Shares required, and proceed according to the subsequent Operation.

A 2		
B 4		
C 5		
D 6		
E 8		
25	2025	:

{

l.

2 : (162 for A, whereof $\frac{1}{2}$ is 81

4 : (324 for B, whereof $\frac{1}{4}$ is 81

5 : (405 for C, whereof $\frac{1}{5}$ is 81

6 : (486 for D, whereof $\frac{1}{6}$ is 81

8 : (648 for E, whereof $\frac{1}{8}$ is 81

2025

Quest. 33. Two Merchants A and B are in Company, the Sum of their Stocks is 300 *l.* the Money of A continuing in Company 9 Months, the Money of B 11 Months, they gain 200 *l.* which they divide equally; the Question is to know how much each Merchant did put in?

Answ. A 165 *l.* B 135 *l.*

Divide 300 into two such Parts which may be in Proportion, as 11 to 9, so will the greater Part be the Stock of A, and the lesser the Stock of B, which Stocks being multiplied by their respective Times, the Products will be equal.

11		
9		
20	300	::

{

11 : 165 for A

9 : 135 for B

Quest. 34. Two Merchants, viz. A and B, are in Company, A put in 325 *l.* more than B, and the Stock of A continued in Company $7\frac{1}{2}$ Months, B put in a certain Sum which is unknown, and it continued in Company $10\frac{3}{4}$ Months: After a certain Time they divided the Gain equally; the Question is, What each Merchant put in?

Answ. B 750 *l.* and A 1075 *l.*

Divide the Product of the Difference of their Stocks multiplied by the Time of A, by the Difference of their Times, so will the Quotient be the Stock of B, which added to 325 *l.* gives the Stock of A.

$$325 \times 7\frac{1}{2} = 2437\frac{1}{2}$$

$$3\frac{1}{4}) 2437\frac{1}{2} \quad (750 \text{ Stock of B.}$$

$$\text{add } 325$$

$$1075 \text{ Stock of A.}$$

Examples of the Rule of Alligation: How the Fineness of Gold and Silver is estimated, v.p. 79, 80

Quest. 35. A Goldsmith has some Gold of 24 Carects, other of 22 Carects, and another sort of 18 Carects, fine; he would so mingle these together, that the Mass mixed may be 60 l. and that the whole Mixture may bear 20 Carects fine. How much of every sort must he take?

Ans. $\left\{ \begin{array}{l} 12 \text{ of } 24 \text{ Carects.} \\ 12 \text{ of } 22 \text{ Carects.} \\ 36 \text{ of } 18 \text{ Carects.} \end{array} \right.$

$$20 \left\{ \begin{array}{l} 24 \\ 22 \\ 18 \end{array} \right. \begin{array}{l} 2 \\ 2 \\ 4 \end{array} + 2 \left| \begin{array}{l} 2 \\ 2 \\ 6 \end{array} \right.$$

$$10 : 60 :: \left\{ \begin{array}{l} 2 : 12 \\ 2 : 12 \\ 6 : 36 \end{array} \right.$$

Note. Some may think the Questions of *Alligation* are capable only of so many several Answers, as there are different Ways to connect the mean Rate or Price with the extreme Rates or Prices; yet it is most certain, that any ordinary Question of *Alligation*, where three or more things are proposed to be mixt in such manner as that Rule requires, is capable of infinite Answers, if Fractions be admitted, and sometimes of many Answers in whole Numbers, which are not discoverable by the common Rule of *Alligation*: So although to the last-mentioned Question, the said Rule of *Alligation* can find but one Answer only, which is before given; yet there are eight other Answers in whole Numbers, viz. these that follow; (the Invention whereof I have shewn in the 19th Question of the 13th Chapter of my second Book of *The Elements of Algebra*.)

Of 24 Carects	18	16	14	10
Of 22 Carects	3	6	9	15
Of 18 Carects	39	38	37	35

Of 24 Carects	8	6	4	2
Of 22 Carects	18	21	24	27
Of 18 Carects	34	33	32	31

Quest. 36. An Apothecary has several Simples, viz. A hot in 3° , B hot in 2° , C temperate, D cold in 2° , and E cold in 4° . Now he desires to make a Medicine of those Simples, in such sort that the Temper of it in respect of Quality may be in 1° . of heat, and the Quantity $8\frac{1}{2}$ Drams; the Demand is, What Quantity of every Simple he must take?

See Chap. 8. of this Appendix.

Ans. $4\frac{1}{2}$ Drams of A, $\frac{1}{2}$ Dram of B, $1\frac{1}{2}$ Dram of C, 1 Dram of D, and 1 Dram of E.

Indices		Drams	
8	6 {	1, 3, 5	9 A
7		1	1 B
5		2, 1	3 C
3		2	2 D
1		2	2 E

17

	Drams	
17 : $8\frac{1}{2}$::	{	2 : $4\frac{1}{2}$ A
		1 : $0\frac{1}{2}$ B
		3 : $1\frac{1}{2}$ C
		2 : 1 D
		2 : 1 E
		$8\frac{1}{2}$

Quest. 37. A Merchant buys 2 sorts of Cloths, viz. of Blacks and of Whites for 68 *l.* 2 *s.* after the Rate of 21 *s.* the Yard for the Blacks, and 12 *s.* the Yard for the Whites, and he takes so much of each Sort, that $\frac{2}{5}$ of the Number of Yards of the Black, are equal to $\frac{7}{6}$ of the White; the Demand is, How many Yards he bought of each sort?

Examples of the Rule of False Position.

Ans. 42 Yards of Black, and 40 Yards of White.

Quest. 38. A certain Person A pays to the Use of B for ever

X x

2500 *l.*

2500 *l.* in present Money, upon this Condition, that B shall pay to A an Annuity or yearly Rent to be continued four Years: the Equality of their Agreement being thus grounded, *viz.* the said 2500 *l.* is supposed to be put forth at Interest for a Year (to commence from the Time of their Agreement) at the Rate of 8 per cent. per annum. Then from the Sum of that Principal and Interest, (arising due at the Year's End,) the first Payment of the Annuity being subtracted, the Remainder is likewise supposed to be put forth at the same Rate of Interest for the second Year; then from the Composed of this Principal and Interest, (due at the second Year's End) the second Payment of the Annuity being subtracted, the Remainder is likewise supposed to be put forth at the same Rate of Interest for the third Year; then from this Principal and Interest the third Payment of the Annuity being subtracted, the Remainder is in like manner supposed to be put forth at the same Rate of Interest for the fourth Year: Lastly, from this Principal and Interest, the fourth and last Payment of the Annuity being subtracted, there must be nothing left; the Question is, What Sum of Money must be yearly paid to satisfy those Conditions?

Ans. 754 $\frac{14117}{17802}$ *l.* as will be manifest by the subsequent Proof.

I. $100 : 108 :: 2500 : 2700$

Subtract the first Payment $754\frac{14117}{17802}$

$$1945\frac{3485}{17802}$$

II. $100 : 108 :: 1945\frac{3485}{17802} : 2100\frac{14325}{17802}$

Subtract the second Payment $754\frac{14117}{17802}$

$$1346\frac{208}{17802}$$

III. $100 : 108 :: 1346\frac{208}{17802} : 1453\frac{12194}{17802}$

Subtract the third Payment $754\frac{14117}{17802}$

$$698\frac{18679}{17802}$$

IV. $100 : 108 :: 698\frac{18679}{17802} : 754\frac{14117}{17802}$

Subtract the last Payment $754\frac{14117}{17802}$

$$000$$

Quest. 39.

Mulae, Asinaeque duos imponit servulus utres

Impletos vius; segnemque ut vidit Asellam

Pondere defessam vestigia figere tarda,

Mula regat; quid chara parens cunctare, gemisque?

Unam

*Unam ex utre tuo mensuram si mihi reddas,
Duplum oneris tunc ipsa feram; sed si tibi tradam
Unam mensuram, fient æqualia utrique
Pondera: mensuras dic dōcte Geometer istas.*

The Sense is this: A *Mule* and an *Ass* carried two unequal Quantities of Wine, each consisting of a certain Number of Measures, in such sort, that if the *Ass* imparted one of her Measures to the *Mule*, then the *Mule*'s Number of Measures so encreased, would be the Double of those which the *Ass* had remaining: But if the *Mule* gave one Measure to the *Ass*, then the *Ass*'s Measures with that encrease would be equal to the *Mule*'s remaining Measures; the Question is, How many Measures each carried?

Ans. The *Mule* 7, and the *Ass* 5.

Quest. 40.

*Æs, ferrum, stannum miscens, aurique metallum,
Sexaginta minas pensantem finge coronam:
Æs aurumque duos simul efficiunto trientes:
Ternos quadrantes stanno mixtum impleat aurum:
At totidem quintos auri vis addita ferro.
Ergo age dic fulvi quantum tibi conjicis auri
Miscendum: dic quantum æris stannique requiras:
Dic quoque sufficiant duri quot pondera ferri:
Præscriptam ut valeas rite efformare coronam.*

The Sense is this: Suppose a Crown that shall weigh 60 lb. is to be made of Gold, Brass, Iron, and Tin, mixed together in such Proportion, that the Weight of the Gold and of the Brass together may be 40 lb. the joint Weight of the Gold and of the Tin 45 lb. and the joint Weight of the Gold and of the Iron 36 lb., the Question is, How much of every one of those four Metals must be taken?

lb.

Answer, $\left\{ \begin{array}{l} 30\frac{1}{2} \text{ of Gold.} \\ 9\frac{1}{2} \text{ of Brass.} \\ 5\frac{1}{2} \text{ of Iron.} \\ 14\frac{1}{2} \text{ of Tin.} \end{array} \right.$

Quest. 41. One being demanded what was the present Hour of the Day, answered, That the time then passed from Noon

X x 2

was

was equal to $\frac{1}{2}$ of $\frac{1}{2}$ of the time remaining until Midnight. The Question is, What a Clock it was? (Supposing the Time between Noon and Midnight to be divided into twelve equal Parts or Hours.

Ans. $\frac{3}{4}$ Hour after Noon.

Quest. 42. A Factor delivers 6 French Crowns and 2 Dollars for 45 Shillings Sterling; also, at another time he delivers 9 French Crowns and 5 Dollars (at the same Rate with the former) for 76 Shillings. The Question is to know the Value of a French Crown, also of a Dollar?

Ans. A Crown was valued at 6 s. 1 d. and a Dollar at 4 s. 3 d.

Quest. 43. A certain Usurer received 36 Dollars for the simple Interest of 186 l. lent for a certain Time unknown; also he received 90 Dollars for the Gain of 360 l. at the same Rate of Interest for a certain Time unknown; now the Sum of the Months in which both the said Numbers of Dollars were gained was twenty Months. The Question is to know in what time, as well the 36 Dollars, as the 90 Dollars, were gained?

Ans. The 36 Dollars were gained in $8\frac{3}{11}$ Months, and the 90 Dollars in $11\frac{1}{11}$ Months, as may be proved by the *Double Rule of Three*.

Which Answer may be discovered by the following Canon found out by the *Algebraick Art*.

Multiply the Dollars first gained, the latter Principal, and the given Time, according to the Rule of continual Multiplication, for a Dividend; then multiply the first Principal by the Dollars last gained; also multiply the latter Principal by the Dollars first gained, and reserve the Sum of these two last Products for a Divisor: Lastly, divide the Dividend first found by the said Divisor, so will the Quotient be the Time wherein the first Number of Dollars was gained, which subtracted from the Time given in the Question, discovers the Time in which the latter Number of Dollars was gained.

$$36 \times 360 \times 20 = 259200$$

$$= 8\frac{3}{11}$$

$$186 \times 90, + 300 \times 36, = 29700$$

And consequently $20 = 8\frac{3}{11} = 11\frac{1}{11}$.

Examples of the Extraction of Roots.

Quest. 44. If 3481 Soldiers are to be placed in a Square Battle, how many are to be set in Rank or in File?

Ans. 59. (for the square Root of 3481 is 59.)

Quest.

Quest. 45. If 4050 Soldiers are to be set in Battle in a Figure, which bears the Form of a long Square, in such manner, that the Number in File may be to the Number in Rank, as 1 to 2, How many Soldiers are to be placed in Rank, and how many in File?

Answ. 90 in Rank, and 45 in File, (found by this Canon or general Rule.)

As the greater Term of the Proportion given is to the lesser, so is the Number of Men to be placed in Battle to a fourth Proportional, whose square Root is the lesser Number sought, (whether it be for the Rank or File :) Also as the lesser Term of the given Proportion is to the greater ; so is the Number of Men to be set in Battle to a fourth Proportional, whose square Root is the greater Number sought (whether it be for the Rank or File.)

$$\begin{array}{lcl} \text{I.} & | & 2 : 1 :: 4050 : 2025 \\ \text{II.} & | & \sqrt{q} \ 2025 = 45 \text{ (Men in File. } \\ \text{III.} & | & 1 : 2 :: 4050 : 8100 \\ \text{IV.} & | & \sqrt{q} \ 8100 = 90 \text{ (Men in Rank. } \end{array}$$

The Proof.

$$45 \times 90 = 4050$$

$$\text{Also } 45 : 90 :: 1 : 2$$

Or when one of the Numbers sought (whether it be for the Rank or File) is found, the other may be discovered by *Division*, viz.

$$\begin{array}{r} 45 \overline{) 4050} \ 90 \\ 90 \overline{) 4050} \ 45 \end{array}$$

Quest. 46. Suppose the Wall of a Garrison to be in height 21 Foot, and the Breadth of the Moat surrounding the said Wall to be 28 Foot ; the Question is, What Length must a Scaling-ladder have to reach from the outermost Side of the Moat to the Top of the Wall?

Answ. 35 Foot, (to wit, the square Root of the Sum of the Squares of 21 and 28.)

$$\begin{array}{r} 21 \times 21 = 441 \\ 28 \times 28 = 784 \end{array}$$

$$\sqrt{q.} \ 2225 = 35$$

Quest. 47. If 100 l. being put forth for Interest at a certain Rate, will at the end of two Years be augmented to 112 $\frac{6}{100}$ l. (compound Interest, or Interest upon Interest being computed) what Principal and Interest will be due at the first Year's End?

Answ. 106 l. (composed of 100 l. Principal, and 6 l. Interest) which

which 106 is a mean Geometrically Proportional between 100 and 112.36, (and may be found by the eighteenth Rule of the fifth Chapter of this Appendix.)

$$100 \times 112.36 = 11236 \text{ (106)}$$

Quest. 48. If 100 *l.* being put forth for Interest at a certain Rate, will at the End of 3 Years be augmented to 115.7625 *l.* (compound Interest being computed,) What Principal and Interest will be due at the first Year's End ?

Ans. 105 *l.* (composed of 100 *l.* Principal, and 5 *l.* Interest) which 105 is the first of two mean proportional Numbers between 100 and 115.7625 *l.* (See the nineteenth Rule of the fifth Chapter of this Appendix.)

Various Practical Questions to exercise Decimal Arithmetick, in the Mensuration of Superficial Figures and Solids.

See the second
Section of the
23d Chapter
of the preceed-
ing Book.

Quest. 49. If the Side of a square Superficies be 3 Foot, what is the Area or Content of that Superficies ? Or (which is the same thing) how many Squares, every one of which is a Foot square, are contained in that Superficies ?

Ans. 9 square Foot, which Content is found out by multiplying the given Side 3 by itself, viz. 3 multiplied by 3 produces 9.

In like manner, if the Side of a square Pavement of Stone be 15.7 Foot, the superficial Content of that Pavement will be 246.49 Foot, that is, 246 Foot and an half very near, (for 15.7 multiplied by itself, produces 246.49.)

Likewise a square Piece of Wainscot, whose Side is 3.24 Yards, will be found to contain 10.4976 Yards, or 10 Yards and an half almost; for 3.24 multiplied by itself, to wit, by 3.24, will produce 10.4976.

Also if the Side of a square Piece of Land be 37.25 Perches, the Content in square Perches (neglecting the Fraction in the Product) will be found 1387, which being reduced (according to the seventh Table in Rule 4, Chapter 7, of the preceeding Book) will give 8 Acres, 2 Roods, and 27 Perches for the Content of that square Piece of Land.

Quest. 50. If a long Square be 8 Foot in length, and 5 Foot in breadth, what is the superficial Content ?

Ans. 40 Foot; which Content is found out by multiplying the Length by the Breadth, viz. 8 multiplied by 5 produces 40. So if one of the Lights of a Glass-Window supposed to be in form of a long Square, has for its Length 3.06 Foot, and

Breadth

Breadth 1.47 Foot, the Content of that Glass will be 4.4982 Foot, or 4 Foot and an half almost, for 3.06 multiplied by 1.47 produces 4.4982.

In like manner, if there be a piece of Wainscot, Plastring, or any other Superficies in form of a long Square, which is in length 6.325 Yards, and in breadth 3.214 Yards; the superficial Content will be found $20.32 +$ Yards, that is, 20 Yards, one quarter of a Yard, and somewhat more, for 6.325 multiplied by 3.214 produces $20.32 +$.

Likewise a piece of Tiling in form of a long Square, whose length is 18.5 Foot, and breadth 11.7 Foot, will be found to contain 216.45 square Foot, which may be reduced to 2.1645 Squares of Tiling, by allowing (according to Custom) 100 square Foot, to one Square of Tiling.

Also if a piece of Land in form of a long Square be 48.75 Perches in length, and 36.25 in breadth, the Area or Content in Perches will be found $1767.18 +$, which 1767 Perches being reduced, will give 11 Acres and 7 Perches for the Content of that piece of Ground.

Quest. 51. If it be required to set forth in a Meadow one Acre of Grass to lie in the Figure of a long Square, and that the length of it be limited or agreed to be 20 Perches, what must the breadth be?

Answ. 8 Perches, which breadth is found out by dividing 160 (the Number of square Perches contained in an Acre) by the given length 20. If two Acres were required, then 320 (to wit, twice 160) must be divided by the given Side, whether it be the length or breadth; so if 7.25 Perches be prescribed for the breadth of two Acres, the length must be $44.13 +$ Perches.

In like manner, if the breadth of a Board be 1.32 Foot, and it be demanded how far one ought to measure along the Side of it to have a superficial Foot, or a Foot square of that Board; divide 1, by the given Breadth, so you'll find in the Quotient this decimal Fraction $.757 +$, which represents three quarters of a Foot, or nine Inches and somewhat more, and so much in length ought to be measured along the side of that Board to make a superficial Foot. Likewise if the breadth of a Board be given in Inches, then 144 (the Number of square Inches contained in a superficial Foot square) being divided by the given Breadth, the Quotient will shew how many Inches ought to be measured along the side of that Board to make a superficial Foot; so the breadth of a Board being 9 Inches, the length forward to make a superficial Foot will be found 16 Inches.

Quest.

Quest. 52. If the three Sides of a piece of Land that lies in form of a Triangle be 15 Perches, 14 Perches, and 13 Perches, what is the Area or Number of square Perches contained in that Triangle?

Ans. 84 Perches, or half an Acre and four Perches, which Content is found out by this Rule, *viz.*

From half the Sum of the three Sides of any plain Triangle, subtract each of the three Sides severally, and note the three Remainders; then multiply the said half Sum and those three Remainders one into the other (according to the Rule of continual Multiplication;) that done, extract the square Root of the last Product, so shall such square Root be the Area or Content of the Triangle.

		Perches
The three Sides of a Triangle	-----	} 15 14 13
The Sum of the three Sides	-----	42
The half of that Sum	-----	21
The three Remainders found out by subtracting every Side from the half Sum	-----	} 6 7 8
The Product arising from the continual Multiplication of the four last Numbers	-----	} 7056
The square Root of which Product is the Content required, to wit,	-----	} 84

Another Example.

		Perches
The three Sides of a Triangle	-----	} 120 . 5 112 . 6 90 . 3
The Sum of the three Sides	-----	323 . 4
The half of that Sum		161 . 7
The three Remainders found by subtracting each Side from the half Sum	-----	} 41 . 2 49 . 1 71 . 4
The Product arising from the continual Multiplication of the four last Numbers	-----	} 23355380 . 1096
The square Root of that Product	-----	4832 . 7 + Where.

Wherefore I conclude, that the Content of a plain Triangle, whose three Sides are 120.5 Perches, 112.6 Perches, and 90.3 Perches, is 4832.7 $\frac{1}{2}$ Perches, which reduced, give 30 Acres and 32 Perches, (the Fraction of a Perch being neglected.)

Now since every irregular Piece of Ground may be divided into Triangles; for a four-sided Field will be divided into two Triangles by one imaginary straight Line leading overthwart from Corner to Corner, called a *Diagonal Line*; a five-sided Field into three Triangles by two *Diagonals*; a six-sided Ground into four Triangles by three *Diagonals*, &c. the Rule before given will be of excellent Use to find out the Contents of large Fields, especially if the Land be of a dear Value; as also when any Controversy arises by reason of the different Admeasurements of Surveyors of Land: For if the Sides of those Triangles be measured in the Field, and their Lengths be agreed on, all Artifts to whom the Reason of the Rule before given is known, will agree in one and the same Content. But yet this way of measuring presupposes that there is no Obstacle, as Water, Wood, or other Impediment, to hinder the measuring of the Sides of those Triangles into which the Field is divided as aforesaid.

Quest. 53. If the Diameter of a Circle be 28.25; What is the Circumference?

Answ. 88.749 $\frac{1}{2}$: For as 113 is in Proportion to 355; or as 1 is to 3.14159, so is the Diameter to the Circumference: Therefore always multiplying the Diameter given by the said 3.14159, the Product will be the Circumference required.

Quest. 54. If the Diameter of a Circle be 28.25, what is the superficial Content of that Circle?

Answ. 626.79 $\frac{1}{2}$: For as 1 is in Proportion to .78539, so is the Square of the Diameter to the superficial Content. Therefore always multiplying the said decimal Fraction .78539 by the Square of the given Diameter (which Square is the Product of the Multiplication of the Diameter by it self,) the Product shall be the superficial Content required.

Quest. 55. If the Diameter of a Circle be 28.25, What is the Side of a Square which may be inscribed within the same Circle?

Answ. 19.975 $\frac{1}{2}$: For the square Root of half the Square of the Diameter, or the square Root of the Double of the Square of the Semi-Diameter, will be the Side of the inscribed Square sought. Otherwise, as 1 is to .707166, so is the Diameter to the Side required. Therefore if you multiply (always) the

said .707166 by the Diameter given, the Product will be the Side of the inscribed Square required.

Quest. 56. If the Circumference of a Circle be 88.75, What is the Diameter?

Answ. 28.249 +: For as 355 is to 113, or as 1 is to .318309, so is the Circumference to the Diameter. Therefore if .318309 be always multiplied by the given Circumference, the Product shall be the Diameter required.

Quest. 57. If the Circumference of a Circle be 88.75, What is the superficial Content of that Circle?

Answ. 626.801 +: For as 1 is to .079578, so is the Square of the Circumference to the superficial Content. If therefore .079578 be always multiplied by the Square of the given Circumference, the Product will be the superficial Content sought.

Quest. 58. If the Circumference of a Circle be 88.75, What is the Side of a Square that may be inscribed within the same Circle?

Answ. 29.975 +: For as 1 is to .225078, so is the Circumference to the Side required: Therefore if 225078 be always multiplied by the Circumference given, the Product will be the Side of the inscribed Square sought.

Quest. 59. If the superficial Content of a Circle be 626.8, What is the Diameter?

Answ. 28.25 +: For as 1 is to 1.27324, so is the Content to the Square of the Diameter. Therefore multiplying (always) 1.27324 by the given Content, the square Root of that Product shall be the Diameter required.

Quest. 60. If the superficial Content of a Circle be 626.8, What is the Circumference?

Answ. 88.75 +: For as 1 is to 12.5664, so is the Content to the Square of the Circumference. If therefore 12.5664 be always multiplied by the given Content, the square Root of the Product will be the Circumference required.

Quest. 61. If the superficial Content of a Circle be 626.8, What is the Side of a Square equal to the same Circle?

Answ. 25.035 +: For the square Root of the given Content is the Side of the Square required.

Quest. 62. If the Side of a Cube be 12 Inches, How many cubical Inches are contained in that Cube?

Answ. 1728. What a Cube is may be well represented by a Dye, which is a little Cube itself, being a Rectangular or square Solid, that has an equal length, breadth, and depth, and is comprehended under six equal Squares: Now if the Side of one of those

those equal Squares (which is also the Side of the Cube,) be 12 Inches, the superficial Content of that Square will be 144 square Inches ; for (according to the preceeding 49th Question) 12 multiplied by 12 produces 144, which multiplied by the Depth 12 Inches, produces 1728 cubical Inches, and such is the solid Content of that Cube, whose Side is 12 Inches : So that by one Foot of Timber or Stone, in whatsoever kind of Solid it be found, is understood a Cube, containing 1728 cubical or Dye-square Inches, and consequently half a Foot solid contains 864 cubick Inches, and a quarter of a Foot solid contains 432 cubick Inches.

In like manner, if the Side of a Cube of Stone be 2.53 Foot, the solid Content of that Cube will be found 16.194 + Foot ; for 253 being multiplied by itself, produces 6.4009 superficial Foot, which Product being multiplied by the said 2.53, will produce 16.194 + solid Foot.

Also if the Side of a Cube of Stone or Wood be 6 Inches, or .5 Foot, the solid Content will be found 216 cubick Inches, or .125 Parts of a Foot solid ; (for 6 multiplied cubically produces 216 ; likewise .5 multiplied cubically produces .125 ;) whence it may be inferr'd, that 8 little Cubes of Stone or Wood, every one of which is half a Foot, or 6 Inches square, are contained in a Foot of Stone or Timber ; for 8 times 216 produces 1728 (being the Number of cubick Inches contained in a Foot solid ;) likewise 8 times .125 produces 1, (to wit, one entire Foot solid.)

Quest. 63. If the Breadth of a squared piece of Timber, supposed to be straight and terminated at both ends by two equal Squares, be 1.55 Foot, the Depth also 1.55 Foot, and the Length 17.33 Foot, How many cubick Feet are contained in that piece of Timber ?

Ans. 41.635 Foot ; that is, 41 Foot and an half, and about half a quarter of a Foot : Which solid Content is found out by this Rule, *viz.* multiply the Breadth 1.55 by the Depth 1.55, the Product will be 2.4025 superficial Foot, which is the Content of the Base, (that is, the Area of either of the two equal Squares at the Ends of the Piece :) Lastly, multiplying the said Base 2.4025 by the Length 17.33, the Product will be 41.635 +, which is the solid Content required.

In like manner, if the Breadth of a squared Piece of Timber, supposed to be straight and terminated at both Ends by two equal long Squares (which are called the Bases) be 2.34 Foot, the Depth 1.61 Foot, and the Length 17.58 Foot, the solid Content will be 66.23 + Foot ; for (as before) multiplying the

Y y 2

Breadth

Breadth by the Depth, and that Product by the Length, the last Product will be the solid Content required.

Quest. 64. If the Breadth, as also the Depth of a squared piece of Timber having equal square Bases, be 1.55 Foot, How far ought one to measure along the Length of that piece of Timber to make a Foot solid?

Answ. .416 parts of a Foot, or 5 Inches very near; which Decimal is thus discovered: *viz.* First find the superficial Content of the Base, which will be 2.4025 (for 1.55 multiplied by 1.55 produces 2.4025:) Then dividing 1 (to wit, 1 solid Foot) by the Base 2.4025, the Quotient will be .416+ or $\frac{416}{1000}$ parts of a Foot, or 5 Inches almost, and so far ought to be measured along the Length of the Piece to make a Foot solid. In like manner, if the Breadth be 2.34 Foot, and the Depth 1.61 Foot, the Length forward along the Piece to make one solid Foot, will be found .265 Parts of a Foot, or three Inches and almost $\frac{1}{3}$ Part of an Inch.

Quest. 65. If a straight squared Piece of Timber be terminated by unequal Bases, whereof one contains 1.92 superficial Foot, the other .85 Foot, and the Length of that piece of Timber be 17.4 Foot, What is the solid Content, or how many cubical Feet are contained in that piece of Timber?

Answ. 23.474 + Foot; (found out by one of Mr. Oughtred's Rules for measuring a Segment of a Pyramid in *Problem* 21, *Chapter* 19, of his *Clavis Mathematic.*) The Rule is this:

Multiply the greater Base by the less, and extract the square Root of that Product; then multiply the Sum of the two Bases and that square Root by one third Part of the Length of the Solid proposed, so will the last Product be the solid Content required.

Example.

The greater Base	_____	_____	1 .92
The lesser Base	_____	_____	0 .85
The Product of the Multiplication of those two Bases	_____	_____	} 1 .6320
The square Root of that Product	_____	_____	1 .2774
The Sum of that square Root and the two Bases--	_____	_____	4 .0474
One third Part of the Length is	_____	_____	5 .8
The Product of the Multiplication of the two last Numbers is the solid Content required	_____	_____	22 .474 +

Quest.

Quest. 66. A Pyramid is a Solid comprehended under plain Surfaces, and from a triangular, quadrangular, or any multangular Base, diminishes equally less and less, till it finish in a Point at the Top ; now if the superficial Content of the Base of a Pyramid be 5.756 Foot, and the Height of it 14.25 Foot, (which Height is the Length of the perpendicular Line that falls from the Top of the Pyramid to the Base ;) What is the solid Content of that Pyramid ?

Answ. 27.341 + Foot ; for if the Area of the Base of a Pyramid be multiplied by one third Part of the Height thereof, the Product will be the solid Content of the Pyramid ; therefore $5.756 \times 4.75 = 27.341$ Foot = the Solidity of the Pyramid proposed.

Note. If a Pyramid be cut into two Segments by a Plane parallel to the Base, one of those Segments will be a Pyramid, and the other will have two unequal Bases ; for the measuring of which latter Segment, a Rule has been already given in the sixty-fifth Question, the Area of each Base being known.

Quest. 67. A Cone is a Solid, which has a Circle for its Base, from whence it grows equally less and less (like a round Steeple of a Church) till it end in a point at the Top ; now if the Area of the Base of a Cone be 5.756 Foot, and the Height of it be 14.25 Foot, What is the solid Content of that Cone ?

Answ. 27.341 Foot ; for if the Area of the Base of a Cone be multiplied by one third Part of the Height thereof, the Product shall be the solid Content of the Cone.

Note. If a Cone be cut into two Segments by a Plane parallel to the Base, one of those Segments will be a Cone, and the other Segment will have two unequal Bases which are Circles ; the Solidity of which latter Segment may be found out by the Rule before given in the 65th Question, the Area of each Base (or Circle) being known.

Quest. 68. A Cylinder is a Solid which may be well represented by a Stone-roll, such as are used in Gardens for the rolling of Walks. Now if the Circumference of a Cylinder be 4.57 Foot, and the Length 3.25 Foot, What is the solid Content of that Cylinder ?

Answ. 5.4 + Foot, thus found out : First, by the Help of the given Circumference 4.57, find out the superficial Content of that Circle, (being the Base of the Cylinder ;) which Content (by the preceeding 57th Question) will be found 1.6619 + Foot ; then multiplying the said 1.6619 by the given Length 3.25, the Product will be 5.402075, which is the solid Content required.

Quest.

Quest. 69. If the Base of a Cylinder be 1.6619 Foot, How much in length of that Cylinder will make a Foot solid?

Answ. .601 Parts of a Foot; for 1 (to wit, 1 solid Foot) being divided by the Base 1.6619, gives in the Quotient the Decimal .601 + for the Length required.

Quest. 70. A Globe is a perfect round Body contained under one Surface; in the middle of the Globe there is a Point called the Center, from whence all straight Lines drawn to the Outside are of equal length, and called Semi-diameters, the Double of any one of which is equal to the Diameter of the Globe; now if the Diameter of a Globe of Stone be 1.75 Foot, How many Feet solid are contained in that Globe?

Answ. 2.807 + Foot; for as 21 is in Proportion to 11, or as 1 is to .5238, so is the Cube of the Diameter to the solid Content of the Globe: Therefore, always multiplying the Cube of the Diameter by the said Decimal .5238, the Product will be the solid Content required: So the Diameter 1.75 being first multiplied by itself, the Product will be 3.0625, which multiplied by the said 1.75, gives in the Product 5.359375, to wit, the Cube of the Diameter, which being multiplied by .5238, the Product thence arising will be 2.807 +, which is the Solidity of the Globe propounded.

Quest. 71. What is the Diameter of a Globe of Stone, which contains 4 cubical or solid Feet?

Answ. 1.96 + Foot; for as 11 is in Proportion to 21, or as 1 is to 1.9090909, so is 4 (the solid Content given) to a fourth Proportional, to wit, 7.636363 +, whose cubick Root is 1.96 +, the Diameter required.

Concerning the Gauging of Vessels.

The easiest and aptest Methods for practice in Gauging, are those which are performed by the Help of Tables, or Gauging-rods purposely composed: However, to give the Reader some light in this matter, I shall here insert one Rule to find out the Number of Gallons contained in a full Tun, Pipe, Hoghead, Barrel, or such like Vessel, according to Mr. *Wingate's* Manner of reducing a Vessel to a Cylinder.

The Rule is this:

Having found the Difference of the two Diameters at the Bung and Head of the Vessel, take $\frac{7}{8}$ of that Difference and add

add to it the lesser Diameter; then square that Sum and reserve the Product; that done, if the Content be required in Wine-Gallons, multiply the Product reserved, this decimal Fraction .0034, and the Length of the Vessel, one into the other (according to the *Rule of continual Multiplication*,) so will the last Product be the Number of Wine-Gallons required: But if the Content be demanded in Ale-Gallons, multiply the Product before reserved, this decimal Fraction .0027, and the Length of the Vessel, one into the other continually, so shall the Product be the Content in Ale-Gallons: This Rule I shall first explain by two Questions, and then shew how it is raised.

Quest. 72. If the Diameter at the Bung of a Vessel be 32 Inches, the Diameter at the Head 28.2 Inches, and the Length 39 Inches, (which Dimensions are said to agree very near with those of an *English* Vessel called a Pipe) What is the Content of that Vessel in Wine-Gallons?

Ans. 126.278 Wine-Gallons; that is, 126 Wine-Gallons, and about a Quart more, (found out by the Rule above-given;) as will be manifest by the following Operation.

Explication.

The Diameter at the Bung	_____	_____	32.0
The Diameter at the Head	_____	_____	28.2
Their Difference	_____	_____	3.8
Which multiplied by $\frac{7}{10}$, that is	_____	_____	0.7
The Product will be	_____	_____	2.66
This added to the lesser Diameter, gives the	}		30.86
mean Diameter			
Which mean Diameter being squared (that	}		952.3396
is, multiplied by itself) produces			
This Product multiplied by	_____	_____	0.0034
The Product thence arising will be	_____	_____	3.2379 +
Which multiplied by the Length of the Vessel--	_____	_____	39.0
The Product is the Number of Wine-Gal-	}		126.278 +
lons sought, viz.			

Quest. 37. If the Diameter at the Bung of a Barrel be 23 Inches, the Diameter at the Head 19.9 Inches, and the Length 27.4 Inches, What is the Content of that Barrel in Ale-Gallons?

Ans. 36.031 Ale-Gallons; that is, 36 Gallons and about a quarter of a Pint more (found out by the preceding Rule.)

Explica-

Explication.

The Diameter at the Bung	_____	_____	23.0
The Diameter at the Head	_____	_____	19.9
Their Difference	_____	_____	3.1
Which multiplied by $\frac{7}{10}$, that is	_____	_____	0.7
The Product will be	_____	_____	2.17
This added to the lesser Diameter, gives the	}		22.07
mean Diameter			
This mean Diameter being squared (that is,	}		487.0849
multiplied by itself) produces			
Which Product multiplied by	_____	_____	0.0027
The Product thence arising is	_____	_____	1.315 +
Which multiplied by the Length of the Vessel---	_____	_____	27.4
The Product is the Number of Ale-Gallons	}		36.031 +
sought, to wit			

The Reason of the Rule.

Two Things are taken for granted in the said Rule, *viz.* First, it is supposed that if $\frac{7}{10}$ of the Difference of the two Diameters at the Bung and Head, be added to the lesser Diameter, the Sum will be an equated or mean Diameter; (near enough for practical Use, though it be not exact:) *viz.* If there be a Cylinder, whose Diameter is equal to that mean Diameter, and whose Length is equal to the Length of the Vessel, that Cylinder shall be equal to the Capacity of the Vessel very near. Secondly, the said Rule presupposes that 231 cubick Inches are equal to a Wine-Gallon, and 282 equal to an Ale-Gallon; concerning which Equalities (especially the latter) Artifts differ somewhat in their Experiments; but according to any Equality which in that particular shall be agreed on, from this that follows a Rule may be framed, and Tables thence calculated for gauging a full Vessel without considerable Errour.

Taking then those two things above-mentioned for granted, we may rightly infer, that if a Cylinder has for its Base a Circle, whose superficial Content is 231 Inches, every Inch in length of that Cylinder will contain 231 cubick Inches, or one entire Wine-Gallon. Now forasmuch as all Circles are in such Proportion one to the other as the Squares of their Diameters, it shall be as 294.11844, (to wit, the Square of the Diameter of that

the Circle, whose superficial Content 231) is to 1, (to wit, the superficial Content 231 considered as the Base of one Wine-Gallon ;) or as 1 is to .0034 : So is the Square of the equated (or any other) Diameter, to the superficial Content of that Circle in Wine-Gallons and parts of a Gallon ; which Content multiplied by the length of the Vessel, will produce its Solidity or Capacity in Wine-Gallons. Therefore the first Part of the preceeding Rule for finding the Number of Wine-Gallons contained in a full Vessel is manifest : And after the same manner, supposing as before 282 Cubick Inches are equal to an Ale-Gallon, the Decimal .0027 prescribed in the said Rule will be found out.

Upon those Grounds Mr. *Wingate* composed his Gauging-Rod ; Mr. *Oughtred* also, in his Circles of Proportion, has delivered another Rule for Gauging, from whence his Gauging-Rod is deduced ; but the particular Constructions of those Rods, and likewise the making of Tables for the same purpose, being handled by several Artists, I shall not insist upon them.

Now if the industrious and more curious Arithmetician, after he is well exercised in vulgar Arithmetick, desires further Knowledge in finding out the Answer of subtil Questions about Numbers, his best Guide will be the admirable *Algebraical* Art, which discovers Rules for the solving of *Problems*, as well Arithmetical as Geometrical, that are above the Reach of any of the Rules of common Arithmetick, or practical Geometry, as may partly appear by the two Rules in the fore-going 52 and 65 Questions, as also by the two following Questions, with which I shall conclude this Chapter.

Quest. 74. To find two Numbers in a given Proportion, suppose the lesser to the greater, as 2 to 3, and such, that if the lesser Number be added to the Square of the greater ; also if the greater Number be added to the Square of the lesser, the two Sums shall be square Numbers, whose Roots are expressible by rational or true Numbers (Fractions being admitted for Numbers.)

Ans. $\frac{1}{12}$ and $\frac{3}{12}$.

The Proof.

The Square of $\frac{3}{4}$ (the greater Number) is $\frac{9}{16}$
 To which adding the lesser Number $\frac{1}{4}$ $\frac{4}{16}$
 The Sum in its least Terms will be $\frac{13}{16}$
 Which is a square Number, whose Root is $\frac{\sqrt{13}}{4}$
 Again, the Square of $\frac{1}{4}$ (the lesser Number) is $\frac{1}{16}$
 To which adding the greater Number $\frac{3}{4}$ $\frac{12}{16}$
 The Sum in its least Terms will be $\frac{13}{16}$
 Which is a square Number, whose Root is $\frac{\sqrt{13}}{4}$

Also the said Numbers $\frac{1}{4}$ and $\frac{3}{4}$ are one to the other, as 2 to 3, wherefore the Question is solved; which Numbers $\frac{1}{4}$ and $\frac{3}{4}$ are found out by this following

Theorem.

If the Fraction $\frac{1}{4}$ be divided into any two Parts; either of those Parts being encreased with the Square of the other Part, will give a Fraction, having a rational square Root.

Wherefore by dividing $\frac{1}{4}$ into the two Fractions $\frac{1}{16}$ and $\frac{3}{16}$, which are in the prescribed Proportion of 2 to 3, those Fractions will satisfy the Conditions in the Question propounded.

Likewise these two Fractions $\frac{7}{16}$ and $\frac{1}{16}$ will answer the Question, and are discovered without extracting any Root; but the manner of finding out the said Theorem and last-mentioned Fractions, I have shewn in the 24th Question of my 3d Book of the Elements of *Algebra*.

Quest. 75. To find three Numbers, such, that the Square of any one of them being added to the other two Numbers, the Sum of such Addition shall be a square Number, whose Root is a rational Number.

Ans. 1, $\frac{8}{3}$, and $\frac{16}{3}$.

The Proof.

First, the Square of the first Number 1 is 1
 To which adding the second and third Numbers $\frac{8}{3}$ and $\frac{16}{3}$, the Sum will be 9
 Which is a square Number, whose Root is 3
 Secondly, the Square of the Number $\frac{8}{3}$ is $\frac{64}{9}$
 To which, adding the first and third Numbers 1 and $\frac{16}{3}$, the Sum in its least Terms will be $\frac{77}{9}$
 Which is a square Number, whose Root is $\frac{\sqrt{77}}{3}$
 Thirdly, the Square of the third Number $\frac{16}{3}$ is $\frac{256}{9}$
 To which, adding the first and second Numbers 1 and $\frac{8}{3}$, the Sum in its least Terms will be $\frac{241}{9}$
 Which is a square Number, whose Root is $\frac{\sqrt{241}}{3}$

Where.

Wherefore it is manifest that the three Numbers 1 , $\frac{8}{3}$, and $1\frac{1}{3}$, will satisfy the Conditions in the Question, which may also be solved by other Numbers; but the manner of finding them out I have shewn in the 32 Question of my 3d Book of the Elements of *Algebra*.

C H A P. XI.

Of Sports and Pastimes.

P R O B L. I.

To discover a Number which any one shall have in his Mind, without requiring him to reveal any part of that or any Number whatsoever.

AFTER any one has thought upon a Number at pleasure, bid him double it, and to that Double bid him add any such even Number as you please to assign; then from the Sum of that Addition let him reject one half, and reserve the other half: Lastly, from this half bid him subtract the Number which he first thought upon; then may you boldly tell him what Number remains in his Mind after that Subtraction is made, for it will always be half the Number which you assigned him to add.

For Example; Suppose he thought upon 6, the double thereof is 12, to which bid him add some even Number at your pleasure, suppose 4, so will the Sum be 16, whereof the half is 8, from which if he subtract 6, (the Number first thought on) the Remainder is 2, (to wit, half the Number 4, which was by you assigned to be added;) which Remainder you discover, notwithstanding all the Operation was performed in his Mind, without his making known any Number whatsoever. *Note*, That the adding of an even Number, as aforesaid, is not of Necessity, but only to avoid a Fraction that will arise, by taking the half of an odd Number.

The Reason of the Rule.

If to the Double of any Number (which Number for distinction sake I call the first) a second Number be added, the half of the Sum must necessarily consist of the said first Number, and

Z z z

half

half the second ; therefore if from the said half Sum the first Number be subtracted, the Remainder must of necessity be half the second Number which was added.

P R O B L. II.

Two Numbers, the one even and the other odd, being proposed unto two Persons, to the end they may (out of your sight) severally chuse one of those Numbers ; to discover which of these Numbers each Person had chosen.

Suppose you have propounded to *Peter* and *John* two Numbers, the one even and the other odd, as 10 and 9, and that each of those Persons is to chuse one of the said Numbers unknown to you. Now to discover which Number each Person made choice of, you must take two Numbers, the one even and the other odd, as 2 and 3 ; then bid *Peter* multiply that Number which he has chosen by 2 ; and cause *John* to multiply that Number which he has pitched upon by 3 ; that done, bid them add the two Products together, and let them make known the Sum to you, or else demand of them whether the said Sum be even or odd, or by any other way more secret endeavour to discover it, by bidding them take the half of the said Sum, for by knowing whether the said Sum be even or odd, you obtain the principal End to be aimed at ; because if the said Sum be an even Number, then infallibly he that multiplied his Number by your odd Number, (to wit, by 3) did chuse the even Number (to wit, 10 ;) but if the said Sum happen to be an odd Number, then he whom you caused to multiply his Number by your odd Number, (to wit, 3) did infallibly chuse the odd Number (to wit, 9.)

For Example ; If *Peter* had made choice of 10, and *John* 9, suppose you required *Peter* to multiply his Number 10 by 2, and *John* to multiply his Number 9 by 3 ; the Products will be 20 and 27, whereof the Sum is 47, which being an odd Number, you may thence conclude that *John*, whom you caused to multiply his Number by 3, chose the odd Number 9, and therefore *Peter* took 10. But if you had ordered *John* to multiply his Number 9 by 2, and *Peter* to multiply his Number 10 by 3, the Products would have been 18 and 30, whereof the Sum is 48, which is an even Number ; from whence you may infer, that he that multiplied his Number by 3 pitched upon the even Number, and therefore *Peter* chose 10, and *John* 9.

Demon-

Demonstration.

The Reason of the said Rule is very easy, and depends principally upon the 28th and 29th Propositions of the 9th Book of *Euclid*; for one may infer from the 21st of the same Book, that an even Number multiplied by any Number whatsoever produces an even Number, but an odd Number is of a different Quality; for if it be multiplied by an even Number, the Product is an even Number, (by the said 28th Proposition;) and if it be multiplied by an odd Number, the Product is odd (by the said 29th Proposition.) Therefore if in making this Sport, it happens that the even Number be multiplied by your odd Number, both the Products will be even, and consequently the Sum shall be infallibly an even Number (by the said 21st Proposition.) But if it so fall out, that you cause the odd Number to be multiplied by your odd Number, that Product will be odd, and the other Product even; therefore the Sum of these two Products shall be an odd Number, as *Clavius* has demonstrated upon the 23 of the 9th of *Euclid*.

P R O B L. III

A certain Number of distinct Things being propounded, to dispose them in such an Order, that casting away always the ninth, or the tenth, or any other that shall be assigned, to a certain Number, those remaining may be such as were first intended to be left.

This Problem is usually proposed in this manner, *viz.* Fifteen *Christians* and fifteen *Turks* being at Sea in one and the same Ship in a terrible Storm, and the Pilot declaring a Necessity of casting the one half of those Persons into the Sea, that the rest might be saved; they all agreed, that the Persons to be cast away should be set out by Lot after this manner, *viz.* the thirty Persons should be placed in a round Form like a Ring, and then beginning to count at one of the Passengers, and proceeding circularly, every ninth Person should be cast into the Sea, until of the thirty Persons there remained only fifteen. The Question is, How those thirty Persons ought to be placed, that the Lot might infallibly fall upon the fifteen *Turks*, and not upon any of the fifteen *Christians*? For the more easy remembring of the Rule to resolve this Question, I shall presuppose the five Vowels, *a, e, i, o, u*, to signify five Numbers, to wit, (*a*) one, (*e*) two,

(*e*) two, (*i*) three, (*o*) four, and (*u*) five ; then will the Rule itself be briefly comprehended in these two following Verses :

*From Numbers, Aid, and Art,
Never will Fame depart.*

In which Verses you are principally to observe the Vowels, with their correspondent Numbers before assigned ; and then beginning with the *Christians*, the Vowel *o* (in *from*) signifies, that four *Christians* are to be placed together ; next unto them the Vowel *u* (in *Num.*) imports that five *Turks* are to be placed together ; in like manner *e* (in *bers*) denotes two *Christians* ; *a* (in *Aid*) one *Turk* ; *i* (in *Aid*) three *Christians* ; *a* (in *and*) one *Turk* ; *a* (in *Art*) one *Christian* ; *e* (in *ne.*) two *Turks* ; *e* (in *ver*) two *Christians* ; *i* (in *will*) three *Turks* ; *a* (in *Fame*) one *Christian* ; *e* (in *Fame*) two *Turks* ; *e* (in *de*) two *Christians* ; *a* (in *part*) one *Turk*.

The Invention of the said Rule, and such like, depends upon the subsequent Demonstration, viz. If the Number of Persons be thirty, let thirty Figures or Cyphers be placed circularly, or else in a right Line as you see,

oooooooooooooooooooooooooooooooooooo

That done, begin to count from the first, and mark the ninth (or what other shall be assigned) by putting a Point or Cross over it ;) then count forward from that which you have marked, and place another Point over the next ninth ; and continue to do the same, beginning again when you shall be at the End (if the Cyphers are placed in a right Line,) and passing over those, which you had already marked, until you have marked the Number required, as in the Example propounded, until you have marked fifteen ; for then all the Cyphers marked shall be those which must be cast away, and the others those that are to remain. Hence it is evident, that if you observe how those Cyphers marked, are disposed among those which are not marked, you will easily make a Rule for any Number whatsoever.

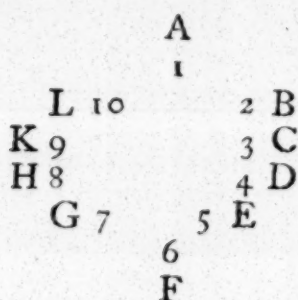
By this Invention (as some conjecture) the famous Historian *Josephus* the *Jew*, preserved his Life very subtilly in the Cave, to which himself and forty of his Countrymen had fled from the furious and conquering *Romans* at the Siege of *Jotapata* : For his said Countrymen having most wickedly resolved to kill one another, rather than yield to their Enemies, he at length

length (when no Arguments that he could use could diffwade them from so horrid an Act) prevailed with them to execute their tragical Design by Lot; and so by the Help of the afore-said Artifice, as we may suppose, himself with one other Person only remaining alive, after the rest were inhumanly murdered, they agreed to put an end to the Lot, and thereby save their Lives. This Story you may see at large in the fourteenth Chapter of the third Book of the History of *Josephus*, of the Wars of the *Jews*.

P R O B L. IV.

Many Numbers which proceed from 1 or Unity, in a Progression, according to the natural Order of Numbers, (such as these, 1, 2, 3, 4, 5, 6, &c. (being placed in a round Form like a Ring; to discover which of those Numbers any one has thought upon.

Let any multitude of Numbers in the afore-said Progression, suppose these ten, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be mark'd upon ten Ivory-counters (or for want thereof upon ten small pieces of Paper) which may be represented by these ten Letters, A, B, C, D, E, F, G, H, K, L, viz. suppose 1 to be writ upon the Counter A, 2 upon B, 3 upon C, &c. Then having placed those Counters circularly as you see (with their blank Faces uppermost, and the Figures underneath, that the



Subtilty of the Sport may the better be concealed) let any one think upon any Number of Units which does not exceed 10; that done, bid him touch one of those Counters at pleasure, and to the Number on the back-side of the Counter touched (which you cannot be ignorant of, having noted well the Place of 1 or A) add secretly in your Mind, the just Number of all the Counters, and reserve the Sum; then bid him imagine in his Mind

Mind the Counter touched to be the Number which he thought, and from that Counter to count backwards, until he has made up the aforefaid Sum, which you reserved, so will his Computation infallibly end on the Counter upon which the Number thought of is marked.

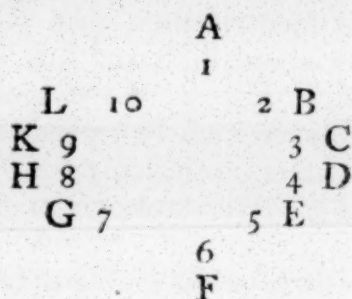
For Example, suppose he thought 7 or G, and that he touched B, to wit, 2, add to 2 the Number of all the Counters, to wit, 10, so the Sum will be 12; then bid him count to 12, beginning at B and going backward, and esteeming B to be the Number thought, to wit, 7; so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H, and lastly, 12 upon the Counter G, which being turned up will shew the Number thought.

The Reason of this Rule is not difficult to be apprehended, two Principles being presupposed, the one is this; to wit, many Counters or Things whatsoever being disposed orderly one after the other, in one continued Line, whether it be right or circular; if you value or name the first Counter to be some Number of Units at pleasure, and continue to count forward according to the natural Order of Numbers, until another Number be named which falls upon the last Counter: Or if you imagine or name the last Counter, to be the same Number of Units as before you put upon the first, and continue to count backwards to the first Counter; I say, that the same Number will be named at the end of both these Computations: For example, in these 9 Letters, A. B. C. D. E. F. G. H. K. if the Letter A be esteemed to be 4, and from thence you count forwards unto K, according to the natural Order of Numbers, the Letter K will fall upon the Number 12. In like manner, if you esteem K to be 4, and count backwards from K to A, the Letter A will likewise fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other Principle is this, to wit, many Counters being disposed in a round Form like a Ring; if you esteem any one of those Counters to be some Number at pleasure, and then from that Counter if you count circularly, until you end upon the Counter where you began, the Number last named will be equal to the Sum of the Number of all the Counters, and of the Number which you put upon the first Counter; for example, if D be one of ten Letters placed in a Circumference, and that
imagining

imagining D to be 7, you begin with it, and count round the whole Circumference, according to the natural Progression of



Numbers till you end with D where you began ; the Number 17, which is composed of 10 and 7, will necessarily fall upon D ; for 9 (which is the Number of Letters in the Circumference besides D) being added to 7 (which was first put upon D) makes 16, to which 1 being added, (because D ends as well as begins the Circumference) the Sum is 17.

Now these two Principles being presupposed, it will not be difficult to apprehend the Reason of the aforesaid Rule in all Cases that can happen ; for imagine that one has thought upon 7, or the Counter G, then that Counter which he shall touch must either be the same Counter G, or some other that preceeds or follows G.

First therefore, supposing the Counter or Number touched to be the same with the Number thought, the Truth of the Rule will be then evident ; for by the Rule given, he will begin to count from the same G to 17, putting 7 upon G, therefore by the second Presupposition the Number 17 will fall upon G.

Secondly, Imagine that he touched a Counter or Number following G the Number thought, as L or 10 ; then according to the Rule adding 10 (the Numbers of all the Counters placed circularly) to 10 or L, the (Counter touched) bid him count backwards to 20 by beginning at L, and esteem L to be 7. Now because by beginning to count at G which is 7, and proceeding to count forward, the Number 10 will fall upon L ; therefore by the first presupposed Principle, if we esteem L to be 7 and count backwards, the Number 10 will infallibly fall upon G, and then the Number 20 shall also fall upon the same G by the second presupposed Principle.

Lastly, Imagine he touched some Number or Counter which preceeds 7 the Number sought, as B or 2 ; then adding 10 to 2, you are to bid him count unto 12, he having first imagined B

A a a

to

to be the Number thought 7; and going backwards to A, L, K, &c. Now because by proceeding to count at B, which is 2, and beginning to count forward to C, D, &c. the Number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards F, E, &c. the Number 7 will fall upon B (by the first presupposed Principle;) therefore when one assumes B to be 7, and counts towards A, L, &c. to any assigned Number, it is in effect as much as when one imagines G to be 2, and counts towards F, E, &c. unto the said assigned Number, for each of those Computations will end in the same Point; but it is manifest (by the second presupposed Principle) that esteeming G to be 2, and counting towards F, E, D, &c. round the whole Circumference; the Number 12 will fall upon the same G. And because G being supposed to be 2, and counting on the same Coast as before, the Number 7 falls upon B; therefore if the Computation be continued on the same Coast from B 7, to 12, the Number 12, will fall upon the same G. So that the Practice of this Sport in all its Cases is demonstrated.

Note. That to the Number of the Counter touched you may not only add the Number of all the Counters once (as the Rule directs) but twice, thrice, or more times: For example, B being touched, you may cause him to count to 12, or to 22; or to 32, 42, &c. the Reason whereof is evident from the second presupposed Principle.

P R O B L. V.

Many Numbers being shewed by Pairs, to wit, two by two, unto any Person, that he may think upon any one of those Pairs at pleasure, to discover the Pair that was thought upon.

Let 20 Numbers, suppose these, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, be writ upon Ivory-Counters (or for want thereof upon small pieces of Paper) to wit, 1 upon one Counter, 2 upon another, 3 upon a third, &c. Then dispose them into Pairs as you see, viz. Suppose 1 and 2 to be one Pair, 3 and 4 to be another Pair, &c.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

and of these Pairs let any one think upon which Pair he pleases. That done, you are to distribute the said 20 Numbers into Ranks, in Form of a long Square, until there be 5 Numbers in length, and 4 in breadth, after this manner; *viz.* lay the three first Numbers, 1, 2 and 3, in a Rank (as you see in the second Figure) from A towards B; then place 4 under 1, and 5 after 3, (in the said Rank A B.) Again, place 6 under 4, and 7 after 5, (in the said Rank A B.) Then place 8 under 6, also 9, 10, 11, on the Right-hand of 4, in the Rank C D. Again, place 12 under 9, and 13 on the Right-hand of 11, in the Rank C D, and 14 under 12. Moreover, place 15, 16, 17, on the Right-hand of 12, in the Rank E F. Lastly, place 18, 19, 20, on the Right-hand of 14, in the Rank G H, so will all the Numbers be ranked as you see in the Table. That done, you are to demand of him that thought upon two Numbers as aforesaid, in what Rank or Ranks the said Numbers happen to be found, *viz.*

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

in which of the Ranks A B, C D, E F, G H, or in which two of the said Ranks: Now if he answer, that the two Numbers he first thought upon are in the first Rank A B, then 1 and 2 will be the Numbers thought or kept in mind; if in the second C D, then 9 and 10 shall be the Numbers thought of; if

A a a z

in

in the third Rank E F, then 15 and 16 will be the Numbers thought: If they are in the fourth Rank G H, then 19 and 20 shall be the Numbers thought; but if he say that the Numbers thought are in different Ranks, then you are heedfully to mark the said Numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which may be called the Keys of the Sport, in regard they serve not only to discover the two Numbers thought, when they are both in one and the same Rank (as aforesaid;) but even when they are in two different Ranks: For in this latter Case, as soon as it hath been declared to you in which two Ranks the two Numbers thought are placed, you are to take the Key of the highest of those two Ranks, and descending in a down-right Line from the first Number of that Key to the lower of the said two Ranks, you'll there find one of the two Numbers thought, and upon the Right-hand of the second Number of the said Key, at the same distance sidewise, from the second Number of the Key, (as one of the Numbers thought was distant from the first Number of the Key,) you will find the other Number thought.

For example, Suppose the two Numbers thought are 7 and 8, and it is declared to you, that they are in the first and fourth Ranks; take then the Key of the highest of those two Ranks; to wit, of the first; which is 1 and 2, and descending down-right from 1 to the fourth Rank, you'll there find 8 one of the Numbers thought: Then seek sidewise on the Right-hand of 2 (the second Number of the Key) a Number as far separated from 2, as 8 is distant from 1, and you'll find 7 the other Number thought.

Again, Suppose he says that the Numbers thought are in the second and third Ranks: Take then the Key of the second Rank which is 9 and 10, and descending down-right from 9 to the third Rank, you shall there find 12, which is one of the Numbers thought; then seek sidewise on the Right-hand of 10, (the second Number of the Key) a Number as far distant from 10, as 12 is from 9, and you'll find 11, which is the other Number thought.

The Reason of this will be apparent, from a serious Consideration of the placing of the Numbers according to the Rules before given: For it is thereby evident, that of the first Numbers coupled two by two, there can never be found more than one Pair in one and the same Rank; and of all the other Pairs, one Number is always found in one Rank, and the other Number in another Rank.

Note

Note also, that this Sport may be practised with divers Persons at once, and not only with 20 Numbers, but with any such multitude of Numbers as is produced by the Multiplication of any two Numbers that differs by 1, or Unity; as 30, which is the Product of 5 multiplied by 6, and 42, which is the Product of the Multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the Numbers in Ranks according to the Directions before given: And for the more easy comprehending of that Order, I have, in the following Table, ranked 30 Numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer Illustration than can be expressed by many Words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

P R O B L. VI.

Three jealous Husbands with their Wives, being ready to pass by Night over a River, do find at the River side a Boat which can carry but two Persons at once, and for want of a Waterman they are necessitated to row themselves over the River at several times: The Question is, How those six Persons shall pass 2 by 2, so that none of the 3 Wives may be found in the Company of 1 or of 2 Men, unless her Husband be present?

They must pass in this manner, viz. First two Women pass, then one of them brings back the Boat and repasses with the third Woman; that done, one of the three Women brings back the Boat, and sitting down upon the Ground with her Husband, permits the other two Men to pass over to find their Wives: Then one of the said Men with his Wife brings back the Boat, and placing her upon the Ground, he takes the other Man, and repasses with him: Lastly, the Woman who is found with the three Men enters the Boat, and at twice goes to fetch over the other two Women.

P R O B L.

P R O B L. VII.

Two merry Companions are to have equal Shares of eight Gallons of Wine, which are in a Vessel containing exactly eight Gallons; now to make this equal Partition they have only two other empty Vessels, of which one contains five Gallons, and the other three; the Question is, How they shall exactly divide the Wine by the Help of those three Vessels?

First, From the Vessel which contains 8 Gallons, and is full of Wine, let 5 Gallons be poured into the empty Vessel of 5, and from this Vessel so filled, let 3 be poured into the empty Vessel of three, so there will remain 2 Gallons within the Vessel of 5. Then let the three Gallons that are within the Vessel of 3 be poured into the Vessel of 8, which will now have 6 Gallons within it; that done, let the two Gallons which are in the Vessel of 5, be put into the empty Vessel of 3: Then of the six Gallons of Wine that are within the Vessel of 8, fill again the 5, and from those 5 pour out one Gallon into the Vessel of 3, which wanted only one Gallon to fill it; so there will remain exactly 4 Gallons within the Vessel of 5, and 4 Gallons within the other two Vessels. This Question may be resolved in another way, but I leave that as an Exercise to the Sagacity of the ingenious Reader.

Now though at first Sight it may be thought by some, that the two last mentioned *Problems* cannot be resolved by any certain Rule, but only by many Tryals: yet by infallible Argumentation and Discourse, the Solution of those Questions may be found out, or else the Impossibility of them, if by chance they should be propounded impossible; as the most ingenious *Gasper Bachet* has manifested in a little Book in the French Tongue, intitled, *Problemes plaisans & delectables qui se font par les nombres*, from which Book I have extracted the Contents of this Chapter.

A
SUPPLEMENT.

CONTAINING

Several Easy Contractions in the Necessary
and Useful Parts of ARITHMETICK,
conducting to a Ready and Expeditious
Way of Accounting.

WITH

Decimal Tables of *Flemish* EXCHANGES,
at the most Usual and Probable Rates of
Exchange.

AS ALSO

Tables of INTEREST, serving at any Rate,
for any Time.

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*The Mensuration of Superficies and Solids, in
various and practical Instances.*

By GEORGE SHELLEY *Writing-Master*, of
Christ's-Hospital.

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TAYLOR. MDCCXXXV.

T O T H E
R E A D E R.

I*T is very well known, That the ingenious Mr. Wingate has excellently explained the Principles of that most useful Science, Arithmetick; in which, together with the incomparable Additions of Mr. Kersey, the Theoretical Part is so largely insisted on, that it is in a manner incapable of any further Improvement.*

But though a Theory may be so settled as to need no Alteration, yet the Practices deduced, are often changed, and much improved.

In this Edition of Mr. Wingate's Arithmetick, I was prevailed upon, to add as a Supplement, some Practical, Compendious, and Easy Rules, not inserted by the Authors; or whatever else might tend to the making it more generally Useful. And which I have endeavoured chiefly in the most necessary Rules of Arithmetick.

The Tables of Simple Interest and of Exchanges were never before Published, and are such as I dare recommend for Practice, beyond any I know extant.

G. SHELLEY.

S E C T.

SECTION I.

MULTIPLICATION.

I. **H**OW to multiply by any Number under 20, so as to bring the whole Operation into one Line.

Rule (1.) Multiply each Figure in the Multiplicand by the unit Figure in the Multiplier, adding to each its back Figure; as in the following Examples.

$$\begin{array}{r} 47947 \\ 13 \\ \hline 623311 \end{array}$$

$$\begin{array}{r} 27942 \\ 17 \\ \hline 475014 \end{array}$$

Which are done thus; viz. 3 times 7 is 21, set down 1 and carry 2; then 3 times 4 is 12, and 2 is 14, and 7 (which is the back Figure to 4) is 21, 1 and carry 2; 3 times 9 is 27, and 2 is 29, and 4 (the back Figure) is 33, 3 and carry 3; 3 times 7 is 21, and 3 is 24, and 9 (the back Figure) is 33, 3 and carry 3; 3 times 4 is 12, and 3 is 15, and 7 is 22, 2 and carry 2, which added to 4 makes 6.

In like manner, the whole Operation of the Multiplication of Numbers between 20 and 30, are brought into one Line, by taking in the Double of the back Figure, and between 20 and 40, the Treble of it, &c.

By which Rule in multiplying by Numbers compounded of those under 20, the Product is readily found thus:

By 112, either multiply by 2 and 11, or by 12 and 1; as,

$$\begin{array}{r} 94267 \\ 112 \\ \hline 188534 \\ 1036937 \\ \hline 10557904 \end{array}$$

$$\begin{array}{r} 94267 \\ 112 \\ \hline 1131204 \\ 94267 \\ \hline 10557904 \end{array}$$

Also in multiplying by 1614, and 1715; in the first I multiply by 14 in one Line, and by 16 in the other: And in the latter by 15 in one Line, and 17 also in the other, as above directed; observing always to place the first Figure of a Line under its own Multiplier; as

B b b

479479

$$\begin{array}{r}
 479479 \\
 1614 \\
 \hline
 6712706 \\
 7671664 \\
 \hline
 773879106
 \end{array}$$

$$\begin{array}{r}
 479479 \\
 1715 \\
 \hline
 7192185 \\
 8151143 \\
 \hline
 822306485
 \end{array}$$

otherwise for any Number under 20, a Method not so troublesome to the Memory.

Rule (2.) *Multiply by the unit Figure, setting that whole Product one Figure back on the right Hand, which added to the Multiplicand, gives the Product, viz.*

$$\begin{array}{r}
 47947 \\
 13 \\
 \hline
 143841 \\
 \hline
 623311
 \end{array}$$

$$\begin{array}{r}
 27642 \\
 17 \\
 \hline
 193494 \\
 \hline
 469814
 \end{array}$$

But for 112, multiply by 12, setting the Product two Figures back, which likewise added to the Multiplicand, gives the Product, as,

$$\begin{array}{r}
 94267 \\
 112 \\
 \hline
 1131204 \\
 \hline
 10557904
 \end{array}$$

Note. If the *Multiplicand* has less significant Figures in it, than the *Multiplier*, work as if it were the *Multiplier*; as in the Examples.

$$\begin{array}{r}
 13000 \\
 874 \\
 \hline
 11362000
 \end{array}$$

$$\begin{array}{r}
 1107 \\
 3758 \\
 \hline
 26306 \\
 413380 \\
 \hline
 4160106
 \end{array}$$

II. How to multiply by the component *Parts* of a Number instead of the *Whole*.

That

That is to multiply 42991 by 7, and that *Product* by 8 is the same as if it were multiplied by 56, the *Product* of 7 by 8. Example,

$$\begin{array}{r} 42991 \\ 7 \\ \hline 300937 \\ 8 \\ \hline 2407496 \end{array}$$

$$\begin{array}{r} 42991 \\ 56 \\ \hline 257946 \\ 214955 \\ \hline 2407496 \end{array}$$

Note. If any odd Numbers are given that cannot be produced by the *Multiplication* of any two Figures, then find out two Figures whose *Product* comes nearest, either under or above it, to which add what is wanting, or from which subtract what is over the given Number ; as,

$$\begin{array}{r} 479 \text{ by } 47 \\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 2874 \\ 8 \\ \hline \end{array}$$

$$22992$$

479 subtracted

$$22513$$

$$\begin{array}{r} 479 \\ 47 \\ \hline \end{array}$$

$$\begin{array}{r} 3353 \\ 1916 \\ \hline \end{array}$$

$$22513$$

$$\begin{array}{r} 927 \text{ by } 29 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3708 \\ 7 \\ \hline \end{array}$$

$$25956$$

927 added

$$26883$$

$$\begin{array}{r} 927 \\ 29 \\ \hline \end{array}$$

$$\begin{array}{r} 8343 \\ 1954 \\ \hline \end{array}$$

$$26883$$

III. To multiply Numbers of several Denominations.

(1.) By whole Numbers.

$$\begin{array}{r} l. \quad s. \quad d. \\ 47 : 18 : 11 \\ \quad \quad 8 \\ \hline \end{array}$$

$$383 : 11 : 04$$

$$\begin{array}{r} C. \quad q. \quad l. \\ 9 : 3 : 15 \\ \quad \quad 6 \\ \hline \end{array}$$

$$59 : 1 : 06$$

(2.) By the Parts instead of the Whole, viz.

B b b z

1a.

Ya.	q.	n.		C.	q.	l.
47	: 3	: 3	by 54	21	: 3	: 08 by 36
		5				6
239	: 2	: 3		130	: 3	: 20
		9				6
2157	: 0	: 3		1785	: 2	: 08

(3.) *By mixt Numbers.*

C.	q.	l.		l.	s.	d.
9	: 3	: 15	by $11\frac{1}{2}$	39	: 18	: 11 by $9\frac{3}{4}$
		11				9
108	: 2	: 25		359	: 10	: 03
4	: 3	: $21\frac{1}{2}$		9	: 19	: $08\frac{3}{4}$
113	: 2	: $18\frac{1}{2}$		369	: 09	: $11\frac{1}{4}$

l.	s.	d.
47	. 18	. 11 by $27\frac{3}{4}$
		4
191	. 15	. 8
		7
1342	. 09	. 8
11	. 19	. 8
1330	. 09	: $11\frac{1}{4}$

In the first of these last Examples, I *multiply* by 11, adding one half of the *Multiplicand* for the $\frac{1}{2}$. In the second I *multiply* by 9, adding a fourth for $\frac{1}{4}$. In the third I *multiply* by 4, and that *Product* by 7, gives 28 times the Sum; from which *subtracting* $\frac{1}{4}$ of the first Sum, the *Remainder* is the *Product* of the given Sum, *multiplied* by $27\frac{3}{4}$.

(4.) *By Numbers of several Denominations.*(1.) *By Aliquot Parts.* (2.) *By Cross Multiplication.*

F.	i.	p.	F.	i.
47	. 08	. 06	47	. 08
by 9	. 04	. 04	9	. 04
429	. 04	. 06	422	. 00
$\frac{1}{3}$ 15	. 10	. 10	06	. 00
$\frac{1}{12}$ 2	. 03	. 10	15	. 06
446	. 07	. 02	Answer	
			444	. 10 . 8

Which are done thus, viz.

In the first, the several *Denominations* in the *Multiplicand*, are multiplied by the *Integers* in the *Multiplier*, to which *Product* add one third of the *Multiplicand*; for four Inches (being $\frac{1}{3}$ of one Foot) and the $\frac{1}{12}$ of that ⁱ for the 4 Parts (being the $\frac{1}{12}$ of 4 Inches) gives the Answer.

In the second, called *Cross Multiplication*, the *Feet* are multiplied by the *Feet*, then (Cross-wise) viz. the 9 *Feet* in the *Multiplier* by the 8 *Inches* in the *Multiplicand*, gives 72 *Inches* or 6 *Feet*, and the 47 *Feet* in the *Multiplicand* being multiplied by 4 *Inches* in the *Multiplier*, give 25 *Foot* 8 *Inches*. Lastly, the *Inches* in both the *Factors*, being multiplied one into the other, make 32 *Parts*, or 2 *Inches*, 8 *Parts*, the *Total* of which is the *Product* of 47 *Foot* 8 *Inches*, by 9 *Foot* 4 *Inches*.

SECTION II.

Of DIVISION.

I. **T**O Divide whole Numbers by the *Component* Parts of a Number.

(1.) Examples having no *Remainders*.

Divide 22992 by 48,
that is by 6, and
that Quotient by 8.

Divide 25956 by 28 or
by 4, and that Quoti-
ent by 7.

$$\begin{array}{r} 6 \overline{) 22992} \\ \hline \end{array}$$

$$\begin{array}{r} 8 \overline{) 3832} \\ \hline \end{array}$$

479 Quot.

$$\begin{array}{r} (4 \overline{) 25956} \\ \hline \end{array}$$

$$\begin{array}{r} \hline 6489 \\ \hline \end{array}$$

927 Quot.

(2.) Examples having *Remainders*.

Divide 47953 by 49

$$\begin{array}{r} 7 \overline{) 47953} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \overline{) 6850.3} \\ \hline \end{array}$$

$$\begin{array}{r} \hline 978.4 \\ \hline \end{array}$$

The Quot. $978\frac{11}{25}$.Divide 47991 into 36
Parts.

$$\begin{array}{r} 4 \overline{) 25956} \\ \hline \end{array}$$

$$\begin{array}{r} 9 \overline{) 11997.3} \\ \hline \end{array}$$

$$\begin{array}{r} \hline 1333.0 \\ \hline \end{array}$$

$$\begin{array}{r|l} 3 & 1 \\ \hline & 12 \end{array}$$

Answer 1333 06

If 21 Pieces of Bag-Holland contain 471 Ells, How much is contained in one Piece?

$$\begin{array}{r} 3 \overline{) 471} \\ \hline \end{array}$$

$$\begin{array}{r} 7 \overline{) 157} \\ \hline \end{array}$$

$$\begin{array}{r} \hline 22.3 \\ \hline \end{array}$$

Answer

$$\begin{array}{r|l} \text{Ells} & 9 \quad 3 \\ \hline & 22.21 \quad 7 \end{array}$$

The true Remainders of which *Divisions* are found by the following Rule.

Multiply the first Divisor by the last Remainder, adding thereto the first Remainder (if any be;) it gives a Numerator, whose Denominator is the whole Number composing the Parts, whose Fraction so arising is a true Remainder, and the very same that would be found in dividing by the Whole.

II. Of dividing Numbers of several *Denomination*, by the *Parts of the Whole*.

l. s. d.

(1.) Example, if 19 . 14 . 10 be divided among 16 Men, What is every Man's Share?

$$\begin{array}{r}
 4 \ 19 \ . \ 14 \ . \ 10 \\
 \hline
 4 \ 4 \ . \ 18 \ . \ 08 \ . \ \frac{1}{2} \\
 \hline
 \text{Answer} \quad 1 \ . \ 04 \ . \ 08 \ . \ \frac{1}{2}
 \end{array}$$

(2.) Example, What is the Weight of one Bale, when 25 weigh 93 . 1 . 01 ?

$$\begin{array}{r}
 \text{C.} \quad \text{q.} \quad \text{l.} \\
 5(\ 93 \ . \ 1 \ . \ 01 \\
 \hline
 5(\ 18 \ . \ 2 \ . \ 17 \ \text{oz.} \\
 \hline
 3 \ . \ 2 \ . \ 25 \ . \ 12 \ . \ \frac{4}{5}
 \end{array}$$

III. To divide Numbers by *broken* or *mixt* Numbers.

Rule. Reduce both the Dividend and the Divisor, into the same Denominator.

(1.) Example, What comes one Yard of Flower'd Damask to, when $13\frac{1}{3}$ Yards cost 21 l. 15 s. 11 d. ? To do this, reduce the Yards, by multiplying by 3 into thirds, which *Product* will be 40; likewise the Sum reduced into thirds, is 65 l. 7 s. 9 d. which being to be divided by 40, work according to the foregoing Rule, taking $\frac{1}{3}$ of $\frac{1}{3}$ for the Answer.

$$\begin{array}{r}
 \frac{1}{3} \ 21 \ . \ 15 \ . \ 11 \qquad \qquad \qquad 13 \\
 \qquad \qquad \qquad 3 \qquad \qquad \qquad 3 \\
 \hline
 5) \ 65 \ . \ 07 \ . \ 09 \qquad \qquad \qquad 40 \\
 \hline
 8) \ 13 \ . \ 01 \ . \ 06 \ . \ \frac{3}{5} \\
 \hline
 \end{array}$$

Answ. $1 \ . \ 12 \ . \ 08 \ . \ \frac{1}{5}$

(2.) Examp. What comes one Piece of Cloath to, when $17\frac{2}{3}$ Pieces cost

$$\begin{array}{r}
 \text{l.} \quad \text{s.} \quad \text{d.} \\
 57 \ . \ 07 \ . \ 09 \qquad \qquad \qquad \frac{2}{3} \\
 \qquad \qquad \qquad 2 \qquad \qquad \qquad 35 \\
 \hline
 3 \ 114 \ . \ 15 \ . \ 06 \\
 \hline
 3 \ 22 \ . \ 19 \ . \ 01 \ . \ \frac{1}{3} \\
 \hline
 \end{array}$$

Answ. $3 \ . \ 05 \ . \ 07 \ . \ \frac{2}{5}$

(3)

(3.) Example, Divide $8792 \cdot \frac{1}{2}$ by $27 \cdot \frac{1}{4}$

$$\begin{array}{r}
 8792 \cdot \frac{1}{2} \quad 55 \\
 \hline
 2 \quad \hline
 5 \quad 17585 \quad \text{The Quot is } 319 \cdot \frac{1}{11} \\
 \hline
 11 \quad 3517 \\
 \hline
 319 \cdot \frac{8}{11}
 \end{array}$$

IV. How to divide Numbers of *several Denominations* by those of *several Denominations*.

Example (1.) How many pieces of Gold, each valued at 2 *l.* 11 *s.* 8 *d.* are contained in 478 *l.* 19 *s.* 10 *d.*? *Answer*, 185 Pieces, as appears by the following Operation.

$$\begin{array}{r}
 2 \cdot 11 \cdot 8 \quad 478 \cdot 19 \cdot 10 \quad (185 \\
 \hline
 2 \cdot 11 \cdot 08 \\
 \hline
 1 \cdot 08 \cdot 4 \\
 \hline
 10 \\
 \hline
 21 \cdot 03 \cdot 4 \\
 20 \cdot 13 \cdot 4 \\
 \hline
 0 \cdot 10 \cdot 0 \\
 \hline
 10 \\
 \hline
 13 \cdot 00 \cdot 4 \\
 12 \cdot 18 \cdot 4 \\
 \hline
 01 \cdot 8 \\
 19 \cdot 10 \text{ added} \\
 \hline
 21 \cdot 6 \text{ Remainder.} \\
 \hline
 \end{array}$$

To do which, First seek how many times 2 . 11 . 8 is contained in 4 *l.* which is once; then setting one in the Quotient, multiply the Divisor by it, the Product of which setting underneath and subtracted from 4 *l.* rest 1 *l.* 8 *s.* 4 *d.*, which multiply by 10, taking in 7 *l.* the next Figure of the Dividend, then taking a new Quotient Figure, you repeat the same

same method as above, until finished, observing at the last to add the odd Money in the Dividend, to the Money remaining, as in the *Example*.

Note. That in adding, multiplying, and dividing of Money, work the second Place in the Shillings, as Angels.

SECTION III. Of REDUCTION.

I. In Money.

POUNDS into Guineas, multiply by 40, which Product divide by 43.

Marks into Pounds, subtract $\frac{1}{2}$.

Pounds into Marks, add $\frac{1}{2}$.

II. In Avoirdupois.

To reduce Hundreds, &c. into Pounds, multiply by 112 into two Lines, as taught in *Multiplication*, taking in the Pounds contained in the odd Weight.

By this Weight Silk is weighed, in which there is a great Pound of 24 Oz. and by which is weighed Raw, Long, Short, China, Morea Silk, &c.

And a small one of 16 Oz. by which Ferret, Filosella, Sleeve-Silk, Fine and Course, &c. are weighed.

To reduce one into the other.

Note. Great Pounds of 24 Oz. into Pounds of 16 Oz. add $\frac{1}{2}$.

Small Pounds of 16 Oz. into Pounds of 24, subtract $\frac{1}{3}$.

III. Liquid Measure.

Of Beer.	{	<i>Firkins into Hogsheads</i>	_____	Take	$\frac{1}{8}$
		<i>Hogsheads into Barrels</i>	_____	Add	$\frac{1}{2}$
		<i>Kilderkins into Hogsheads</i>	_____	Take	$\frac{1}{3}$
		<i>Barrels into Tuns</i>	_____	Take	$\frac{1}{5}$

54 Gallons one Hogshead of Beer.

C c c

Of

Of Wine.	{	Hogsheads into Tierces	_____	_____	Add $\frac{1}{2}$
		Tierces into Hogsheads	_____	_____	Sub. $\frac{1}{3}$
		Tierces into Pipes	_____	_____	Take $\frac{1}{4}$
		Pipes into Hogsheads	_____	_____	Double it.

63 Gallons one Hogshead of Wine.

IV. Cloth-Measure.

Of which there are two Sorts most in use among us, a *Yard*, and an *Ell*, both subdivided into 4 *Quarters*, and each *Quarter* into 4 *Nails*. But the *Nail* of an *Ell* is $\frac{1}{4}$ greater than the *Nail* of a *Yard*; whence it is, that 20 *Nails*, or 5 *Quarters* of a *Yard* are contained in one *Ell English*.

Yards into Ells English	_____	Subtract $\frac{1}{4}$ part
Ells English into Yards	_____	Add $\frac{1}{4}$ part
Yards into Flemish	_____	Add $\frac{1}{3}$ part
Ells Flemish into Yards	_____	Subtract $\frac{1}{4}$ part
Ells English into Flemish	_____	Add 0 and divide by 6
Ells Flemish into English	_____	Multiply by 6, and cut off the last Figure.

Of Tare and Tret, &c.

Called beyond Sea, *The Courtesies of London*, because not practised in any other Place; concerning which, observe, That

Gross is the Weight of the Commodity, and that which contains it without any Deduction.

Tare is an Allowance for that which contains the Commodity, whether it be *Bag*, *Barrel*, *Sack*, *Frail*, &c. and is either,

I. At so much per *Bag*, *Barrel*, *Frail*, &c.

II. At so much per *Cent.* or,

III. By so much of the *Gross Weight*, called *Invoice Tare*.

I. When the Tare is so much per *Bag*, as *Almonds*; per *Barrel*, as *Oil in Candy Barrels*; per *Frail*, as *Raisins*, &c.

Rule. Multiply the Number of the said *Bags*, *Barrels*, *Frails*, *Hogsheads*, &c. by the Allowance of Tare, which Product subtract from the Gross, and (when no Tret is mentioned) the Remainder is Neat.

Example,

Example, What is the *Neat Weight* of 38 Hogsheads of *Tobacco*, weighing *Gross* 102 . 2 . 17, *Tare* per Hoghead 70 l.

C.	q.	l.	
102	2	17	38
112			2660
1297			
11497	Gross		
2660	Tare		
8837	Neat		

Or which is more short, though not according to *Rule*, multiply the Hogsheads by 5, dividing the Product by 8, will give the *Tare* of the Whole in *Hundreds*, *Quarters*, and *Pounds*.

38	C.	q.	l.	
5	102	2	17	Gross
8)	190	23	3	00 Tare
	23 $\frac{3}{4}$	78	3	17 Neat, equal to 8837 l.

II. When the *Tare* is at so much *per cent.* and that,

(1.) If an *Aliquot Part*, or *Parts* of the *Hundred Weight*;

As *Figs*, *Almonds*, *Argol*, &c.

Caroteels, and *Butts* of *Currans*, &c.

Oil in uncertain *Casks*, &c.

14 }
16 } per Cent.
18 }

Rule. Divide the whole *Gross* by the said *Part* or *Parts* that the *Tare* is of an *Hundred*, gives the *Tare* of the *Whole*; for such a *Part* of the *Gross* must be the *Tare* of the *Whole*, as the given *Tare* is of an *Hundred*.

Example, What is the *Neat Weight* of 15 Barrels of *Argol*, *Gross* C. 51 . 2 . 18. *Tare* 14 *per Cent.* which being the 8th of an *Hundred*, take the 8th of the *Gross* for *Tare*.

	C.	q.	l.	
8)	51	2	18	Gross
	6	1	23	4 Tare
	45	0	22	12 Neat.
	C c c 2			

Example,

Example, (2.) How many Neat Hundreds in 18 Quarter-Rolls of Currans, each Gross 8 . 3 . 10 Tare 21 l. per Cent.

$$\begin{array}{r}
 \text{C. } q. \text{ l.} \\
 8 . 3 . 10 \text{ each} \\
 \hline
 3 \\
 26 . 2 . 02 \\
 \hline
 6 \\
 \hline
 \text{l. } \frac{1}{8}) 159 . 0 . 12 \text{ oz. Gross} \\
 14 \quad \frac{1}{2}) \hline
 7 \quad 19 . 3 . 15 . 8 \\
 \quad 9 . 3 . 21 . 12 \\
 \hline
 29 . 3 . 09 . 04 \text{ Tare} \\
 \hline
 129 . 1 . 02 . 12 \text{ Neat} \\
 \hline
 \end{array}$$

Tare
 For $\left. \begin{array}{c} 2 \\ 4 \\ 7 \\ 1 \end{array} \right\}$ Divide the Gross by $\left. \begin{array}{c} 7 \\ 4 \\ 4 \\ 2 \end{array} \right\}$ and that Quote by $\left. \begin{array}{c} 8 \\ 7 \\ 4 \\ 7 \end{array} \right\}$ gives the *Tare*.

2. But if the *Tare* is not readily found to be an *Aliquot Part* or *Parts* of a *Hundred*;

Rule. Multiply the Pounds Gross by the Allowance of *Tare*, dividing the Product by 112, the Quotient is the *Tare* of the Whole.

Example, What is the *Neat Produce* of 12 Barrels of Potashes, each Gross 203 l. Tare 10 l. per cent.

$$\begin{array}{r}
 203 \\
 12 \\
 \hline
 2436 \\
 217 . \frac{1}{2} \text{ Tare} \\
 \hline
 2218 . \frac{1}{2} \text{ Neat} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 112) 24360 \\
 196 \\
 840 \\
 \hline
 56 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 203 \\
 12 \\
 \hline
 2436 \\
 217 . \frac{1}{2} \text{ Tare} \\
 \hline
 2218 . \frac{1}{2} \text{ Neat} \\
 \hline
 \end{array}$$

III. If the Allowance for *Tare* is not by the *Hundred*, but according to the *Gross Weight*, as *Sumach* in *Bags*, *Bales*, or *Fangots* of *Aleppo*, *Cyprus*, and *Smyrna Silk*, &c. such *Tare* is called *Invoice Tare*; of which the *Book of Rates*, in the Table of Allowances for *Tare*, says, *viz.*

Cyprus, and Smyrna Silk.

Bales	{	about, or above 300 <i>l.</i>	{	allowed	{	<i>l.</i> 16	}
		from 300 to 200		for <i>Tare</i>		14	
		from 200 downwards				12	

Tobacco from Virginia.

Hogsheads	{	5 C. and upwards	{	allowed	{	<i>l.</i> 100	}
		from 4 to 5,		for <i>Tare</i>		90	
		from 3 to 4,				80	
		under 5 Hundred				70	

Example, What is the *Neat Weight* of four Hogsheads of *Virginia Tobacco*, weighing *Gross* and Number'd? *viz.*

N ^o .	C.	q.	l.		<i>l.</i>
27	2	2	19	{	70
9	5	0	15		100
4	3	2	11		80
19	4	2	19		90

16 . 0 . 08 *Gross* 112) 340 (3
3 . 0 . 04 *Tare* 4

13 . 0 . 04 *Neat*, and so of *Silk*, &c.

Also Sugar from India.

In Casks and Canisters,

In Chests, and Casks, *St. Thom.*

{ *Tare* { $\frac{1}{2}$
 $\frac{1}{3}$

Tret, is an allowance of 4 *l.* per 104, on some sort of Goods, (to the Buyer by the Merchant) for the *Dust* and *Waste* in them; which when allowed, after you have subtracted the *Tare* from the *Gross*, the Remainder is *Subtil* or *Suttle*.

Rule.

Rule. *This Suttle take out, and divide by 26, because the Allowance of 4 l. per 104, is $\frac{1}{26}$ Part, and the Quote is the Tret, which Tret subtract from the Suttle, and the Neat remains.*

Example, How many Pound Neat in 12 C. 3 q. 12 l. Gross, Tare 2 l. per Cent. Tret 4 l. per 104?

$$\begin{array}{r} 26) \quad 1415 \quad (54 \\ \hline \quad 115 \\ \hline \quad \quad 11 \\ \hline \end{array}$$

$$\begin{array}{r} \text{C.} \quad \text{q.} \quad \text{l.} \\ 12 \quad . \quad 3 \quad . \quad 12 \\ \hline 112 \\ 8) \quad 1440 \text{ Gross} \\ 7) \quad 180 \\ \hline 25 \quad + \quad \text{Tare} \\ \hline 1415 \text{ Suttle} \\ 54 \text{ Tret} \\ \hline 1361 \text{ Neat} \end{array}$$

Otherwise to find and deduct Tret.

Rule. *Add 2 Cyphers to the Suttle Pounds, which Number divide by 104, and the Quote is the Neat Pounds.*

Example, What is the Neat Weight of 4 Par. of Spanish Tobacco, containing? viz.

N^o C. q. l.

4 1 . 0 . 04

7 0 . 3 . 25

9 1 . 0 . 05

3 0 . 2 . 26

Tare 14 per Cent.

Tret 4 per 104 l.

3 . 3 . 04

112

8) 424 Grofs
53 Tare

104) 37100 (356 + Neat
590
700
76

Cloff, is another Allowance of 2 l. Weight, to Citizens of London, on every Draught above three hundred Weight, on some Goods, as Galls, Madder, Sumack, Argol, &c.

Neat, is the pure Weight of the Commodity, when all given Allowances are deducted.

As for other Allowances not so common, as Break, Damage, &c. the first being generally at so much per Cask, Bale, Bag, &c. the other so much in the whole, can have no Difficulty.

Example, What is the Neat Weight of 7 Hogsheads Grofs, 28 C. 1 q. 19 l. Tare 16 per Cent. Break 8 per Hogshead, Damage 98 $\frac{1}{2}$?

7) C. q. l.
28 . 1 . 19 Grofs

4 . 0 . 06 . 11 Tare

24 . 1 . 12 . 5

— . 2 . 00 . 0 Break

23 . 3 . 12 . 5

3 . 14 . 8 Damage

22 . 3 . 25 . 13 Neat

In Oil the Tare is of two Sorts.

(1.) On Candy Barrels — — at 29 l. per Barrel.

(2.) On uncertain Casks — — at 18 l. per Cent.

The *Neat Pounds* of which are found by the fore-going Rules. But to reduce those *Pounds* into *Gallons*;

Rule. Multiply those *Pounds Neat* by 2, the *Product* of which divide by 3, and that *Quote* by 5, gives *Neat Gallons*, 7 l. $\frac{1}{2}$, making a Gallon.

Example, Oil, Gross 127 C. 3 q. 7 l. Tare 18 l. per Cent. How many *Neat Gallons*?

C.	q.	l.
127	3	7
112		
1615		
7) 14315 Gross		
8) 2045		
255		
2300 Tare		
12015 Neat Pounds		
2		
3) 24030		
5) 8010		
1602 Gallons		

SECTION IV.

Of PRACTICE.

THE Variety of this Rule may be reduced to these two Heads, viz.

(1.) In respect to the *Rate* or *Price* of Goods.

(2.) The given *Number*, or *Quantity* of such Goods.

Of the Price, when

{	a Fraction.	} whether <i>d.</i> or <i>d. & far.</i>
	Farthings.	
	* An Aliquot Part of a <i>s.</i>	
	* Not an Aliquot Part.	
	Between one <i>s.</i> and two	
	* An Aliquot Part of a <i>l.</i>	
* Not an Aliquot Part.		
* Between one Pound and two		
having a Fraction annexed.		

Of the Given Number, when

{	t.	} It has odd W. or M. in it.
	* It has a Fraction annexed.	
	It is not of the same Name with that on which the Price is set.	
	It is of less Denomination than it.	

Mr. Kersey having in the second Chapter of his *Appendix* treated of those Heads, to which an *Asterisk* is prefixed, I shall therefore omit what he has said.

A Table of such Pence as are an Aliquot Part of a Pound.

d.	
2	$\frac{1}{120}$
3	$\frac{1}{80}$
4	$\frac{1}{60}$
6	$\frac{1}{40}$
8	$\frac{1}{30}$

} is } of a Pound.

I. Of the Given Price.

1. When it is a Fraction.

Rule. Multiply the given Number by the Numerator, the Product of which divided by the Denominator, gives the Answer in the same Denomination with the Integer, of which the Price is a Fraction.

Example. What comes the Whitening of 479 Ells of *Osnaburghs* to, at $\frac{5}{8}$ of a Penny an Ell.

$$\begin{array}{r}
 479 \\
 \times 5 \\
 \hline
 2395 \\
 8) \quad \quad \quad \\
 \hline
 299 \text{ . } \frac{3}{8} \\
 \hline
 \text{Answ. } 24 \text{ . } 11 \text{ . } \frac{3}{8} \\
 \text{D d d}
 \end{array}$$

H. When

II. When the Price is Farthings only.

Rule. Reduce the Farthings into 3 Pence, which divided by 80, produce Pounds.

Example, What come 6472 Yards of Tape to, at 1 Farthing, or 3 Farthings per Yard:

$$\begin{array}{r} 6472 \text{ at } \frac{1}{4} \text{ per Yard} \\ 12) \underline{\hspace{2cm}} \\ 53|9 \cdot 4 \\ 80) \underline{\hspace{2cm}} \\ 6 \cdot 14 \cdot 10 \end{array}$$

$$\begin{array}{r} 6472 \text{ at } \frac{3}{4} \text{ per Yard} \\ 4) \underline{\hspace{2cm}} \\ 161|8 \\ 8|0 \underline{\hspace{2cm}} \\ 20 : 04 : 7 \end{array}$$

To value the Remainder in dividing by any Number consisting of a Digit and a Cypher, as 80 . 60 . 40, &c.

Rule. What Pounds remain, turn into Shillings, (taking in the Double of the Figure cut off) which Shillings being divided by the Digit in the Divisor, give the odd Shillings; what Shillings remain from thence, turn likewise into Pence, which divided also by the Digit, give the odd Pence; and so in the Farthings. Otherwise,

What remains in the Pounds, reckon as Tens, which add to the Unit cut off, the Sum of which divided by half the Digit, gives Shillings; what remains from thence, turn into Pence, which divided also by the half, gives Pence, &c.

III. When the Price is an Aliquot Part of a Shilling, the Rule is, To divide the given Number by the same Part, and the Quote is the Answer in Shillings. But since those Parts of a Shilling are also the Aliquot Parts of a Pound:

Rule. Divide the given Number by the said Parts of a Pound, the Quote is the Answer in Pounds, the Remainder being valued as above.

Example, What come 472 Oz. to, at 4 d. or 8 d. per Oz?

$$6|0) 47|2 \text{ at } 4 \text{ d. per Oz.} \quad 3|0) 47|2 \text{ at } 8 \text{ d. per Oz.}$$

Ans. 7 . 17 . 4

Ans. 15 . 14 . 8

IV. If the Price be not an Aliquot Part of a Shilling, as 5, 7, 9, 10, and 11 Pence.

Rule. Take the greatest even Part of a Pound, and for what is wanting, take Parts out of that Part, which added to the other, gives the Facit.

Example. What come 645 l. of Sugar to, at 7 d. or 9 d. per Pound?

$$4|0)$$

4|0) 64|5 at 7 d. per Pound.

$$\begin{array}{r} \frac{1}{2}) \quad 16 \cdot 02 \cdot 0 \\ \quad 2 \cdot 13 \cdot 9 \\ \hline \end{array}$$

18 . 16 . 3 Facit An. l.

3|0) 64|5 at 9d. per Pound.

$$\begin{array}{r} \frac{1}{3}) \quad 21 \cdot 10 \\ \quad 2 \cdot 13 \cdot 9 \\ \hline \end{array}$$

24 . 03 . 9

V. When any *Farthings* are added to any of the forementioned Prices, the

Rule is the same with the foregoing.

Example. What comes 735 Dozen of Buttons to, at 7 d $\frac{1}{2}$, or 6 d. $\frac{3}{4}$ per Dozen ?

4|0) 73|5 at 6 d. $\frac{3}{4}$ per Dozen.

$$\begin{array}{r} \frac{1}{4}) \quad 18 \cdot 07 \cdot 06 \\ \quad 2 \cdot 05 \cdot 11 \cdot \frac{1}{4} \\ \hline \end{array}$$

20 . 13 . 05 . $\frac{1}{4}$ 4|0) 73|5 at 7 d $\frac{1}{2}$ per Oz.

$$\begin{array}{r} \frac{1}{4}) \quad 18 \cdot 07 \cdot 06 \\ \quad 4 \cdot 11 \cdot 10 \cdot \frac{1}{2} \\ \hline \end{array}$$

22 . 19 . 04 . $\frac{1}{2}$

VI. If it were here required at such a *Profit* in the *Shilling*, to know what *Profit per Cent*.

Rule. Divide 100 l. by the *Parts* that the said *Profit* is of a *Shilling*, the *Total* of which is the *Answer*.

Example. What *Profit per Cent*. when I gain 4 d. $\frac{1}{2}$, or 7 d. per *Shilling* ?

$$4 \frac{1}{2} \quad 100$$

$$\begin{array}{r} \frac{1}{2} \frac{1}{8} \quad 33 : 06 : 08 \\ \quad 4 : 03 : 04 \\ \hline \end{array}$$

37 : 10 : 00

$$4 \frac{1}{3} \quad 100 \text{ at } 7$$

$$\begin{array}{r} 3 \frac{1}{4} \quad 33 : 06 : 08 \\ \quad 25 : 00 : 00 \\ \hline \end{array}$$

58 : 06 : 08

Otherwise 4 d. $\frac{1}{2}$ being the $\frac{3}{8}$, and the 7 d. the $\frac{7}{8}$ of 2 s.

Multiply 100 by the *Numerator*, and divide that *Product* by the *Denominator*, according to the first *Rule*, gives the *Facit* ; as

$$\begin{array}{r} 100 \\ 3 \\ \hline \end{array}$$

$$8) \quad 300$$

Answ. 37 . 6

$$\begin{array}{r} 100 \\ 7 \\ \hline \end{array}$$

$$12) \quad 700$$

Facit 58 . 4

D d d 2

VII. When

VII. When the Price is any Number of Pence, under two Shillings, as 14, 15, 16, 18, 21, 22 Pence.

Rule. Reduce the said Prices either into 2 d. 3 d. 4 d. 6 d. or 8 Pence, and these into Pounds at one Operation, by the third Rule.

Gallons	Bushels
2751 at 14 d. per Gallon.	375 at 15 d. per Bushel.
7	5
12 0 1925 7 2 Pence	8 0) 188 5 3 Pence
Facit 160 . 09 . 06.	Facit 23 . 08 . 09.

Dozen	Pair
297 at 16 d. per Dozen.	736 at 18 d. per Pair.
2	3
3 0 59 4	4 3 220 8
Answ. 19 . 16	Facit 55 . 04

L.	Ells at 22 d. per Ell.
715 at 21 d. per l.	917
7	11
8 0 500 5	11 0) 1008 7
Facit 62 . 11 . 3	Answ. 84 . 01 . 02

For 13, 17, and 19 Pence.

Rule. Multiply the given Number by the said Pence, (as taught in Multiplication;) which Product divided by 12 and 20, gives the Facit.

Examples.

463 l. of Copper, at 17 d. l.	Ells of Long Cloth, at 19 d.
463	479
17	19
12) 7871	12) 9101
20) 65 5 . 11	2 0 75 8 . 5
Answ. 32 . 15 . 11	37 . 18 . 5

VIII. If

VIII. If any of the last fore-going Prices had Farthings annexed.

Rule. *Then take Parts, for the Pence and Farthings above a Shilling, out of a Shilling, which added to the given Number, the Total is the Answer in Shillings ; as,*

Example. What do I spend per Annum, at 19 d. $\frac{1}{2}$, or 18 d. $\frac{3}{4}$ per Day ?

$$\begin{array}{r} 365 \text{ Days at } 19 \text{ d. } \frac{1}{2} \text{ each} \\ 6 \quad \frac{1}{2} \quad 365 \quad 6 \quad \frac{1}{2} \\ 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad 182 \cdot 6 \cdot \frac{3}{4} \quad \frac{1}{2} \\ \quad \quad 45 \cdot 7 \cdot \frac{1}{2} \end{array}$$

$$\begin{array}{r} 2|0 \quad 59|3 \cdot 1 \cdot \frac{1}{2} \\ \hline 29 \cdot 13 \cdot 1 \cdot \frac{1}{2} \end{array}$$

$$365 \text{ Days at } 18 \text{ d. } \frac{3}{4} \text{ per Day.}$$

$$\begin{array}{r} 365 \\ 182 \cdot 06 \\ 22 \cdot 09 \cdot \frac{3}{4} \end{array}$$

$$\begin{array}{r} 2|0 \quad 57|0 \cdot 03 \cdot \frac{3}{4} \\ \hline 28 \cdot 10 \cdot 03 \cdot \frac{3}{4} \end{array}$$

IX. If the Price be an even Number of Shillings, as 2, 6, 8, 12, 14, 16, 18, 22, &c.

Rule. *Multiply the given Number by the half of the Price, setting the Double of the unit Figure for Shillings.*

Example. What come 379 Yards of Broad-Cloth to, at 18 s. or 22 s. per Yard ?

$$\begin{array}{r} 379 \text{ at } 18 \text{ s.} \\ 9 \\ \hline 3411 \cdot 2 \end{array}$$

$$\begin{array}{r} 379 \text{ at } 22 \text{ s.} \\ 11 \\ \hline 3469 \cdot 18 \end{array}$$

See Mr. Kersey's 5th and 6th Rule of the second Chapter.

X. When the Price is an odd Number of Shillings.

Rule. *Multiply the given Number by the said Shillings, dividing the Product by 20.*

Example. What come 796 Small Grofs to, at 13 s. or 17 s. per Grofs ?

$$\begin{array}{r} 796 \text{ at } 13 \text{ s.} \\ 13 \\ \hline 2|0 \quad 1034|8 \\ \hline 517 \cdot 08 \end{array}$$

$$\begin{array}{r} 796 \text{ at } 17 \text{ s.} \\ 17 \\ \hline 2|0 \quad 1353|2 \\ \hline 676 \cdot 12 \end{array}$$

Or

Or rather according to Mr. Kersey's 8th Head, in the Rule of Practice.

XI. When the Price is Shillings and Pence, and such Shillings and Pence, are the same Figure.

Rule. Multiply the given Quantity by the Shillings, taking always the $\frac{1}{20}$ of the Product for the Pence, the Total of which divided by 20, gives the Answer in Pounds.

Example. What comes 731 Ells of Holland to, at 7 s. 7 d. or 11 s. 11 d. per Ell?

$$\begin{array}{r}
 731 \text{ Ells at } 7 \text{ s. } 7 \text{ d.} \\
 \underline{7} \\
 5117 \\
 426 \cdot 05 \\
 \hline
 210 \ 55413 \cdot 05 \\
 \hline
 277 \cdot 03 \cdot 05
 \end{array}$$

$$\begin{array}{r}
 731 \text{ Ells at } 11 \text{ s. } 11 \text{ d.} \\
 \underline{11} \\
 8041 \\
 670 \cdot 01 \\
 \hline
 210 \ 87111 \cdot 01 \\
 \hline
 435 \cdot 11 \cdot 01
 \end{array}$$

XII. When the Price is between one Pound and two.

Rule. Work after the manner of the 8th Rule, i. e. only for the Money above a Pound, adding thereto the given Quantity.

Example. What come 224 l. of Cochineal to, at 27 s. 6 d. per l.

$$\begin{array}{r}
 \text{5.} \quad \text{224} \\
 \text{5} \quad \frac{1}{4}) \quad \underline{224} \\
 \quad \quad 56 \\
 \quad \quad 28 \\
 \quad \quad \hline
 \quad \quad 308
 \end{array}$$

$$\begin{array}{r}
 \text{224} \\
 \underline{224} \\
 8) \quad 2464 \\
 \hline
 308 \text{ Answer}
 \end{array}$$

XIII. When the Price has a Fraction annexed.

Rule. Work for the Price according to the fore-going Rules, and for the Fraction, either value it as directed in the first Rule: Or if an Aliquot Part of the Money fore-going, take such Part for it.

Example (1.) What come 479 l. of Tobacco to, at 22 d. $\frac{1}{4}$ per l.

$$\begin{array}{r}
 479 \\
 11 \overline{) 479} \\
 \underline{11} \\
 12 \overline{) 0} \quad 526 \overline{) 9} \\
 \underline{43} \cdot 18 \cdot 2 \\
 \underline{14} \cdot 11 \cdot \frac{1}{8} \\
 \text{Answer } 44 \cdot 13 \cdot 01 \cdot \frac{5}{8} \quad 14 \cdot 11 \cdot \frac{5}{8}
 \end{array}$$

Example, (2.) How much Sterling for 7439 French Crowns Exchange, at 55 Pence $\frac{1}{8}$ per Crown?

$$\begin{array}{r}
 \text{s. } d. \quad 7439 \\
 \hline
 2 \quad 6 \quad \frac{1}{8} \quad 929 \cdot 17 \cdot 06 \\
 1 \quad 8 \quad \frac{1}{16} \quad 619 \cdot 18 \cdot 04 \\
 \quad 5 \quad \frac{1}{4} \quad 154 \cdot 19 \cdot 07 \\
 \quad \frac{5}{8} \quad \frac{1}{8} \quad 19 \cdot 07 \cdot 05 \cdot \frac{1}{2} \\
 \hline
 \text{Answer } 1724 \cdot 02 \cdot 10 \cdot \frac{3}{4}
 \end{array}$$

II. Of the Given Quantity.

I. When it has odd Weight or Measure added,

Rule. Value the whole Number as before directed, then take Parts for such odd Weight or Measure out of an Integer; dividing the given Price by such Parts, the Total of which added to the Value of the whole Number gives the Amount.

Example. What come 249 C. 2 q. 18 l. to, at 10 s. 4 d. per Hundred?

$$\begin{array}{r}
 294 \\
 5 \overline{) 294} \\
 \hline
 \frac{1}{10} \quad 147 \cdot 00 \\
 \quad 4 \cdot 18 \\
 \quad \quad 06 \cdot 09 \cdot \frac{1}{2} \\
 \hline
 152 \cdot 04 \cdot 09 \cdot \frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{s. } d. \\
 \frac{1}{2} \quad 10 \cdot 4 \\
 \hline
 \frac{1}{7} \quad 5 \cdot 2 \\
 \frac{1}{8} \quad 1 \cdot 5 \cdot \frac{1}{2} \\
 \quad 0 \cdot 2 \cdot 0 \\
 \hline
 6 \cdot 9 \cdot \frac{1}{2}
 \end{array}$$

II. When the given Number is not of the same Name with the Integer on which the Price is.

Rule. It must be reduced to the same according to the Rules of Reduction.

Ex.

Ex. What come 49 Thousand
to, at 17 s. 4 d. per Hundred ?

$$\begin{array}{r} 490 \\ 9 \\ \hline 441 . 00 \\ 16 . 06 . 8 \\ \hline 424 . 13 . 04 \end{array}$$

10 l. 11 oz. of Silk to, at
2 s. 8 d. per Oz. ?

$$\begin{array}{r} 10 . 11 \\ 16 \\ \hline 3) 171 \\ 57 \text{ Answ. } 22 \text{ l. } 16 \text{ s.} \\ \hline 22 . 8 \end{array}$$

III. When it has a Fraction annexed ;

Rule. *Value the whole Number, according to the Rules afore-
going, and for the Fraction, multiply the Price by the Numerator,
and that Product divided by the Denominator, gives the Value of
the Fraction, which added to the Value of the whole Number,
gives the Answer.*

Example, What come 372 Ells $\frac{3}{4}$ of Bag-Holland to, at
5 s. 10 d ?

$$\begin{array}{r} \frac{3}{4} 372 \\ \hline \frac{1}{2} 93 \\ 15 . 10 \\ 3 . 6 \\ \hline 108 . 13 . 6 \end{array} \quad \begin{array}{r} 5 . 10 \\ 3 \\ \hline 5) 17 . 06 \\ 3 . 06 \\ \hline \end{array}$$

IV. When the given Quantity is of a less Denomination
than that on which the Price is given :

Rule. *Take Parts for the less Name out of the greater, divi-
ding the Price by such Parts.*

Ex. At 7 l. 10 s. per
Tun, what per Barrel ?

$$7) 7 . 10$$

Answ. $1 . 01 . 5 . \frac{1}{7}$

How much a Gallon, at 38 s.
3 d. per Hoghead ?

$$9) 38 . 3$$

$$7) 4 . 3$$

Answ. $0 . 7 . \frac{3}{7}$

What comes a Cheeshire-Cheese to, At 19 l. 7 s. 8 d. Hun-
Weight 44 l. at 37 s. 9 d. Hundred? dred, How much a Pound?

$$\begin{array}{r} \frac{1}{4} 37 . 9 \\ \hline \frac{1}{7} 9 . 5 . \frac{1}{4} \\ 4 . 8 . \frac{1}{2} \\ \hline \end{array}$$

Answ. $14 . 9 . \frac{1}{4}$

$$\begin{array}{r} l. \quad s. \quad d. \\ 16 \frac{1}{7} 19 . 07 . 08 \\ 2 \frac{1}{8} 2 . 15 . 04 . \frac{1}{2} \\ 1 \frac{1}{2} 0 . 06 . 11 . 0 \\ \hline \end{array}$$

Answ. $0 . 03 . 05 .$

The

The Quantities of particular Goods in Wholesale-Trade, found in the Book of Rates, with concise and ready Ways of casting up the same.

I. Goods sold by Sixscore to the Hundred.

Lamb-Skins, Barlings, Balks Great, Middle, and Small; Clap-Boards, Pipe-Boards, Bow-Staves, Cap-ravens, Deal-Boards, Spars of all Sorts, Cod, Cole, Ling, and all Sorts of Stock Fish; with many Sorts of Linnen, viz. Hamburgh, Silesia, Irish, Muscovia, Westphalia, Hanover, &c.

To cast up Goods of this Quantity.

Rule. Take half the Pence of the Price of one, which gives you the Value of 120 in Pounds, Shillings, &c.

$$\begin{array}{r} 120 \\ \hline 240 \end{array} \text{ being the } \frac{1}{2} \text{ of a l. Sterling.}$$

Example. What come 120 Ells of Canvas to, at 5 d. $\frac{1}{2}$ per Ell?

$$\begin{array}{r} 2) 5 \text{ . } \frac{1}{2} \\ \hline 2 \text{ . } 15 \end{array} \text{ Answer } 2 \text{ l. } 15 \text{ s.}$$

Example (2.) At 7 : 15 s. per Hundred of Deals, How much each?

$$\begin{array}{r} 7 \text{ . } 15 \\ \hline 2 \end{array} \text{ Answer } 1 \text{ l. } 15 \text{ s. } \frac{1}{2}.$$

N. B. That you esteem every Penny (which remains in the first Example) as 20 s. and every 20 s. as one Penny, (in the second.) As

In Example the first, there remains one Penny, which reckoning 20 s. and accordingly the Half-penny in the given Price for 10 s. both these make 30 s. which divided by 2, gives 15 s. and so the Answer is found to be 2 l. 15 s.

II. Goods sold by the Thousand, are

Paste-Boards for Books, Tennis-Balls, Lemmons, Oranges, Teazels, Flanders-Paving and Pan-Tiles, Lantern-Horns, Barrel-Hoops, and Boards; Quills, Lamperns, Squirrel-Skins, Ox-Bones, Yards of Lift, &c.

E e e

To

To cast up Goods of this Quantity.

Rule. Multiply the Pence that one costs by 50, and the Product divided by 12, gives the Value of a Thousand.

$$\frac{1000}{240} \text{ being the } \frac{1}{12} \text{ or } \frac{1}{2} \text{ of } 20 \text{ s.}$$

Example, (1.) What comes 1000 of Flanders Tiles to, at 3 d. each?

$$\begin{array}{r} 3 \\ 50 \\ \hline 12 \overline{) 150} \\ \text{Answer l. } 12 \text{ . } 10 \end{array}$$

Example, (2.) If 1000 of Squirrel-Skins cost 11 l. 05 s. What per Skin?

$$\begin{array}{r} 11 \text{ . } 05 \\ 12 \\ \hline 5 \overline{) 135} \text{ . } 00 \\ \hline 2 \text{ . } \frac{1}{2} \text{ per Skin.} \end{array}$$

III. Goods sold by the Great Grofs, containing 12 Small Grofs, or 144 Dozen, viz.

Buttons of Metal, Glafs, Thread, Silk, or Hair; Beads of Bone, Box, Glafs, or Wood; Cap-Hooks, Chefs-Men; Tobacco-Pipes, Combs, of Light or Box-Wood, Thread or Silk-Points, Playing-Cards, &c.

Goods of this Quantity are cast up thus, viz.

Rule. Multiply the Pence of the Price of one Dozen by 3, and the Product divided by 5, gives the Value of a Great Grofs in Pounds, &c.

$$\frac{144}{240}$$

240 being the $\frac{3}{5}$ of a Pound Sterling.

Example, (1.) What comes a Great Grofs of Silk-Buttons to, at 8 d. $\frac{1}{2}$ per Dozen?

$$\begin{array}{r} 8 \text{ . } \frac{1}{2} \\ 3 \\ \hline 25 \text{ . } \frac{1}{2} \end{array}$$

5) 25 . $\frac{1}{2}$ Answer 5 l. 2 s.

5 . 2

Example, (2.) What comes one Dozen to, when a Great Gros costs 7 l. 8 s.

7 . 8

5

3) 39 . 0 Answer 13 d. $\frac{1}{3}$

13 . $\frac{1}{3}$

IV. Goods sold by the Small Gros of 12 Dozen, viz.

Tobacco and Pepper-Boxes, Nest and Tinder-Boxes, Buckles of some Sorts, Wash-Balls, Inkhorns, Bodkins, several sorts of Knives, Comb-Cases, &c.

Goods of this Quantity may be cast up by the last preceeding Rule, seeing there are as many Particulars in a small Gros, as Dozens in a great one.

V. Goods sold by the Fivescore to the Hundred.

Annotto, Capers, Safflowers, Thrums, Ginger, Cloves, Indico, Cross-Bow-Thread, Pack-Thread, Kids or Goats Hair, Quick-Silver, Cotton-Wool, English Hard Wax, Brass and Latten-Work, as Chafing-dishes, And-irons, Laver-cocks, &c.

To cast up these Goods,

Rule. Multiply the Pence of the Price of one Pound Weight by 5, dividing the Product by 12.

100

210 being the $\frac{5}{12}$ of a l.

Example, (1.) What comes 100 Weight of China-Roots to, at 15 d. per l.?

15

5

12) 75 Answer 6 l. 3 s. 4 d.

6 . 3

Example, (2.) What comes 1 Pound to, when 100 Pounds costs 14 l. 15 s.?

$$\begin{array}{r} l. \quad s. \\ 14 \cdot 15 \\ 12 \end{array}$$

$$\begin{array}{r} 5) 177 \cdot 00 \\ \hline \end{array}$$

Answ. $35 \cdot \frac{2}{3}$ or 2 s. 11 d. $\frac{2}{3}$

To value Goods sold by the common Hundred of 112 Pounds to the Hundred.

Rule. Multiply the Pence that one Pound costs by 7, and divide the Product by 15; or multiply by 14, and divide by 30 gives the Value $\frac{1}{4}\frac{1}{5}$, being the $\frac{1}{4}\frac{1}{5}$, or $\frac{1}{5}\frac{1}{4}$ of a Pound.

Example (1.) What comes a Hundred Weights to, at 8 d. $\frac{1}{4}$ per Pound?

$$\begin{array}{r} 8 \cdot \frac{1}{4} \\ 7 \\ \hline 15) 61 \cdot \frac{1}{4} d. \\ \hline \end{array}$$

$$4 \cdot 1 \cdot 8$$

Example (2.) At 9 l. per Hundred, What comes 1 Pound to?

$$\begin{array}{r} 9 \\ 15 \\ \hline 7) 135 \\ \hline 19 \cdot \frac{1}{4} \end{array}$$

To cast up Goods of this Quantity in mind, See Mr. Kersey's 18th Rule of the second Chapter.

Now if it be further required to know what any Number (not exceeding Five-score) of the aforesaid Quantities in Wholesale-Trade (or any other) amount to at any Rate, the most practical and easiest Way is by the following Rule.

Rule. Multiply the Price of one by the Parts of the given Quantity (as taught in Multiplication.)

Examples.

What comes 12 C. $\frac{1}{2}$ to, at
7 l. 11 s. 8 d. per Hundred?

At 7 s. 7 d. $\frac{1}{2}$ per Gross,
What comes 46 Gross to?

$$\begin{array}{r} 7 . 11 . 8 \\ 12 . \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 91 . 00 . 0 \\ 3 . 15 . 10 \\ \hline \end{array}$$

$$\begin{array}{r} 94 . 15 . 10 \\ \hline \end{array}$$

$$\begin{array}{r} 4 . 7 . \frac{1}{2} \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 23 . 01 . \frac{1}{2} \\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 10 . 08 . 01 . \frac{1}{2} \\ 04 . 07 . \frac{1}{2} \\ \hline \end{array}$$

$$10 . 12 . 09 . 0$$

What comes 2900 to, at
2 l. 14 s. 7 d. per Hundred?

$$\begin{array}{r} 2 . 14 . 07 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8 . 03 . 09 \\ 10 \\ \hline \end{array}$$

$$81 . 17 . 06$$

$$2 . 14 . 07$$

$$79 . 02 . 11$$

33 Hundred, at 19 s. 7 d.
per Hundred?

$$\begin{array}{r} 19 . 07 \\ 11 \\ \hline \end{array}$$

$$\begin{array}{r} 10 . 15 . 05 \\ 3 \\ \hline \end{array}$$

$$32 . 06 . 03$$

If instead of knowing the Value or Worth of Goods at a certain Price, it be required to know what Quantity of Goods can be bought for a certain Sum of Money.

If the Price be even Shillings,

Rule. Add a Cypher to the Pounds, and divide by $\frac{1}{2}$ the Price.

Example 1. How many lb. Oz. and Drums, can I buy for 479 l. at 6 s. per lb.?

$$3) 1790 . \text{Oz. Dra.}$$

$$\text{Answer, lb. } 2596 . 10 . 10 . \frac{2}{3}$$

Example 2. At 8 s. per Yard, How many Yards, Quarters, and Nails, can I buy for 315 l.

$$4) 3150$$

$$\text{Yards, Quarters.}$$

$$787 . \frac{1}{2} \text{ Answ. } 787 . 2$$

At 14 s. 3 d. per Yard, How much per Ell? Add $\frac{1}{4}$ part.

$$\begin{array}{r} 4) 14 . 3 \\ 3 . 6 . \frac{3}{4} \\ \hline \end{array}$$

$$\text{Answ. } 17 . 9 . \frac{3}{4}$$

What

What cost 1 Pound, when $\frac{2}{3}$ cost 16 s. 8 d. Add $\frac{1}{3}$ the Price?

$$\begin{array}{r} 16 \text{ s. } 8 \\ 8 \text{ s. } 4 \\ \hline \end{array}$$

Ans^w. 25 . 0

What come 47 Pounds to, when $\frac{3}{4}$ cost 6 d. i. e. 8 d. per Pound?

$$\begin{array}{r} 3|0) 4|7 \\ \hline \end{array}$$

Ans^w. 1 . 11 . 4

How much can I buy for 11 d. when 24 lb. 11 oz. cost 11 s.?

$$\begin{array}{r} \frac{1}{12}) 24 \text{ s. } 11 \text{ dr.} \\ \hline \end{array}$$

Ans^w. 2 . 00 . 14 $\frac{2}{3}$

If 22|0 Yards cost 57 l. What is that per Yard?

$$\begin{array}{r} 11) 57 \\ \hline \text{d.} \end{array}$$

Ans^w. 5 . 2 . $\frac{2}{3}$

Here I divide 57 l. by 11, seeing there is the like Reason between 11 and 57, as there would have been between 220 and 1140, the Product of 57 by 20.

SECTION V.

Of Simple INTEREST.

TO find the Interest of any Sum at any Rate for any Time given.

I. By Months.

In this the Year is taken as containing 12 Months of 30 Days each; which wants $\frac{1}{12}$ of a Day, and therefore the precise Interest cannot be found by this Method.

Yet because 'tis so much in use among Traders, so little differing from Truth, and so ready in Operation, I shall give three several Ways of performing it.

The first and usual Way :

Rule. Multiply the Principal by the Rate of Interest per Cent. per Annum, and the Product (casting off two Figures on the Right-Hand) is the Interest for a Year: From whence it is found for any Time required.

Example

Example 1. What is the Interest of 479 l. 18 s. for 2 Years, at 4 l. $\frac{1}{2}$ per Cent. per Annum?

$$\begin{array}{r}
 479 \cdot 18 \\
 \underline{4 \frac{1}{2}} \\
 1919 \cdot 12 \\
 239 \cdot 19 \\
 \hline
 21 \overline{) 59 \cdot 11} \\
 \underline{20} \\
 11 \overline{) 91} \\
 \underline{13} \\
 10 \overline{) 92} \\
 \underline{4} \\
 3 \overline{) 68}
 \end{array}$$

l. s. d.

$$\begin{array}{r}
 21 \cdot 11 \cdot 10 \frac{1}{2} \\
 \underline{2}
 \end{array}$$

Answ. 43 . 03 . 09 $\frac{1}{2}$

Example 2. At 6 per Cent. per Annum, What is the Interest of 571 l. 15 s. for 8 Months?

$$\begin{array}{r}
 571 \cdot 15 \\
 \underline{6 \cdot 6 \frac{1}{2}} \\
 34 \overline{) 30 \cdot 10} \quad 2 \cdot 3 \quad 17 \cdot 03 \cdot 00 \cdot \frac{1}{2} \\
 \underline{20} \quad \quad \quad 5 \cdot 14 \cdot 04 \cdot 0 \\
 6 \overline{) 10} \\
 \underline{1} \overline{) 20}
 \end{array}$$

Answ. 22 . 17 . 04 . $\frac{1}{2}$

Note. By this Rule, *Brokage, Commission, Provision, Storage, Insurance, &c.* are work'd, having no respect to Time.

Example 1. Unto what comes the Commission of 356 l. 17 s. 11 d. $\frac{1}{2}$, at 2 $\frac{1}{4}$ per Cent.?

$$\begin{array}{r}
 359 \cdot 17 \cdot 11 \cdot \frac{1}{2} \\
 \hline
 \frac{1}{4} \quad 719 \cdot 15 \cdot 11 \cdot 0 \\
 129 \cdot 18 \cdot 11 \cdot \frac{3}{4} \\
 \hline
 6 \quad 49 \cdot 14 \cdot 10 \cdot \frac{3}{4} \\
 \quad 20 \\
 \hline
 9 \quad 94 \\
 \quad 12 \\
 \hline
 11 \quad 38 \\
 \quad 4 \\
 \hline
 1 \quad 55
 \end{array}$$

Example 2. At $1 \frac{1}{4}$ per Cent. What comes the Brokage of 198 l. 11 s. 7 d. to?

$$\begin{array}{r}
 4 \text{ l. } 198 \cdot 11 \cdot 7 \\
 49 \cdot 12 \cdot 10 \cdot \frac{3}{4} \\
 \hline
 2 \quad 48 \cdot 4 \cdot 5 \cdot \frac{3}{4} \\
 \quad 20 \\
 \hline
 9 \quad 64 \\
 \quad 12 \\
 \hline
 7 \quad 73 \\
 \quad 4 \\
 \hline
 2 \quad 95
 \end{array}$$

Answ. 2 l. 9 s. 7 d. $\frac{1}{2}$

But Questions of this nature are more concisely and readily worked by this.

Rule. Divide the given Sum by the Aliquot Part, that the Rate of the Commission or Brokage, &c. is of 100 l. As for Instance, in the last Examples.

Example 1. The Commission being $2 \frac{1}{2}$ per Cent. which is the 40th Part of 100 l. Therefore

$$4 \overline{) 10} \quad 25 \overline{) 9} \text{ l. } 17 \text{ s. } 11 \text{ d. } \frac{1}{2}$$

Answ. 6 . 09 . 11 . $\frac{1}{4}$

Example 2. The Brokage being $1 \frac{1}{4}$ per Cent. which is the 80th Part of a 100 l. Therefore

8|0) 19|8 l. 11 s. 7 d.

Answ. 2 . 09 . 07 . $\frac{1}{2}$

The second Method.

Rule. Multiply the Principal, by the Interest of 100 l. for the Time required, and the Product (striking off two Figures on the Right-hand) is the Answer?

Example 1. What is the Interest of 479 l. 18 s. for two Years, at $4\frac{1}{2}$ per Cent. per Annum.

The Interest of 100 l. for two Years, at $4\frac{1}{2}$ per Cent. is 9 l. Therefore

$$\begin{array}{r}
 479 \cdot 18 \\
 9 \\
 \hline
 43 \mid 19 \cdot 02 \\
 20 \\
 \hline
 3 \mid 82 \\
 12 \\
 \hline
 9 \mid 84 \\
 4 \\
 \hline
 3 \mid 36
 \end{array}$$

Answ. 43 l. 03 s. 09 d. $\frac{3}{4}$

Example. What is the Interest of 571 l. 15 s. for 8 Months, at 6 per Cent. per Annum?

The Interest of 100 l. for 8 Months, at 6 per Cent. is 4 l. Therefore

$$\begin{array}{r}
 571 \text{ l. } 15 \text{ s.} \\
 4 \\
 \hline
 22 \mid 87 \cdot 00 \\
 20 \\
 \hline
 17 \mid 40 \\
 12 \\
 \hline
 4 \mid 80 \\
 4 \\
 \hline
 3 \mid 20
 \end{array}$$

Answ. 22 l. 17 s. 04 d. $\frac{3}{4}$

Note. If it be for 3 Months, at 5 per Cent. per Annum, then you need only add $\frac{1}{4}$ of the Principal to itself, working as before directed, &c.

The Third Method.

Rule. Since the Interest of any Principal,

$$\text{at } \left\{ \begin{array}{l} 5 \text{ l.} \\ 6 \\ 4 \\ 3 \end{array} \right\} \text{ per Cent. for } \left\{ \begin{array}{l} 12 \\ 10 \\ 15 \\ 20 \end{array} \right\} \text{ Months.}$$

is just so many Shillings as it has Pounds. Therefore,

(1.) What is 379 l. 15 s. for 12 Months, at 5 per Cent. ?

(2.) What is 461 l. 12 s. 6 d. for 10 Months, at 6 per Cent. ?

$$2|0 \ 37|9 \ . \ 15$$

$$2|0 \ 41|6 \ . \ 12 \ . \ 6 \ d.$$

$$\text{Answ. } 18 \ . \ 19 \ . \ 9$$

$$\text{Answ. } 20 \ . \ 16 \ . \ 7 \ \frac{1}{2}$$

(3.) What is 427 l. 13 s. 9 d. for 20 Months, at 3 per Cent. ?

(4.) What is 927 l. 11 s. 3 d. for 15 Months, at 4 per Cent. ?

$$2|0 \ 42|7 \ . \ 13 \ . \ 9$$

$$2|0 \ 92|7 \ . \ 11 \ . \ 3$$

$$\text{Answ. } 21 \ . \ 7 \ . \ 8 \ \frac{1}{2}$$

$$\text{Answ. } 46 \ . \ 7 \ . \ 6 \ \frac{1}{2}$$

Hence, to find the Interest of a given Principal, for any other Time.

Rule. Take Parts, for the Time required, out of the Time affixed.

Thus, if the Interest of 379 l. 15 s. for 12 Months, at 5 per Cent. is so many Shillings, then for 6 Months, it is the $\frac{1}{2}$ of so many Shillings, or so many Six-pences; and for 4 Months the $\frac{1}{3}$ of it, or so many Groats, as the Principal has Pounds.

Example 1. What is the Interest of 279 l. 11 s. for 4 Months, at 5 per Cent. ?

$$2|0) \ 27|9 \ . \ 11$$

$$3) \ 13 \ . \ 19 \ . \ 6 \ \frac{1}{2}$$

$$\text{Answ. } 4 \ . \ 13 \ . \ 2$$

Example 2. What is the Interest of 197 l. 11 s. for 2 Months, at 6 per Cent. ?

$$2|0) \ 19|7 \ . \ 11$$

$$5) \ 2 \ . \ 17 \ . \ 6 \ \frac{1}{2}$$

$$\text{Answ. } 1 \ . \ 19 \ . \ 6$$

Example

Example 3. What is 49*l.* 17*s.* 10*d.* for 5 Months, at 4 per Cent. ?

$$\begin{array}{r} 2|0) \quad 49|1 \cdot 17 \cdot 10 \\ \hline 3) \quad 24 \cdot 11 \cdot 10 \frac{3}{4} \\ \hline \end{array}$$

Answ. 8 . 3 . 11 $\frac{1}{2}$

Example 4. What is 279*l.* 11*s.* 8*d.* for 7 $\frac{1}{2}$ Months, at 4 per Cent. ?

$$\begin{array}{r} 2|0) \quad 279 \cdot 11 \cdot 8 \\ \hline 2) \quad 13 \cdot 19 \cdot 7 \\ \hline \end{array}$$

Answ. 6 . 19 . 1 $\frac{1}{2}$

For what odd Money is adjoined to the Principal, value every 20*s.* as 12*d.* and lesser Sums in proportion.

SECTION VI.

II. Interest for Days.

To find the exact Number of Days between any two given Times, without any Book or Table.

This is done by carrying in Memory that old Distich, viz:
Thirty Days has *September, April, June, and November* ;
February has Twenty-eight alone ; all the Rest have Thirty one.

And according to this reckoning, the Year contains 365 Days ; therefore, for Instance,

To find the Number of Days between the 7th of *June* 1701, and the 15th of *September* 1703 : Say thus,

From the 7th of *June* 1701, to the 7th of *June* 1703, is 365

2

730

or two Years.

From the 7th of *June* 1703, to the End of the same Month is

Then in $\left\{ \begin{array}{l} \text{July} \text{---} 31 \\ \text{August} \text{---} 31 \end{array} \right.$

From which, to the 15th of *September*, is $\text{---} \text{---} 15$
F f f 2 So

So that from the 7th of June 1701, to the 15th } Days 830
of September 1703, is _____

Note. That 1 Day must be added to every 4 Years contained in the Time. For every fourth Year, being Leap-Year, February has then 29 Days.

After this manner may any Number of Days, even to $\frac{1}{2}$ or $\frac{1}{4}$ of a Day, be exactly and readily discovered. Now

To find the Interest of any Sum, at any Rate, for any Number of Days, observe these two General Methods.

First Rule. Multiply the Pence of the Principal by the Number of Days, and the Product by the Rate of Interest for a Dividend, which divide by 36500, the Quotient is the Answer in Pence.

Otherwise, Take the Product of the Pence of the Principal, multiplied by the Days, for a Dividend, and the Quotient of 36500, divided by the Rate of Interest, for a Divisor, and the Quotient of that Division is the Answer.

Example 1. What comes the Interest of 192 l. 14 s. 10 d. for 830 Days, at 5 per Cent. per Annum ?

$$\begin{array}{r}
 20 \\
 \hline
 3854 \\
 12 \\
 \hline
 46258 \\
 \hline
 830 \\
 \hline
 1387740 \\
 370064 \\
 \hline
 38394140 \\
 5 \\
 \hline
 365100 \quad 1919707100 \quad (5259 \\
 947 \quad \quad \quad 4318.3 \\
 \hline
 2170 \quad \quad \quad 21.18.3 \text{ Answ.} \\
 \hline
 3457 \\
 \hline
 17200
 \end{array}$$

Example

Sect. 6.

Of Simple Interest.

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Example 2. What comes the Interest of 311 l. 13 s. 4 d. to, for 315 Days, at 6 per Cent. per Annum?

$$\begin{array}{r}
 311\text{ l } 13\text{ s. } 4\text{ d.} \\
 20 \\
 \hline
 6233 \\
 12 \\
 \hline
 74800 \\
 315 \\
 \hline
 1122000 \\
 224400 \\
 \hline
 6083) \quad 23562000 \quad (3873 \\
 \hline
 53130 \quad 32 \overline{) 2} . 9 \\
 \hline
 44660 \quad 16 . 2 . 9 \\
 20790 \\
 \hline
 2541
 \end{array}$$

The second, which I received from my Friend Mr. Ralph Snow, is,

Rule. Multiply the Pence in the given Principal, by the Number of Days, divide the Product always by 7300, and the Quotient is the Interest, in Pence, of that Principal, for that Time, at 5 per Cent. per Annum.

Thus the Interest of 192 l. 14 s. 10 d. for 830 d. at 5 per Cent.

$$\begin{array}{r}
 192 \text{ l. } 14 \text{ s. } 10 \text{ d.} \\
 20 \\
 \hline
 3853 \\
 12 \\
 \hline
 46258 \\
 830 \\
 \hline
 1387740 \\
 370064 \\
 \hline
 73|00) 383941|40 \quad (5259 \text{ Interest in Pence at } 5 \text{ per} \\
 189 \quad \text{Cent.} \\
 \hline
 434 \\
 \hline
 691 \\
 \hline
 34
 \end{array}$$

Whence, to find the Interest of the same Principal, for the same Time, at the Rate of 3, 4, 5, 6, 7, &c. *per Cent.*

Add so many Fifths as the Rate of Interest exceeds 5: Or subtract so many Fifths as it is less than 5.

The Fifths of any Number are readily found, by multiplying double the Numerator, and cutting off the unit Figure of the Product.

Because so many Fifths of any Number is the same with twice so many Tenths of that Number.

Now 5359 *d.* being the Interest of 192 *l.* 14 *s.* 10 *d.* for 830 Days, (at 5 *per Cent.* as in the last Precedent) let it be required, to know the Interest of the same Principal, for the same time, at any other Rate, suppose 3, 4 $\frac{1}{2}$, or 6 *per Cent. per Annum*, then *viz.*

Ex. 1. $\frac{4}{187}$, or $\frac{2}{93}$ being subtracted, refts
the Interest likewise in Pence, at 3 per Cent.

Ex. 2. One Tenth subtracted, refts
the Interest at $4\frac{1}{2}$ per Cent.

Ex. 3. $\frac{2}{10}$, $\frac{1}{5}$, being added, gives
the Interest at 6 per Cent.
And the like for any other Rate whatever.

$$\begin{array}{r}
 5259 \\
 \underline{4} \\
 2103 \cdot 6 \\
 \underline{3155 \cdot 4} \\
 5259 \\
 \underline{525 \cdot 9} \\
 4733 \cdot 1 \\
 \underline{5259} \\
 1051 \cdot 8 \\
 \underline{6311 \cdot}
 \end{array}$$

Of Exchequer-Notes and Tallies.

Chequer-Notes of five and twenty Pounds did formerly bear five Farthings, but now a Penny per Day Interest.

Tallies have been at 6, but now at 5 per Cent. per Annum, the Interest payable every three Months.

Example. Unto what comes the Interest of two 25 l. Notes, from the 27th of November 1702 to this Day, being the 4th of January 1712--13?

The Days found, according to the Direction given in the beginning of this Section, are 403, which at 2 d. per Day Interest, is 3 l. 7 s. 2 d. as, viz.

$$\begin{array}{r}
 12|0) \ 40|3 \\
 \underline{00} \\
 3 \cdot 7 \cdot 2
 \end{array}$$

Example 1. What is the quarterly Interest of a Tally, viz. Numb. 2704, struck June 11. 1702, for 579 l. at 6 per Cent.?

$$\begin{array}{r}
 \frac{1}{2}) \quad 579 \\
 \underline{289} \quad 10 \\
 8 \mid 68 \quad 10 \\
 \underline{\quad} \quad 20 \\
 13 \mid 70 \\
 \underline{\quad} \quad 12 \\
 8 \mid 40 \\
 \underline{\quad} \quad 4
 \end{array}$$

1|60 *Anfw.* 8 l. 13 s. 8 d. $\frac{1}{2}$

Example 2. What comes the Interest to, until the 10th of January 173 $\frac{3}{4}$, of two Tallies, the Number, Principal, and Time, of whose Orders are, *viz.*

N^o 3081, from August 5, 1702, for 914 l. 5 s. at 6 per Cent. per Annum.

$$\begin{array}{r}
 20 \\
 \hline
 18285 \\
 12 \\
 \hline
 219420 \\
 523 \text{ Days} \\
 \hline
 658260 \\
 \text{Anfw. } 78 \text{ l. } 12 \text{ s. } 438840 \\
 1097100 \\
 \hline
 73) \quad 1147566 \mid 60 \quad (15720 \\
 417 \quad \quad \quad 3144 \\
 525 \quad 12) \quad 18864 \\
 \hline
 146 \quad \quad \quad 157 \mid 2 \\
 \hline
 6 \quad \quad \quad 78 \cdot 12
 \end{array}$$

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N^o 5792, struck *May* 22, 1703, for 379 *l.* 10 *s.* at 5 *per*
Cent. per Annum.

$$\begin{array}{r}
 20 \\
 \hline
 7590 \\
 12 \\
 \hline
 91080 \\
 233 \\
 \hline
 273340 \\
 273240 \\
 182160 \\
 \hline
 73) 212516 | 40 \quad (2907 \\
 662 \qquad \qquad 242 . 3 \\
 \hline
 516 \qquad \qquad 12.2 . 3 \\
 (5
 \end{array}$$

G g g

T A

T A B L E S

O F

Simple Interest.

Serving at any Rate for any Time.

SEC-

SECTION VII.

Days	At 3 per Cent.	At 4 per Cent.	At 5 per Cent.
1	,000082192	,000109589	,000136986
2	,000164384	,000219178	,000273972
3	,000246576	,000328767	,000410958
4	,000328768	,000438356	,000547944
5	,000410960	,000547945	,000684930
6	,000493152	,000657534	,000821916
7	,000575344	,000767123	,000958902
8	,000657536	,000876712	,001095888
9	,000739728	,000986301	,001232874
10	,000821920	,001095890	,001369860
11	,000904112	,001205479	,001506846
12	,000986304	,001315068	,001643832
13	,001068496	,001424657	,001780818
14	,001150688	,001534246	,001917804
15	,001232880	,001643835	,002054790
16	,001315072	,001753424	,002191776
17	,001397264	,001863013	,002328762
18	,001479456	,001972602	,002465748
19	,001561648	,002082192	,002602734
20	,001643840	,002191781	,002739720
21	,001726032	,002301370	,002876706
22	,001808224	,002410959	,003013692
23	,001890416	,002520548	,003150678
24	,001972608	,002630137	,003287664
25	,002054800	,002739726	,003424650
26	,002136992	,002849315	,003561636
27	,002219184	,002958904	,003698622
28	,002301376	,003068493	,003835608
29	,002383568	,003178082	,003972594
30	,002465760	,003287671	,004109580
31	,002547952	,003397260	,004246566
32	,002630144	,003506849	,004385552
33	,002712336	,003616438	,004522538
34	,002794528	,003726027	,004659524
35	,002876720	,003835616	,004786510
36	,002958912	,003945205	,004923496

Days	At 6 per Cent.	At 7 per Cent.	At 8 per Cent.
1	,000164383	,000191781	,000219178
2	,000328766	,000383562	,000438356
3	,000493149	,000575343	,000657534
4	,000657532	,000767124	,000876712
5	,000821915	,000958905	,001095890
6	,000986298	,001150686	,001315068
7	,001150681	,001342467	,001534246
8	,001315664	,001534248	,001753424
9	,001479447	,001726029	,001972602
10	,001643836	,001917810	,002191780
11	,001808219	,002109591	,002410958
12	,001972602	,002301372	,002630136
13	,002136985	,002493153	,002849314
14	,002301368	,002684934	,003068492
15	,002465751	,002876715	,003287670
16	,002630134	,003068496	,003506848
17	,002794517	,003260277	,003726026
18	,002958890	,003452058	,003945204
19	,003123253	,003643839	,004164382
20	,003287636	,003835620	,004383560
21	,003452019	,004027401	,004602748
22	,003616402	,004219182	,004821926
23	,003780785	,004410963	,005041104
24	,003945168	,004602744	,005260282
25	,004109551	,004794525	,005479460
26	,004273934	,004986306	,005698648
27	,004438317	,005178097	,005917826
28	,004602600	,005369878	,006136004
29	,004766983	,005561659	,006355182
30	,004931366	,005753440	,006575360
31	,005095749	,005945221	,006794538
32	,005260132	,006137002	,007013716
33	,005425515	,006328783	,007122894
34	,005588798	,006520564	,007342072
35	,005753181	,006712345	,007561250
36	,005867564	,006904126	,007780428

Days

Days	At 3 per Cent.	At 4 per Cent.	At 5 per Cent.
37	,003041096	,004054794	,005060482
38	,003123288	,004164382	,005197468
39	,003205479	,004273972	,006337454
40	,003287671	,004383561	,006474440
41	,003369863	,004493150	,006611426
42	,003452055	,004602739	,006748412
43	,003534246	,004712328	,006885398
44	,003616438	,004821917	,007022384
45	,003698629	,004931506	,007159370
46	,003780821	,005041095	,007296356
47	,003863013	,005150684	,007433342
48	,003945205	,005260273	,007573328
49	,004027397	,005369862	,007810314
50	,004109589	,005479451	,007947300
51	,004191780	,005589040	,008084286
52	,004273972	,005698629	,008228272
53	,004356164	,005808218	,008358258
54	,004438355	,005917807	,008495244
55	,004520547	,006027396	,008632230
56	,004602739	,006136985	,008769216
57	,004684931	,006246574	,008906202
58	,004767123	,006356163	,009043188
59	,004843314	,006465752	,009180174
60	,004935406	,006575341	,009316160
61	,005013698	,006684930	,009453146
62	,005095890	,006794519	,009590132
63	,005178082	,006904108	,009727118
64	,005260274	,007013697	,009864104
65	,005342465	,007123286	,010001090
66	,005424657	,007232875	,010138076
67	,005506849	,007342464	,010275062
68	,005589041	,007452053	,010412048
69	,005671233	,007561642	,010549034
70	,005753424	,007671231	,010686020
71	,005835616	,007780820	,010823006

Days

Days	At 6 per Cent.	At 7 per Cent.	At 8 per Cent.
37	,006082191	,007095907	,008109589
38	,006246574	,007287688	,008328767
39	,006410957	,007479468	,008547945
40	,006575340	,007671249	,008767123
41	,006739723	,007863030	,008986301
42	,006904106	,008054811	,009205479
43	,007068489	,008246592	,009424657
44	,007232872	,008438373	,009643835
45	,007397255	,008630154	,009863013
46	,007561638	,008821935	,010082191
47	,007726021	,009013716	,010301369
48	,007890404	,009205497	,010520547
49	,008054777	,009397278	,010739725
50	,008219170	,009589059	,010958903
51	,008383553	,009780840	,011178081
52	,008547946	,009972621	,011397259
53	,008712329	,010164402	,011616437
54	,008876712	,010356183	,011835615
55	,009041095	,010547954	,012054793
56	,009205478	,010739735	,012273971
57	,009369861	,010931526	,012493149
58	,009534244	,011123307	,012712327
59	,009698627	,011315188	,012931505
60	,009863010	,011506969	,013150683
61	,010027393	,011698750	,013369861
62	,010191786	,011890531	,013589039
63	,010356169	,012082312	,013808217
64	,010520552	,012274103	,014027395
65	,010684935	,012465884	,014246573
66	,010849318	,012657625	,014465751
67	,011013701	,012849406	,014684929
68	,011178084	,013041187	,014904107
69	,011342467	,013232968	,015123285
70	,011506850	,013424049	,015342463
71	,011671233	,013615750	,015561641

Days

Days	At 3 per Cent.	At 4 per Cent.	At 5 per Cent.
72	,005917808	,007190409	,010959992
73	,006000000	,007299998	,011096978
74	,006082192	,007409587	,411233964
75	,006164383	,007519176	,012370950
76	,006249575	,007628765	,013507936
77	,006328767	,007738758	,013644922
78	,006410959	,007848343	,013781908
79	,006493151	,007957932	,013918894
80	,006575343	,008067521	,014045880
81	,006657535	,008177110	,014182566
82	,006739727	,008286699	,015119552
83	,006821919	,008396288	,015256538
84	,006904111	,008505877	,015385524
85	,006986313	,008615466	,015522510
86	,007068495	,008725055	,015669496
87	,207150687	,008835014	,015806482
88	,007232879	,008944603	,015943468
89	,007315071	,009054192	,016080454
90	,007397263	,009163781	,016217440
91	,007479455	,009273380	,016354428
—	---Months---	---Months---	---Months---
3	,007590000	,010000000	,012500000
6	,015000000	,020000000	,025000000
9	,022500000	,030000000	,037500000
—	-Years, &c.-	-Years, &c.-	-Years, &c.-
1	,030000000	,040000000	,050000000
1 $\frac{1}{4}$	,037500000	,050000000	,062500000
1 $\frac{1}{2}$	,045000000	,060000000	,075000000
1 $\frac{3}{4}$	,052500000	,070000000	,087500000
2	,060000000	,080000000	,100000000
2 $\frac{1}{4}$	,067500000	,090000000	,112500000
2 $\frac{1}{2}$	,075000000	,100000000	,125000000
2 $\frac{3}{4}$	,082500000	,110000000	,137500000
3	,090000000	,120000000	,150000000
3 $\frac{1}{4}$	,097500000	,130000000	,162500000
3 $\frac{1}{2}$	,105000000	,140000000	,173000000
3 $\frac{3}{4}$	,112500000	,150000000	,187500000

Days

Days	At 6 per Cent.	At 7 per Cent.	At 8 per Cent.
72	,011835616	,013807531	,015780819
73	,011999999	,014009312	,015999997
74	,012164381	,014192093	,015219175
75	,012328765	,014383874	,015438353
76	,012493148	,014575655	,015657531
77	,012657531	,014767436	,015876709
78	,012821914	,014959217	,016095987
79	,012986297	,015150998	,016315165
80	,013150680	,015342779	,016614343
81	,013315063	,015534560	,016833521
82	,013479446	,015726541	,017052699
83	,013643831	,015918522	,017271877
84	,013808214	,016100303	,017491055
85	,013972597	,016292084	,017710233
86	,014136990	,016483865	,017929411
87	,014301363	,016675646	,020148589
88	,014465746	,016867327	,020367767
89	,014630129	,017059208	,020586945
90	,014794512	,017250989	,020806123
91	,019550895	,017442770	,021025301
—	—Months—	—Months—	—Months—
3	,015000000	,017500000	,020000000
6	,030000000	,035000000	,040000000
9	,045000000	,052500000	,060000000
—	—Years, &c.—	—Years, &c.—	—Years, &c.—
1	,060000000	,070000000	,080000000
1 $\frac{1}{4}$	,075000000	,087500000	,100000000
1 $\frac{1}{2}$	,090000000	,105000000	,120000000
1 $\frac{3}{4}$	,105000000	,122500000	,140000000
2	,120000000	,140000000	,160000000
2 $\frac{1}{4}$	,135000000	,157500000	,180000000
2 $\frac{1}{2}$	,150000000	,175000000	,200000000
2 $\frac{3}{4}$	,165000000	,192500000	,220000000
3	,180000000	,210000000	,240000000
3 $\frac{1}{4}$	,195000000	,227500000	,260000000
3 $\frac{1}{2}$	,100000000	,245000000	,280000000
3 $\frac{3}{4}$	,225000000	,262500000	,300000000

Expli-

Explication.

The first Column of every Page being Time, against it stands the Interest of one Pound for that Time, at the Rate *per Cent.* specified on the Top of the Column, and at the same View you have the Interest of 10 *l.* 100 *l.* 1000 *l.* and all in one Line. Let the Interest of 500 *l.* for 73 Days at 5 *per Cent.* be required; turn to 73 Days, and against that, under the Rate of 5 *per Cent.* it appears that

<i>l.</i>			<i>l.</i>	<i>s.</i>	<i>d.</i>
1 is	.010	or			2 . $\frac{1}{4}$
10 is	.10	or		02 . 0 . 0	
100 is	1.00	or	1 . 00 . 0 . 0		
	1 . 11	or	1 . 02 . 2 . $\frac{1}{4}$		

So that one Pound after that manner being found to be the Interest of an 100 *l.* at 5 *per Cent. per Annum* for 73 Days, 5 times that Money, to wit, 5 *l.* must therefore be the Interest of 500 *l.* Again, if the Interest of 111 *l.* at the same Rate and for the same Time were required; take the Total of the above Sums for an answer, being 1 . 11, or 1 *l.* 2 *s.* 2 *d.* $\frac{1}{4}$. So that let the given Principal be what Number of Pounds, Shillings, and Pence it will, it may be either taken in Parts, as 111 *l.* above, or by multiplying the Interest of one Pound by the given Principal; so that no Sum at any of the inserted Rates or Time can require more than one Multiplication. And although this is calculated only for three Years $\frac{3}{4}$, beyond which time Simple Interest seldom happens, yet the Interest from thence for any longer Time may as easily and readily be found, *viz.*

Example 1. Unto what amounts the Interest of 79 *l.* 15 *s.* for 6 Years $\frac{1}{2}$ and 73 Days, at 6 *per Cent. per Annum*?

Years

.180	}	the Interest of	}	3	3 $\frac{1}{2}$	73 Days.
.210						
.012						

.402 the Interest of one Pound at the given Rate and Time, which multiplied by the Principal, produces the Answer.

$$\begin{array}{r}
 79 \cdot 75 \\
 .402 \\
 \hline
 15950 \\
 31900 \\
 \hline
 32,05950 \quad \text{Answ. } 36 \text{ l. } 01 \text{ s. } 2 \text{ d. } +.
 \end{array}$$

Likewise may the Interest of any Sum at any other Rate *per Cent.* whatsoever, be readily found according to the Directions in the last Head, in the Rule of Interest for Days.

Example 2. What comes the Interest of 432 l. 10 s. for 1 Year $\frac{1}{4}$ at 4 l. $\frac{1}{2}$ *per Cent. per Annum*?

First take the Interest of one Pound for the given Time at 5 *per Cent. per Annum*, which is $\frac{1}{20}$ of the Pound, or .0625
 From which subtracting one Tenth ——— 625

gives the Interest of one Pound at 4 $\frac{1}{2}$ *per Cent.* ——— 5625
 Multiply'd by the given Principal ——— 432,5

$$\begin{array}{r}
 28125 \\
 11250 \\
 16875 \\
 22500 \\
 \hline
 \end{array}$$

Answ. 24 l. 06 s. 06 d. $\frac{3}{4}$

24,328125

SECTION VIII.

OF EXCHANGE.

1. **I**N London, and throughout England, Accounts are kept in Pounds, Shillings, and Pence *Sterling*, and are cast up as *Flemish* and *French* Money are, by 12 and 20, and the Exchange made with all places by Pence, *i. e.* giving so many Pence *Sterling* for the Pieces on which the Exchange is made, except

1. *Portugal*, and with it in Shillings and Pence, on their *Milrea*.

2. *Ant-*

2. *Antwerp, Hamburgh, &c.* Countries bordering upon *Flanders* and *Holland*, and with them by the Pound of 20 s. *Sterling*.

3. *Ireland*, and with it by the Hundred Pound.

II. The principal Places where the Course of Exchange is settled, and in what Denomination they exchange with *London*, you may find in the 3d Chapter of Mr. *Kersey's* Appendix; but the Places to which there is a more constant Course of Exchange from *London*, are

In *Italy*,

Genoa, Leghorn, Venice, &c.

Genoa and *Leghorn* exchange with *London* by the Dollar, or Piece of Eight; *Venice* by the Ducat.

Genoa and *Leghorn* keep their Accounts in Livres, Solz, and Deniers, 5 Livres a Piece of Eight at *Genoa*, and 6 at *Leghorn*.

The Par of the Dollar, or Piece of Eight, with *London*, is 54 d. *Sterling*.

In *Spain*,

Madrid, Cadiz, Bilboa, &c.

Exchange also by the Piece of Eight.

The Par being the same as *Genoa* and *Leghorn*.

In *Portugal*,

Lisbon and *Oporto, &c.*

Exchange on the *Milrea*.

The Par about 6 s. 8 d. $\frac{1}{2}$ *Sterling*.

In *France*,

Paris, Lyons, and Rouen.

Exchange by the Crown.

But keep their Accounts in Livres, Solz, and Deniers.

12 Deniers	} make	{	1 Sol.
12 Solz			1 Livre.
3 Livres			1 Crown.

The Par of the *French* Crown with *London* is 4 s. 6 d. *Sterling*.

In the *Netherlands*,

Antwerp, Brussels, Amsterdam, Rotterdam, Hamburgh, &c.

Exchange with *London* by Shillings and Pence *Flemish*. Accounts are in these Places kept in *Flemish* Pounds, Shillings, and Pence; by some in Guilders, Stivers, and Pennicks. The *Flemish* Pounds, Shillings, and Pence, are divided as our Money, viz. one Pound into 20 s. and one Shilling into 12 d. But,

H h h 2

6 Pen-

16 Pennicks,	} one {	Stiver	} the Par with Lon.	0 s. 1 d. $\frac{1}{2}$
6 Stivers,		Flemish Shilling		0 : 7 : $\frac{1}{2}$
20 Stivers,		Guilder		2 : 0 : 0
6 Guilders,		Flemish Pound		12 : 0 : 0
30 Stivers,		Common Dol.		3 : 0 : 0
50 Stivers,		Specie Dollar		5 : 0 : 0
63 Stivers,		Ducatoon		6 : 3 : $\frac{1}{2}$

The Par between the *Flemish* Pound and Pound Sterling, is 33 s. 4 d. But the Course of Exchange may, and does vary some Shillings.

In the Arithmetical Part, or casting up of Exchanges, I shall only instance in those with *France* and *Flanders*; for as *London* exchanges with *France* by the Pence, so it does with all Places in *Spain* and *Italy*; and as it exchanges with *Flanders*, at *Brussels* and *Antwerp*, so it does with *Germany* and *Holland*, at *Hamburg* and *Amsterdam*, &c. by the Pound Sterling, for their *Flemish* Shillings and Pence.

French Exchange.

The most common Ways of casting these up are two, viz.

Rules. Either multiply the given Crowns by the Pence of Exchange, taking Parts for the Farthings, (if any be;) otherwise, Take the Aliquot Parts for the Pence of the Exchange out of a Pound, dividing the given Crowns by those Parts, the Total of which is the Answer.

Example. How much Sterling must I pay here to receive in *France* 479 Crowns, Exchange at 52 d. Sterling per Crown?

$$\begin{array}{r}
 4 \text{ s. } \frac{1}{2} \text{ 479 Crowns} \qquad 479 \\
 4 \text{ } \frac{1}{2} \text{ 95 : 16} \qquad \qquad \qquad 52 \\
 \hline
 \qquad \qquad \qquad 7 : 19 : 8 \qquad \qquad \qquad 958 \\
 \hline
 \qquad \qquad \qquad 103 : 15 : 8 \qquad \qquad \qquad 2395 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad \qquad 12) \ 23908 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad \qquad 210 \ 20715 : 8 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad \qquad 103 : 15 : 8
 \end{array}$$

So that all Exchanges made with *London* upon the Ducat, Dollar, Ducatoon, Florin, Milrea, &c. are worked after the same manner, and are no more than so many several Questions in

in the Rule of Three, or of Practice ; for if the above Crowns were Ells, &c. you must pay as much for them at such a Price, as Sterling, for so many Crowns at such Exchange. If the Exchange is at so many Pence and a Fraction of a Penny, see my second Example of the 13th Rule in Practice.

Of Flemish Exchange.

First, To reduce Sterling into Flemish.

Rule. Take the Aliquot Parts for what the Rate of the Exchange is above a Pound, dividing the given Sterling by the said Parts, the Total of which, with the given Sterling, is the Flemish Money.

Example 1. One in Antwerp delivering Money by Exchange for London at 35 s. 6 d. Flemish per Pound Sterling, How much must he pay there to receive here 597 l. Sterling.

$$\begin{array}{r}
 10 \text{ s. } \frac{1}{2} \quad 597 \\
 5 \quad \frac{1}{2} \quad 298 : 10 \\
 6 \quad \frac{1}{5} \quad 149 \cdot 05 \\
 \quad \quad 14 : 18 : 6 \\
 \hline
 1059 : 13 : 6
 \end{array}$$

Answer l. 1059 : 13 : 6 Flemish.

Example 2. A Merchant at Rotterdam has a Bill drawn on him for the Value of 673 l. 16 s. 8 d. Sterling, Exchange at 33 s. 4 d. Flemish per Pound Sterling, How much must he pay there?

$$\begin{array}{r}
 10 \text{ s. } \frac{1}{2} \quad 673 : 16 : 8 \\
 3 : 4 \quad \frac{1}{8} \quad 336 : 18 : 4 \\
 \quad \quad 112 : 06 : 1 \frac{1}{4} \\
 1123 : 01 : 1 \frac{1}{4} \quad \text{Answ. 1123 l. 1 s. 1 d. } \frac{1}{4} \text{ Flemish.}
 \end{array}$$

Secondly, To reduce Sterling into Guilders.

Rule. Reduce the Sterling to Flemish, by the foregoing Rule, which Flemish Money multiply by 6.

Example. If 397 l. 15 s. Sterling, Exchange at 34 s. 8 d. Flemish per Pound Sterling, were remitted to Amsterdam, How many Guilders, Stivers, &c. may be received at the said Rate of Exchange?

$$\begin{array}{r}
 10 \text{ s. } \frac{1}{2} \quad 397 : 15 \\
 4 \quad \frac{1}{3} \quad 198 : 17 : 06 \\
 1 \quad \frac{1}{8} \quad 79 : 11 : 00 \\
 \hline
 13 : 05 : 02
 \end{array}$$

$$\begin{array}{r}
 689 : 08 : 08 \\
 \hline
 6
 \end{array}$$

Guil. Stiv.

Answ. 4136 : 12

4136 : 12 : 00

Thirdly, To reduce Flemish to Sterling.

Rule. Reduce the given Flemish Sum, either into Pence, two Pences, three Pences, Groats, or Six-pences, &c. which make a Dividend; then reduce the Rate of Exchange to the same, which take for a Divisor, and the Quote of that Division will be the Answer in Pounds Sterling, the Remainder being valued according to Mr. Wingate's 5th Rule of the 7th Chapter.

Example 1. A Merchant at Bruffels delivers 579 l. 10 s. Flemish, Exchange at 35 s. 6 d. to receive at London one Pound Sterling, How much Sterling must he receive?

$$\begin{array}{r}
 35 : 6 \quad 579 \text{ l. } 10 \text{ s.} \\
 \hline
 2 \quad 40 \\
 \hline
 71 : 0 + \quad 71) 23180 (326 : 09 : 06 + \\
 \hline
 80 \\
 \hline
 460
 \end{array}$$

Answ. 326 : 09 : 06

34

Example 2. How much Sterling must be here paid to receive in Hamburgh 836 l. 8 s. 4 d. Flemish, Exchange at 36 s. 8 d. Flemish per Pound Sterling.

$$\begin{array}{r}
 36 : 8 \quad 836 \text{ l. } 8 \text{ s. } 4 \text{ d.} \\
 \hline
 6 \quad 60 \\
 \hline
 1110 \quad 5018 \frac{1}{2} \text{ Groats}
 \end{array}$$

Answ. 456 l. 4 s. 6 d. $\frac{6}{11}$

Fourthly, To reduce Guilders, Stivers and Pennicks into Sterling.

Rule. Reduce the said Guilders, Stivers, and Pennicks into Pence, Two-Pences, or Groats, which divide by the given Rate of Exchange (when reduced to the same Name,) and the Quotient will be Pounds Sterling.

Note. That 40 Flemish Pence make one Guilder.

E. 28

Example 1. A Merchant at *Rotterdam* delivering 1489 *Guild.* 10 *Stiv.* 8 *Pen.* for *London*, Exchange at 35 s. 7 d. I demand how much Sterling he may draw the Bill for ?

$$\begin{array}{r}
 1489 : 10 : 8 \\
 \underline{40} \\
 427) 59581 \quad (139 : 10 : 8 + \\
 \underline{1688} \\
 4071 \\
 \underline{228} \quad \text{Answ. } 139 : 10 : 8
 \end{array}$$

Example 2. How much Sterling is equivalent to 4372 : 10, Exchange 36 s. 4 d. *Flemish* per Pound Sterling ?

$$\begin{array}{r}
 3 \\
 \hline
 109 \text{ Groats} \qquad 109) 43725 \quad (401 : 2 : 11 + \\
 \qquad \qquad \qquad \underline{125} \\
 \qquad \qquad \qquad 16
 \end{array}$$

Or more short.

When the Pence of the Exchange are so many, as that 40 (which are the *Flemish* Pence in one *Guilder*) is an Aliquot Part thereof.

Rule. Divide the *Guilders*, *Stivers*, &c. by that Aliquot Part, and the Quote is the Sterling-Money in Pounds, Shillings and Pence; but to reduce Sterling into *Guilders*, *Stivers*, &c. multiply by the aforesaid Part.

Example 1. How much Sterling, at 36 s. 8 d. *Flemish* per Pound Sterling, must be paid for 1587 : 10 *Stivers* ?

$$\begin{array}{r}
 11) 1587 : 10 \\
 \hline
 144 : 06 : 04 : \frac{4}{11} \quad \text{Answ. } 144 : 06 : 04 \frac{4}{11}
 \end{array}$$

Example 2. Exchange at 33 s. 4 d. *Flemish* per Pound Sterling, How many *Guilders*, *Stivers*, &c. may I draw for 179 l. 17 s. 10 d. Sterling ?

$$\begin{array}{r}
 179 : 17 : 10 \\
 \text{Guild. Stiv. Penn.} \quad \underline{10} \\
 \text{Answ. } 1798 : 18 : 05 \quad 1798 : 18 : 05
 \end{array}$$

Here

Here note, That when the Rate of Exchange is at Par, then the Proportion between their Money and Ours, is the same as between their Cloth-Measure and Ours; for as five of their Ells make three of Ours, so, at such Exchange, five of their Flemish Pounds make three Pounds Sterling, or 500 l. Flemish, 300 l. Sterling.

Of Gain or Loss by Exchange.

Note. That when Exchange is made with Italy, France, Spain, &c. with which, as aforesaid, London exchanges for so many Pence for their Pieces, the Gain thereby is so much the more, by how much the Course of Exchange runs low; because it is evident, that I can receive more Ducats for 500 l. when the Exchange is at 53 d. than at 56 per Piece.

But when I remit for Flanders, or Holland, with which London exchanges on the Pound Sterling, the higher the Exchange is above Par, the more we have the Advantage of Gain; for according to reason, I may receive more Flemish Money for 1000 l. Sterling, when the Exchange is at 36 s. 8 d. than at 35 s. 4 d. for at 35 s. 4 d. I gain but 2 s. per Pound; but at 36 s. 8 d. Flemish, I gain by Exchange 3 s. 4 d. for every Pound Sterling I remit.

SECTION IX.

*Decimal Tables for FLEMISH Exchanges,
at the most usual Rates of Exchange.*

Rate of Exchan.	The Value of Pounds Ster- ling in Pounds <i>Flemish</i> .	The Value of Pounds <i>Fle- mish</i> in Pounds Sterling.
<u>s. d.</u>		
32—	I .6	.625
I	I .6041666	.6233766
2	I .6083333	.6217616
3	I .6125 . . .	.6201550
4	I .6166666	.6185567
5	I .6208333	.6169665
6	I .625	.6153846
7	I .6291666	.6138107
8	I .6333333	.6122449
9	I .6375 . . .	.6106870
10	I .6416666	.6091370
<u>s. d.</u>		
33—	I .6458333	.6075949
I	I .65	.6060606
1	I .6541666	.6045343
2	I .6583333	.6030150
3	I .6625 . . .	.6015037
4	I .6666666	.6
5	I .6708333	.5985037
6	I .675	.5970149
7	I .6791666	.5955335
8	I .6833333	.5940594
9	I .6875 . . .	.5925925
10	I .6916666	.5911330
11	I .6958333	.5896806
34—	I .7	.5882352
I	I .7041666	.5867970
2	I .7083333	.5853658
3	I .7125 . . .	.5839416
4	I .7166666	.5825242

I i i

Rate

Rate of Exchan.	The Value of Pounds Ster- ling in Guild- ers.	The Value of Guilders in Pounds Ster- ling.
<u>s.</u> <u>d.</u>		
32—	9 .6	.10416666
1	9 .625	.10389610
2	9 .65	.10362694
3	9 .675	.10235916
4	9 .7	.10309278
5	9 .725	.10283775
6	9 .75	.10256410
7	9 .775	.10230178
8	9 .8	.10204081
9	9 .825	.10178116
10	9 .83	.10152283
11	9 .875	.10126581
33—	9 .9	.10101010
1	9 .925	.10075571
2	9 .95	.10050250
3	9 .975	.10025061
4	10.0	.1
5	10.025	.09975061
6	10.05	.09950248
7	10.075	.09925558
8	10.1	.09900990
9	10.125	.09876541
10	10.150	.09852216
11	10.175	.09828011
34—	10.2	.09803921
1	10.225	.09779951
2	10.25	.09756006
3	10.275	.09732360
4	10.3	.09708736

Rate

Rate of Exchan.		The Value of Pounds Ster- ling in Pounds <i>Flemish</i> .	The Value of Pounds <i>Fle- mish</i> in Pounds Sterling.
<u>s.</u>	<u>d.</u>		
34	5	I .7208333	.5811138
	6	I .725....	.5797101
	7	I .7291666	.5783132
	8	I .7333333	.5769230
	9	I .7375...	.5755395
	10	I .7416666	.5741621
	11	I .7458333	.5727923
35	—	I .75.....	.5714285
	I	I .7541666	.5700712
	2	I .7583333	.5687309
	3	I .7625...	.5673758
	4	I .7666666	.5660377
	5	I .7708333	.5647058
	6	I .775....	.5633802
	7	I .7791666	.5620609
	8	I .7833333	.5607476
	9	I .7875...	.5594405
	10	I .7916666	.5581395
	11	I .7958333	.5568445
36	—	I .8.....	.5555555
	I	I .8041666	.5542725
	2	I .8083333	.5529954
	3	I .8125...	.5517241
	4	I .8166666	.5504587
	5	I .8208333	.5491990
	6	I .825....	.5479452
	7	I .8291666	.5466970
	8	I .8333333	.5454545
	9	I .8375...	.5442176
	10	I .8416666	.5429864
	11	I .8458333	.5417607

Rate of Exchan.		The Value of Pounds Sterling in Guilders.	The Value of Guilders in Pounds Sterl.
s.	d.		
34	5	10.325	.09685231
	6	10.35	.09661835
	7	10.375	.09638553
	8	10.4	.09615383
	9	10.425	.09592325
	10	10.45	.09569368
	11	10.475	.09546538
35	—	10.5	.09523808
	1	10.525	.09501186
	2	10.55	.09478681
	3	10.575	.09456263
	4	10.6	.09433961
	5	10.625	.09411763
	6	10.65	.09389671
	7	10.675	.09367681
	8	10.7	.09345793
	9	10.725	.09324008
	10	10.75	.09302325
	11	10.775	.09280741
36	—	10.8	.09259259
	1	10.825	.09237875
	2	10.85	.09216590
	3	10.875	.09195402
	4	10.9	.09174311
	5	10.925	.09153316
	6	10.95	.09132420
	7	10.975	.09111616
	8	11.0	.09090909
	9	11.025	.09070293
	10	11.05	.09049773
	11	11.075	.09029345

Rate

Rate of Exchan.		The Value of Pounds Sterl. in Pounds <i>Flemish</i> .	The Value of Pounds <i>Fl.</i> in Pounds Sterl.
<u>s.</u>	<u>d.</u>		
37	—	I .85	.5405405
	I	I .8541666	.5393258
	2	I .8583333	.5381166
	3	I .8625 . . .	.5369127
	4	I .8666666	.5357143
	5	I .8708333	.5345211
	6	I .875	.5333333
	7	I .8791666	.5321508
	8	I .8833333	.5309734
	9	I .8875 . . .	.5298013
	10	I .8916666	.5286343
	11	I .8958333	.5274725
38	—	I .9	.5263157
	I	I .9041666	.5251641
	2	I .9083333	.5240174
	3	I .9125 . . .	.5228758
	4	I .9166666	.5217391

Rate

Rate of Exchan.	The Value of Pounds Sterling in Guilders.	The Value of Guilders in Pounds Sterl.
<u>s.</u> <u>d.</u>		
37 —	11 . 1	.09009009
1	11 . 125 . .	.08988763
2	11 . 15 . . .	.08968610
3	11 . 175 . .	.08948545
4	11 . 2	.08928571
5	11 . 225 . .	.08908685
6	11 . 25 . . .	.08888888
7	11 . 275 . .	.08869180
8	11 . 3	.08849556
9	11 . 325 . .	.08830021
10	11 . 35 . . .	.08810571
11	11 . 375 . .	.08791208
38 —	11 . 4	.08771928
1	11 . 425 . .	.08752735
2	11 . 45 . . .	.08733624
3	11 . 475 . .	.08714596
4	11 . 5	.08695651

The Explication.

The first Column shews the Rate of the Exchange from 32 s. to 38 s. 4 d. and is extended no farther; for a Regulation being made of the Coin, it is not probable it will rise higher, or fall lower than those Limitations.

2. The second Column on the Left-hand, shews the Value of Pounds Sterling, express'd in Pounds *Flemish*, whereby, at the first View, you discern what is the Amount of 1, 10, 100, 1000, &c. to ten Millions of Sterling, in Pounds *Flemish*, at any of the insert'd Rates of Exchange: For Example, if the Exchange is at 35 s. 2 d. one Pound Sterling, being equivalent to one Pound fifteen Shillings and Two-pence at one view; therefore it appears that

<i>l.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	
10 <i>l.</i> Sterling makes	17	11	8	} <i>Flem.</i>
100	175	16	8	
1000	1758	6	8 &c.	

All which *Flemish* Monies are found in the Line against the said given Rate of Exchange: But if the Sterling-Money, for which the Value in Pounds *Flemish* is required, consists of odd Pounds, or hath Shillings, &c. annexed; then multiply the *Flemish* Money, at the given Exchange, by the Sum in Sterling, and the Product answers the Question. Thus 67 *l.* 10 *s.* Sterling, multiplied by 1,75833, the *Flemish* Money at 35 : 2, makes 118 *l.* 13 *s.* 9 *d.* *Flemish*.

3. The third Column on the Left-hand Page shews the Value or Amount of Pounds *Flemish* in Sterling, after the same manner as the second shews the Value or Amount of Pounds Sterling in *Flemish* Money :

For Instance.

At the aforesaid Exchange of 35 *s.* 2 *d.* I see that one Pound *Flemish* being equivalent to .5687209, or *l.* 11 *s.* 4 *d.* $\frac{1}{2}$ } Therefore 10 *Flem.* makes 5.687209, or 5 : 13 : 9 : } Sterling
 100 *l.* 56.87209, or 56 : 17 : 6 : } very
 and 1000 568.7209, or 568 : 14 : 5 : } near

But if your *Flemish* Money, the Value of which is required in Sterling, consists of odd Pounds, or hath Shillings, &c. annex'd, multiply the same by the Sterling, as before directed, and the Product of such Multiplication answers the Question ; so the Exchange being at 35 *s.* 2 *d.* 137 *l.* 10 *s.*

Flemish, multiplied by .5687209 Sterling, produces 78 *l.* 19912375, or 78 *l.* 4 *s.* + Sterling.

4. The 5th Column, or the second on the Right-hand Page, shews the Value or Amount of Pounds Sterling in Guilders, or what Number of Guilders I ought to receive, or pay, for any Sum of Sterling, at any of the inserted Rates of Exchange ;

For Example.

At the aforesaid Exchange of 35 *s.* 2 *d.* it appears that 10 Guilders, 10 Stivers, and 8 Pennicks, are equivalent to one Pound Sterling ; as likewise, that

<i>Guild.</i>	<i>Stiv.</i>		<i>l.</i>
105 ; 10		are to	10
1055		to	100
10550		to	1000

} Sterling.

If the Sterling-Money is in odd Pounds, or hath Shillings, &c. annexed, you are to multiply the Guilders corresponding to the Rate of Exchange by your Sterling-Sum, as above directed ; so 132 *l.* 10 *s.* Sterling, Exchange at 35 *s.* 2 *d.* will be found to make 1397 Guilders, 17 Stivers, 8 Pennicks.

V. The

V. The last Column shews the Value of Guilders in Pounds Sterling, or how much Sterling I ought to receive, or pay, for any Number of Guilders, at any of the specify'd Rates of Exchange.

So a the aforesaid Rate of Exchange, viz. 35 s. 2 d. one Guilder being worth .09478681, or 1 s. 10 d. $\frac{1}{4}$ Sterling ;

Therefore,

		l.	s.	d.	
10 make	.9478681, or		18	11	$\frac{1}{2}$ Sterling
100	9.478681, or		9	09	$\frac{1}{4}$ } very
1000	94.78681, or		94	15	$\frac{1}{4}$ } near

If your Guilders be in odd Numbers, or have Stivers annex'd, multiply them also by the Value in Sterling corresponding to the Rate of Exchange, and the Product will answer your Demand.

Guild. Stiv. Penn.

So 1327 : 17 : 8, Exchange at 35 s. 2 d. being multiply'd by .09478681, amounts to 132 l. 10 s. Sterling, as above said.

Note. The Reason of the Tables being calculated so high, is, That you may be more exact in long Sums, tho' you'll seldom have occasion to make use of more than five or six Figures. And the Dots placed to the Right-hand are inserted for the more easy and safe Observation of large even Sums. But if the casting up of odd Sums should be thought as troublesome to be effected by these Tables, as by the common Method of doing them, (tho' in all cases performed by one single Multiplication) yet even Sums most commonly happening in Drawing and Remitting by Exchange, the Usefulness of these Tables will be sufficiently manifest.

SECTION X. Of MEASURING.

THE various Kinds of which, are Three, viz.

I. *Lincol.* called (by Workmen *Running Measure*, in that it respects *Length* only, the Parts of which are

12 Inches,	}	{	1 Foot,
3 Feet,			1 Yard,
16 $\frac{1}{2}$ Feet,			1 Rod.

II. *Su-*

II. *Superficial, or Square Measure*, in that it respects Length and Breadth, the Parts are, viz.

144 Inches,	}	1 Foot,
72 Inches,		$\frac{1}{2}$ a Foot,
36 Inches,		$\frac{1}{4}$ of a Foot,
48 Inches,		$\frac{1}{3}$ of a Foot,
18 Inches,		$\frac{1}{2}$ a quarter of a Foot,
272 $\frac{1}{4}$ Feet,	}	1 Rod,
136 $\frac{1}{8}$ Feet,		$\frac{1}{2}$ a Rod.
Inches	}	
1226, or 9 Feet,		1 Yard.

Note. That in dividing by 272 $\frac{1}{4}$, which reduces *Superficial Feet* to *Square Rods*, Workmen take no notice of the $\frac{1}{4}$ of the Foot, dividing only by 272, which gives the Content a little too much.

III. *Solid, or Cube Measure*, respecting Length, Breadth, and Thickness; whose Parts are,

Inches		
1728,	}	1 Foot,
1296,		$\frac{3}{4}$ of a Foot,
864,		$\frac{1}{2}$ of a Foot,
432, &c.		$\frac{1}{4}$ of a Foot,
27 Feet,		1 Yard.

Of Superficial Measure.

A General Rule.

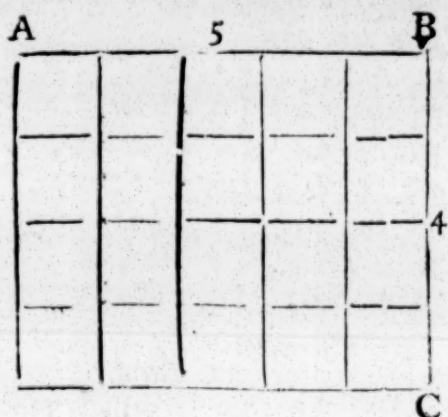
Multiply the Length by the Breadth, and the Product is the Superficial Content.

Demonstration.

For if a Line, A B, being divided into any Number of equal Parts, be moved constantly at right Angles with another Line, B C, divided into any other Number of such equal Parts, as are in A B; it will, when advanced to any Part of B C, describe thereby as many equal Squares as there are Parts in it: And if moved on farther, to the next Part, it will describe as many more Squares, and so the same Number of Squares will be as often described, as there are Parts in the Line, B C; and consequently there must be as many Squares in the Surface, made by these Lines, as there are Units in the Product of the Parts of the two Sides.

K k k

Now



Now the Square of any thing is found three several Ways, viz.

1. By Decimals,
2. By Aliquot Parts,
3. By cross Multiplication.

All which I shall promiscuously use in the following Examples. But the latter of the Three is used by most Workmen, except Glaziers, whose Foot being decimally divided, they cast up the Content of their Work by Decimals.

How to multiply by the two last, see my 4th Head in Multiplication.

Note. The Content of the Work, is given either by the Foot, the Yard, the Square, or the Rod, viz.

I. By the Foot.

Glazing and Masonry.

Dimensions are taken in Feet and Inches, the Content given per Foot square.

In Glazing.

Example. How many Feet of Glazing are there in that Pane of Glass which is 4 Feet 5, and 2 Feet 42.

$$\begin{array}{r}
 3,42 \\
 4,5 \\
 \hline
 1210 \\
 968 \\
 \hline
 10,890
 \end{array}$$

Answer, 10 f. 890

In Masonry.

Example. Suppose a Yard pav'd with Free-stone, viz. Length 22 Feet 4 Inches, Breadth 19 Feet 7 Inches, How many Square Feet?

$$\begin{array}{r}
 22 . 4 \\
 19 . 7 \\
 \hline
 424 . 4 \\
 7 . 5 . 4 \\
 5 . 7 \\
 \hline
 \end{array}$$

Feet In. Part.

$$437 . 4 . 4 \text{ Facit. } 437 . 4 . 4$$

But if only the Breadth was given, and it were required how much in Length will make a Foot Square, see Mr. Kersey's 51st Question of his 10th Chapter.

II. By the Yard.

Painting, Joynery, Plastering, &c.

Dimensions are taken in Feet and Inches; but the Content given in Square Yards, found by dividing the Superficial Feet by 9.

In Painting.

Feet Inch.

Example. The Height of a Room being 12 . 4, and 84 Feet 11 Inches about, How many Square Yards?

$$\begin{array}{r}
 84 . 11 \text{ about} \\
 12 . 4 \text{ high} \\
 \hline
 1019 . 0 \\
 21 . 3 . 8 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 9) 1047 . 3 . 8 \quad \text{Ya. Feet. Inch. Part.} \\
 116 : 3 . 3 . 8 \text{ Answ. } 116 . 3 . 3 . 8
 \end{array}$$

Joynery.

Example. How many Yards of Wainscot does that Room take up, whose Height is 12 Feet 3 Inches, and Compass 104 Feet 6 Inches?

Feet Inch.

$$\begin{array}{r}
 104 . 6 \\
 12 . 3 \\
 \hline
 1241 . 0 \\
 06 . \\
 26 . 1 \\
 \hline
 \end{array}$$

$$9) 1280 . 1 . 6$$

Yar. Feet. Inch. Part.

$$\begin{array}{r}
 142 . 2 . 1 . 6 \text{ Answ. } 142 . 2 . 1 . 6 \\
 \text{K k k z} \quad \text{Plastering.}
 \end{array}$$

Plastering.

Example. How many Yards in a Cieling 47 Feet 4 Inches, 7 Parts long, and 18 Feet broad?

$$\begin{array}{r}
 47 \cdot 4 \cdot 7 \text{ long} \\
 18 \text{ broad} \\
 \hline
 284 \cdot 3 \cdot 6 \\
 3 \\
 9) 852 \cdot 10 \cdot 6 \\
 \hline
 94 \cdot 6 \cdot 10 \cdot 6 \quad \text{Yar. Feet. Inch. Part.} \\
 \text{Facit } 94 \cdot 6 \cdot 10 \cdot 6
 \end{array}$$

Under this Head you may further note, that if you multiply the $\frac{1}{2}$ Part of the Length, by the $\frac{1}{2}$ of the Breadth, the Product will be the Content in *Square Yards*, it being the same to take $\frac{1}{2}$ of Lineal, as $\frac{1}{2}$ of Superficial Measure.

III. By the Square.

Partitioning, Flooring, Roofing, Tiling.

Dimensions are taken in *Feet*, and *Inches*, but the Content or Value is by the *Square* of 10 *Feet*, done by cutting off two *Figures* of the *Superficial Feet* towards the Right-Hand.

Partitioning.

Example. How many Squares are contained in that Work which is 199 Feet 10 Inches in length, and 10 Feet 7 Inches high?

$$\begin{array}{r}
 199 \cdot 10 \\
 10 \cdot 7 \\
 \hline
 1998 \cdot 4 \\
 66 \cdot 7 \cdot 4 \\
 49 \cdot 11 \cdot 6 \\
 \hline
 21,14 \cdot 10 \cdot 10 \text{ Squares Feet. Inch. Parts.} \\
 \text{Answ. } 21 \cdot 14 \cdot 10 \cdot 10
 \end{array}$$

From which sometimes the Content of Doors and other Vacancies are deducted.

Flooring.

Example. A Floor being 49 Feet, 7 Inches, 4 Parts long, 26 Feet 6 Inches broad, How many Squares?

$$\begin{array}{r}
 49 \cdot 7 \cdot 4 \\
 26 \cdot 6 \cdot 0 \\
 \hline
 1289 \cdot 10 \cdot 8 \\
 24 \cdot 9 \cdot 8 \\
 \hline
 \end{array}$$

Answ. 13,14 . 8 . 4

Roofing.

Example. Suppose an House 18 Feet 4 Inches in Front, and 37 Feet 10 Inches in Depth, How many Squares in the Roof?

$$\begin{array}{r}
 37 \cdot 10 \\
 3 \\
 \hline
 113 \cdot 6 \\
 6 \\
 \hline
 \frac{1}{2}) 681 \cdot 0 \\
 12 \cdot 7 \cdot 4 \\
 \hline
 \frac{1}{2}) 693 \cdot 7 \cdot 4 \\
 346 \cdot 9 \cdot 8 \text{ added} \\
 \hline
 \end{array}$$

Squares. Feet Inch.

$$10 \cdot 40 \cdot 5 \quad 10,40 \cdot 5 \cdot 0$$

Here note, after you have multiply'd the Depth by the Front, and so found the Content of the Ground-plot in Feet, half of those being added, gives the Content of a Pitch-Roof, which reduce into Squares.

IV. By the Rod, as in

Brick-Work.

Dimensions are taken in Feet and Inches, and the Content, given per the Square of a Rod of 16 $\frac{1}{2}$ Foot.

The Content of this Work is found in the same manner as all the other, to wit, by multiplying the Length by the Breadth, and only differs from the former in two things.

1. When you have so found the Superficial Content in Feet, divide it by 272, the Square Feet in one Rod, the Quote is Rods, the Content being always so given.

2. That when 'tis more or less than a Brick and $\frac{1}{2}$ thick; it must be reduced to that Thickness, being always measured by the Square Rod of a Brick and $\frac{1}{2}$. To do which observe,

Rule. Multiply the Product of the Length and Height of the Wall by as many half Bricks as it contains in Thickness, which Product divided by 3, gives the Content of the Wall, 1 Brick and

and $\frac{1}{2}$ thick, which superficial Feet divide by 272, the Quote is Square Rods reduced to 1 Brick and $\frac{1}{2}$; or more short by the following Table.

Brick	
1 Subtract	$\frac{1}{3}$
2 Add	$\frac{1}{3}$
3 Multiply by 2	} Reduce the said Thicknes to 1 Brick and $\frac{1}{2}$.
4 $\frac{1}{2}$ Multiply by 3	
6 Multiply by 4	

Ex. 1. Suppose a Wall of a Garden 231 Feet about, and 13 Feet 4 Inches high, and 1 Brick $\frac{1}{2}$ thick, How many Rods?

$$\begin{array}{r}
 213 \\
 13 \cdot 4 \\
 \hline
 2769 \cdot 0 \\
 71 \cdot 0 \\
 \hline
 272) 2840 \cdot 0 \quad (10 \cdot 120 \\
 120
 \end{array}$$

Ex. 2. How many Rods of Brick-work in that Wall which is 40 Feet 7 Inches long, and 11 Feet high, being one Brick thick?

$$\begin{array}{r}
 40 \cdot 7 \\
 11 \cdot 0 \\
 \hline
 3) 446 \cdot 5 \\
 148 \cdot 9 \cdot 8
 \end{array}$$

$$\begin{array}{r}
 272) 297 \cdot 7 \cdot 4 \\
 1 \cdot 25 \cdot 7 \cdot 4 \quad \text{Rods Feet Inches Part.} \\
 \text{Answer, } 1 \cdot 25 : 7 \cdot 4
 \end{array}$$

Ex. 3. If a Wall be 254 Feet about, and 12 Feet 7 Inches high, and 3 Bricks thick, How many Rods?

$$\begin{array}{r}
 254 \\
 12 \cdot 7 \\
 \hline
 3048 \cdot 0 \\
 127 \cdot 0 \\
 21 \cdot 2
 \end{array}$$

$$\begin{array}{r}
 \text{Rods, Feet, Inch. } 3196 \cdot 2 \\
 \text{Ans. } 23 : 136 : 4 \quad 2 \quad \text{Rods, Feet, Inches.} \\
 272) 6392 : 4 \quad (23 : 136 : 4 \\
 952 \\
 136
 \end{array}$$

Of solid Measure.

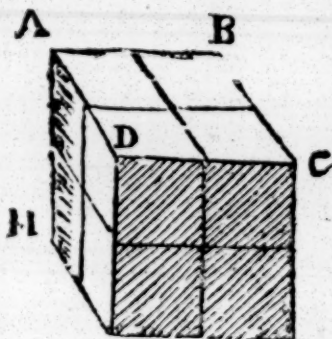
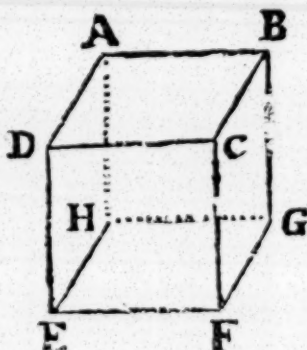
What respects Length, Breadth, and Thickness.

Rule. Multiply the Length by the Breadth, and that Product by the Depth.

Otherwise, Multiply the Area of the Base by the Height, the Product gives the solid Content, which if in Feet, divide by 27, the solid Feet in one Yard, which reduces them to solid Yards, if in Inches, by 1728, gives solid Feet, &c.

Demonstration.

For a Surface A B C D being imagined to move, suppose upwards or downwards, so that its Points A, B, C, D, describe the Lines A H, B G, C F, D E, and consequently, each Line therein describes other Surfaces, &c. there will be generated by this Motion, a Magnitude called a Solid, having three Dimensions, viz. Length, Breadth, and Thickness: Therefore that Surface A B C D, being divided into little square Area's, and moving the Height A H, divided by a like Measure for Length, may be conceived to produce as many small Cubes, as is the Number of the little Area's, in the Surface A B C D, multiplied by the Number of the Divisions in the Height A H.

*Timber and Stone.*

The Content given by the *Solid Foot*.

Example 1. How many solid Feet are contained in that Piece of Timber, whose Length is $17\frac{1}{2}$ Feet, Breadth 23 Inches, and Depth 2 Feet 7 Inches?

Ans. 86 Feet, 7 Inches, 9 Parts +

Ex. 2. A Stone $\left\{ \begin{array}{l} \text{Length} \quad 5 : 9 \\ \text{Breadth} \quad 3 : 11 \\ \text{Depth} \quad 2 : 8 \end{array} \right\}$ How many solid Feet?

$$\begin{array}{r}
 3 . 11 \\
 5 . 9 \\
 \hline
 19 . 07 \\
 1 . 11 . 6 \\
 0 . 11 . 9 \\
 \hline
 22 . 6 . 3 \\
 2 . 8 . 0 \\
 \hline
 45 . 0 . 6 \\
 11 . 3 . 1 . 6 \quad \text{Feet, Inches, Parts.} \\
 3 . 9 . 0 . 6 \quad \text{Ans. } 60 : 00 : 8 \\
 \hline
 60 . 0 . 8 . 0
 \end{array}$$

Digging.

The Content given by the Solid Yard.

Ex. 3. A Vault digged 9 Feet deep, $4\frac{1}{2}$ Feet long, and 9 Inches broad, How many solid Yards?

$$\begin{array}{r}
 4 . 6 \text{ Length} \\
 9 . 0 \text{ Depth} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 40 . 6 \\
 3 . 9 \text{ Breadth} \\
 121 . 6 \\
 20 . 3 \\
 12 . 1 . 6 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 27) 151 . 10 . 6 \text{ Yards, Feet, Inches, Parts.} \\
 \text{Answ. } 5 . 16 . 10 . 6
 \end{array}$$

Example 4. How many Yards of Digging will there be in a Cellar that is 25 Feet 4 Inches long, 15 . 8 broad, and $7\frac{1}{2}$ Feet deep?

Answ. 110 Yards, 6 Feet, 8 Inches.

For the Mensuration of round and square Pillars, Pyramids, Cones, Globes, Cylinders, &c. See Mr. Kersey's 10th Chapter of his Appendix.

Soli Deo Gloria.



9.

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